An Adaptive Control Scheme for Robot Manipulators with Flexible Joints

Q.-H. Max Meng
Department of Electrical Engineering
Lakehead University
Thunder Bay, Ontario
CANADA P7B 5E1

W.-S. Lu
Department of Electrical & Computer Engineering
University of Victoria
Victoria, B.C.
CANADA V8W 3P6

Abstract — In this paper, an adaptive control scheme for robot manipulators with flexible joints is proposed. It is based on a previous result on adaptive control of rigid robot manipulators using the regressor dynamics and combined with a linear damping term to handle the joint flexibility. The proposed control scheme has the advantages that it can be efficiently implemented using the recursive regressor dynamics; it does not need acceleration measurements of either the robot joint or the motor shaft; and it can handle the dynamic uncertainties of the robot system, such as the load and the last link dynamics parameters. A computer simulation is included to illustrate the proposed control scheme.

I. Introduction

A large volume of literature reporting work on the control of rigid robot manipulators has been available for the past two decades. In comparison, not too much work has been done on the control of robot manipulators with flexible components, especially flexible joints. Previous work has revealed that joint flexible modes on robots with transmission systems, such as gear-boxes using harmonic drives, typically exhibit resonance at frequencies lower than the lowest frequency of the flexible link resonance. Joint flexibility, therefore, exhibits a much important effect on the robot system performance. For a robot manipulator to carry out demanding tasks with high performance, such as the space robots performing services in space, joint flexibility should be taking into consideration in both modeling and control of robot manipulators.

Following the pioneer work reported in [1,2], which pointed out the importance of studying the effects of the flexible components in a robot manipulator with DC servo motors and gear-box drives on the dynamics modeling and control of robot manipulators, various approaches have been proposed to control robot manipulators with flexible joints. In previously reported work, different control strategies based on static and dynamic feedback linearization [3,4], on singular perturbation theory [5], on the concept of integral manifold [6,7], on a simple PD regulator [8], and on adaptive control techniques [9,10] have been proposed.

In this paper, an adaptive control scheme for robot manipulators with flexible joints is proposed. It is based on a previous result on adaptive control of rigid robot manipulators using the regressor dynamics [11] and combined with a linear damping term to handle the joint flexibility. The proposed control scheme has the advantages that it can be efficiently implemented using the recursive regressor dynamics; it does not need acceleration measurements of either the robot joint or the motor shaft; and it can handle the dynamic uncertainties of the robot system, such as the load and the last link dynamics parameters.

This paper is organized as follows. First the dynamic equations of a rigid robot with rigid as well as flexible joints are first derived in the next two sections. The flexibility, mainly introduced through the transmission system, is modeled as a linear torsional spring. The dynamic model is formed by two sets of equations, one of them describes the rigid robot dynamics and the other the motor drive dynamics. They are dynamically linked through the linear torsional spring model. In sections IV and V, the proposed control scheme is presented. For a typical robot system with flexible joints, the joint stiffness is relatively large and the joint damping is relatively small in comparison with other parameters in the system. This can lead to strong resonant behavior using rigid control schemes unless the control bandwidth is severely restricted. An intuitive control scheme, therefore, should control the rigid body dynamics of the robot while effectively damping out the flexible oscillations exhibited at the joints. Inspired by this observation, an adaptive control scheme for the control of robot manipulators with flexible joints is proposed, which is based on our previous work on adaptive control of rigid robot manipulators using the regressor dynamics and combined with a linear damping term to handle the joint oscillations. The proposed control scheme has the advantages that it can be efficiently implemented using the recursive regressor dynamics computationally; it does not need acceleration measurements of either the robot joint or the motor shaft; and it can handle the dynamic uncertainties of the robot system, such as the load and the last link dynamics. Finally in section VI, a simulation study is included to illustrate the proposed control scheme.

II. Dynamics of a Rigid Robot

When the Newton-Euler or Lagrangian dynamics equations are evaluated symbolically for any rigid robot manipulator, the dynamic equation of the manipulator can be written in the form

$$\boldsymbol{\tau} = \mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) \qquad (1)$$

where $\mathbf{H}(\mathbf{q})$ is the $n \times n$ mass matrix of the manipulator, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ is an $n \times 1$ vector of centrifugal and Coriolis terms, and $\mathbf{g}(\mathbf{q})$ is an $n \times 1$ vector of gravity terms.

In general, entries of $\mathbf{H}(\mathbf{q})$, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$, and $\mathbf{g}(\mathbf{q})$ are complicated functions of $\mathbf{q}$ and $\dot{\mathbf{q}}$. Although the dynamics equations of a robot manipulator (1) are coupled, nonlinear equations in general, they have several properties which are found useful in facilitating analysis and design of adaptive control system. In what follows these properties will be stated without proofs. Derivations and proofs of these properties can be found in, for example, [12,13].

Property 1 The mass matrix $\mathbf{H}(\mathbf{q})$ in equation (1) is symmetric, positive definite, and both $\mathbf{H}(\mathbf{q})$ and $\mathbf{H}^{-1}(\mathbf{q})$ are uniformly bounded for $\mathbf{q} \in \mathbf{R}^n$.

Property 2 All robot dynamic parameters such as link masses, moments of inertia, etc., appear as coefficients of known functions of $\mathbf{q}$, $\dot{\mathbf{q}}$, and $\ddot{\mathbf{q}}$. By defining each coefficient as a separate parameter, we can write (1) as

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\boldsymbol{\theta} = \boldsymbol{\tau} \quad (2)$$

where $\mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})$ is an $n \times r$ matrix of known functions which has been known as the *manipulator regressor*, and $\boldsymbol{\theta}$ is an r dimensional vector of known and unknown dynamic parameters.

Property 3 Matrix

$$\mathbf{S}(\mathbf{q}, \dot{\mathbf{q}}) = \dot{\mathbf{H}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$$

is skew symmetric, i.e., it satisfies

$$\mathbf{S}(\mathbf{q}, \dot{\mathbf{q}}) = -\mathbf{S}(\mathbf{q}, \dot{\mathbf{q}})^{\mathbf{T}}$$

or equivalently,

$$\mathbf{x}^{\mathbf{T}}\mathbf{S}\mathbf{x} = \mathbf{x}^{\mathbf{T}}(\dot{\mathbf{H}} - 2\mathbf{C})\mathbf{x} = \mathbf{0}.$$

III. Dynamics of a Flexible-Joint Robot

The dynamics of an n degree-of-freedom flexible joint robot can be described by a pair of second-order nonlinear differential equations [10]

$$\begin{aligned} \mathbf{H}(\mathbf{q}_1)\ddot{\mathbf{q}}_1 + \mathbf{C}(\mathbf{q}_1, \dot{\mathbf{q}}_1)\dot{\mathbf{q}}_1 + \mathbf{g}(\mathbf{q}_1) + \mathbf{K}(\mathbf{q}_1 - \mathbf{q}_2) &= \mathbf{0} \\ \mathbf{M}\ddot{\mathbf{q}}_2 - \mathbf{K}(\mathbf{q}_1 - \mathbf{q}_2) &= \mathbf{u} \end{aligned} \qquad (3)$$

where $\mathbf{q}_1$ is the joint position vector, $\mathbf{q}_2$ is the actuator position vector and $\mathbf{u}$ is the control vector; $\mathbf{M}$ is the actuator inertia matrix; and $\mathbf{K}$ is the motor shaft stiffness matrix. While $\mathbf{H}$, $\mathbf{C}$, and $\mathbf{g}$ are nonlinear functions, $\mathbf{M}$ and $\mathbf{K}$ are constant positive definite diagonal matrices.

This model of a flexible joint robot reduces to Eqn. (1) as the joint stiffness $\mathbf{K}$ tends to infinity and shares many of the properties of Eqn. (1). However, there is no longer an independent control input for each degree-of-freedom. As a result, many adaptive control algorithms for rigid robots do not extend immediately to Eqns. (3).

IV. Adaptive Control for Rigid Case

Consider the equation of motion (1). Define

$$\dot{\mathbf{q}}_r = \dot{\mathbf{q}}_d - \boldsymbol{\Lambda}\tilde{\mathbf{q}} \qquad (4)$$

where $\dot{\mathbf{q}}_d$ is the desired velocity, $\tilde{\mathbf{q}} = \mathbf{q} - \mathbf{q}_\mathbf{d}$, and $\boldsymbol{\Lambda} > 0$, and the control torque $\boldsymbol{\tau}$ is assigned as

$$\boldsymbol{\tau} = \mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}_\mathbf{r})\hat{\boldsymbol{\theta}} - \boldsymbol{\Gamma}\mathbf{s} \qquad (5)$$

with $\mathbf{s} = \dot{\mathbf{q}} - \mathbf{q}_\mathbf{r}$ and $\hat{\boldsymbol{\theta}}$ is the estimate of $\boldsymbol{\theta}$. If we choose $\boldsymbol{\Lambda} = \lambda\mathbf{I}_\mathbf{n}$, and $\boldsymbol{\Gamma} = \lambda\hat{\mathbf{H}}(\mathbf{q})$, then the error dynamics of the system becomes

$$\ddot{\tilde{\mathbf{q}}} + 2\lambda\dot{\tilde{\mathbf{q}}} + \lambda^2\tilde{\mathbf{q}} = \hat{\mathbf{H}}^{-1}\mathbf{Y}\tilde{\boldsymbol{\theta}} \qquad (6)$$

where $\tilde{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}} - \boldsymbol{\theta}$. If the estimate is updated by

$$\dot{\tilde{\boldsymbol{\theta}}} = \dot{\hat{\boldsymbol{\theta}}} = -\mathbf{W}\mathbf{Y}^{\mathbf{T}}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}_\mathbf{r})\mathbf{s}, \qquad \mathbf{W} > \mathbf{0} \qquad (7)$$

the Lyapunov function candidate

$$\mathcal{V} = \frac{1}{2}(\mathbf{s}^{\mathbf{T}}\mathbf{H}\mathbf{s} + \tilde{\boldsymbol{\theta}}^{\mathbf{T}}\mathbf{W}^{-1}\tilde{\boldsymbol{\theta}})$$

and its time derivative along trajectories of (1) as

$$\dot{\mathcal{V}} = -\mathbf{s}^{\mathbf{T}}(\boldsymbol{\Gamma} - \bar{\mathbf{C}}(\mathbf{q}, \dot{\mathbf{q}}))\mathbf{s}$$

with symmetric matrix $\bar{\mathbf{C}}$ defined as

$$\bar{\mathbf{C}}(\mathbf{q}, \dot{\mathbf{q}}) = \frac{1}{2}(\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{C}^{\mathbf{T}}(\mathbf{q}, \dot{\mathbf{q}}))$$

guarantees the position and velocity tracking errors converge to zero if we choose $\boldsymbol{\Gamma} > \bar{\mathbf{C}}$, which is also clear from Eqn. (6) since the estimate update law (7) will drive $\tilde{\boldsymbol{\theta}} = \mathbf{0}$ and therefore $\tilde{\mathbf{q}}$ and $\dot{\tilde{\mathbf{q}}}$ approach to zero.

V. Adaptive Control for Flexible Case

Inspired by the control scheme for flexible joint robot manipulators presented in [10], the control law (5) and (7) that achieves robust tracking of the reference trajectory for rigid robot manipulators can be modified easily to control the flexible joint system of Eqns. (3). Specifically, let the control law be defined as

$$\mathbf{u} = \mathbf{u}_{\mathbf{s}} + \mathbf{u}_{\mathbf{f}} \tag{8}$$

where $\mathbf{u}_{\mathbf{s}}$ is chosen as the control law (5) and $\mathbf{u}_{\mathbf{f}}$, which handles the flexible joint resonance, is chosen as

$$\mathbf{u}_{\mathbf{f}} = \mathbf{K}_{\mathbf{v}}(\dot{\mathbf{q}}_1 - \dot{\mathbf{q}}_2) \tag{9}$$

where $\mathbf{K}_{\mathbf{v}}$ is a constant diagonal matrix. Control law (8) provides a stable adaptive control scheme for flexible joint robots. The parameter updating is done according to (7), which means that only the parameter of the rigid model are updated in this scheme. The joint stiffness and motor inertia need to be known. Normally these parameters can be identified with sufficient accuracy off-line and will not change with varying payloads.

VI. Simulation Results

Simulations are carried out on a single link flexible joint robot performing trajectory tracking using the proposed adaptive control algorithms. The trajectory is shown in Fig. 1(a) and Fig. 2(a). First the adaptive control scheme for rigid robot is applied to the flexible joint robot. The tracking performance is shown in Fig. 1(a) while the tracking error is shown in Fig. 1(b). Next the adaptive control scheme for flexible joint robot is then applied to the flexible joint robot. The tracking performance is depicted in Fig. 2(a) while the tracking error is in Fig. 2(b). Apparently the simple control law in Eqn. (8), which combines an adaptive control algorithm for rigid robots with a linear damping term works fine to control a flexible joint robot.

VII. References

[1] L. M. Sweet and M. C. Good, "Redefinition of the robot motion control problem: Effects of plant dynamics, drive system constraints, and user requirements," in *Proc. of 23rd IEEE CDC*, Las Vegas, NV, pp. 724-730, 1984.

[2] M. C. Good, L. M. Sweet, and K. L Strobed, "Dynamic models for control system design of integrated robot and drive systems," *ASME J. Dyna. Sys. Meas. and Contr.*, Vol. 107, pp. 53-59, 1985.

[3] M. W. Spong, " Modeling and control of elastic joint robot," *ASME J. Dyna. Sys. Meas. and Contr.*, Vol. 109, pp. 310-319, 1987.

[4] A. DeLuca, A. Isidori, and F. Nicolo, "Control of robot arm with elastic joints via nonlinear dynamic feedback," in *Proc. 24th IEEE CDC*, pp. 1671-1679, Ft. Lauderdale, 1985.

[5] R. Marino and S. Nicosia, "Singular perturbation techniques in the adaptive control of elastic robots," in *Proc. IFAC Symp. Robot Control*, pp. 11-16, Barcelona, 1988.

[6] K. Khorasani and P. V. Kokotovic, " Feed-back linearization of a flexible manipulator near its rigid body manifold," *Systems and control letters*, Vol. 6, pp. 187-192, 1985.

[7] M. W. Spong, K. Khorasani, and P. V. Kokotovic, "An integral manifold approach to the feedback control of flexible joint robots," *IEEE J. Robotics and Automation*, vol. 3, pp. 291-300, 1987.

[8] P. Tomei, "A simple PD controller for robot manipulators," *IEEE Trans. Robotics and Automation*, Vol. 7, pp. 565-570, 1991.

[9] M. W. Spong, "Adaptive control of flexible joint manipulators," *Systems and Control Letters*, Vol. 13, pp. 15-21, 1987.

[10] F. Ghorbel, J. Y. Hung, and M. W. Spong, "Adaptive control of flexible-joint manipulators," *IEEE Control Systems Magazine*, vol. 9, no. 7, pp. 9-13, 1989.

[11] W.-S. Lu and Q.-H. Meng, "Regressor formulation of robot dynamics: computation and applications," to appear in *IEEE Trans. Robotics and Automation*, 1993.

[12] P. K. Khosla and T. Kanade, "Parameter identification of robot dynamics," in *Proc. 24th IEEE CDC*, Ft. Lauderdale, pp. 1754-1760, 1985.

[13] C. H. An, C. G. Atkeson, and J. M. Hollerbach, "Estimation of inertial parameters of rigid body links of manipulators," in *Proc. IEEE 24th CDC*, Ft. Lauderdale, 1985.

Fig. 1(a) Trajectory tracking of flexible joint robot using adaptive control algorithm for rigid robot

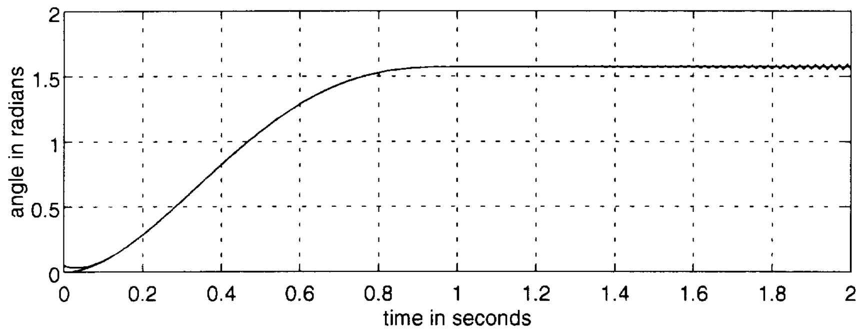

Fig. 1(b) Tracking error of flexible joint robot using adaptive control algorithm for rigid robot

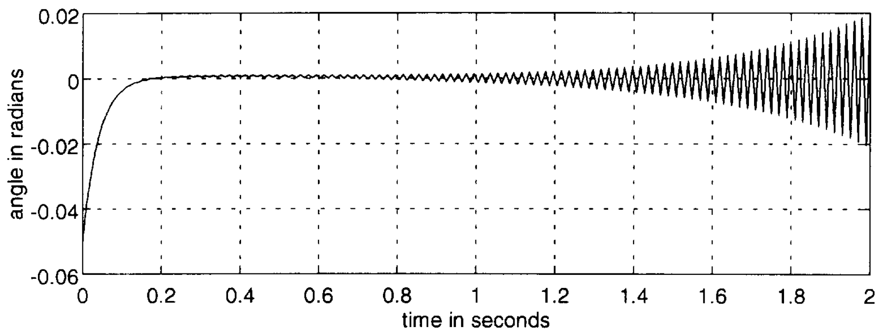

Fig. 2(a) Trajectory tracking of flexible joint robot using adaptive control algorithm for flexible joint robot

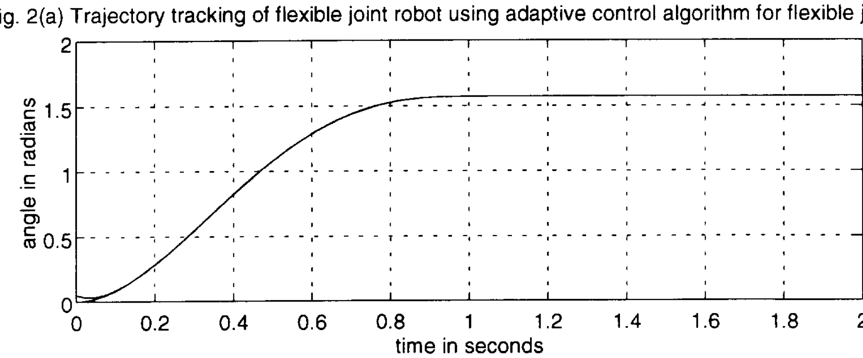

Fig. 2(b) Tracking error of flexible joint robot using adaptive control algorithm for flexible joint robot