DETECTION OF EDGES OF NOISY IMAGES BY 1-D AND 2-D LINEAR FIR DIGITAL FILTERS

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ABSTRACT

Image edge detection is the process of identifying sharp changes in light intensity of images. Differentiation is obviously an operation to display such changes. Unfortunately, differentiation is very sensitive to noise as can be seen from its magnitude response. Therefore, in the process of edge detection, differentiation should be regularized before it is applied to noisy images. As a result, edges in a noisy image are usually obtained by two steps — a pre-smoothing step and a differentiation step. Simultaneous consideration of these two steps is employed in designing edge detectors in this paper. Following this perspective, several image edge detection algorithms are developed by means of low-order one-dimensional (1-D) and two-dimensional (2-D) finite impulse response (FIR) filters. The filters obtained are linear and shift-invariant, and can be implemented efficiently by discrete convolution, or discrete Fourier transform (DFT).

1. INTRODUCTION

In this paper, we refer edges in an image to the discontinuities of the image in light intensity and we are interested primarily in detecting edges of noisy images. Edge detection is essential in image understanding as edges represents a large portion of characteristics contained in an image. Most of the existing analytic approaches to edge detection is based on the concept of differentiation [1]. The disadvantage of using a differential operator is its high sensitivity to noise. It is therefore a common practise [2] that before applying the differential operator, the image need to be smoothed with a lowpass filter to suppress noise. Edges are then obtained using a standard differential operator. In this paper, new edge detection algorithms are developed based on several FIR filter approaches. The FIR edge detectors are designed such that they have the ability to differentiate the edges and to smooth the noises in a noisy image simultaneously. Intuitively, for a given cutoff frequency ω_c, an FIR edge detector is designed such that its frequency (or magnitude) response is zero in the frequency range $[\omega_c, \pi]$ and is equal to the frequency (or magnitude) response of a differentiator in the frequency range $[0, \omega_c]$.

Three types of FIR edge detectors, namely 1-D first-order FIR edge detectors, 2-D second-order FIR edge detectors and 2-D first-order FIR edge detectors, are described in Sections 2, 3 and 4, respectively. Experimental results are also presented at the end of each section.

2. 1-D FIRST-ORDER FIR EDGE DETECTORS

A nonrecursive digital filter of length N can be represented [3] by the transfer function

$$H(z) = \sum_{k=0}^{N-1} h(k) z^{-k} \tag{1}$$

where $h(k)$ is the impulse response of the filter. In the design of digital differentiators the frequency response is required to be antisymmetric. As most conventional first-order digital differentiators in edge detection are of odd order, the length N is chosen to be odd in this paper for comparison purpose. Thus we have

$$h(k) = -h(N-1-k) \quad \text{for} \quad n = 0, 1, \cdots, (N-3)/2$$

and $h(\frac{N-1}{2}) = 0$. Consequently the frequency response of the filter is given by [4]

$$H(\omega) = M(\omega) e^{-j\theta(\omega)} \tag{2}$$

$$\begin{aligned} M(\omega) &= \sum_{k=1}^{\frac{N-1}{2}} c(k) sin(k\omega) \\ \theta(\omega) &= -\frac{(N-1)}{2}\omega \end{aligned} \tag{3}$$

$$c(k) = 2h[\frac{(N-1)}{2} - k], \quad k = 1, 2, \cdots, (N-1)/2 \tag{4}$$

and ω is the normalized frequency varying from $-\pi$ to π. For processing bandlimited signals, the ideal frequency response of a first order digital differentiator with linear phase response is given by

$$D(\omega) = j\omega e^{-j\omega\frac{(N-1)}{2}} \tag{5}$$

Our objective is to design a linear phase FIR filter that has both the ability to discriminate the light intensity changes in an image and to attenuate the high frequency components due to the presence of noise in the image. Hence the ideal frequency response of such differentiator is of the form

$$I(\omega) = \begin{cases} j\omega e^{j\omega\frac{(N-1)}{2}}, & 0 \le \omega \le \omega_c \\ 0 & \omega_c \le \omega \le \pi \end{cases} \tag{6}$$

where ω_c is the cutoff frequency. In a least pth design, a first-order digital differentiator is designed where coefficients $c(k)$ in (3) are chosen such that the error function

$$e(c) = \int_0^{\omega_c} [M(\omega) - \omega]^{2p} d\omega$$

is minimized.

In particular, with $p = 1$, it can be shown that the optimal $c = [c(1) \;\; c(2) \;\; \cdots \;\; c(\frac{N-1}{2})]^T$ is given by

$$c = Q^{-1}b \tag{7}$$

where

$$Q(k,l) = \begin{cases} \frac{sin[(k-l)\omega_c]}{k-l} - \frac{sin[(k+l)\omega_c]}{k+l} & k \neq l \\ \omega_c - \frac{sin(2k\omega_c)}{2k} & k = l \end{cases}$$

is a square matrix of dimension $(N-1)/2$, and b with

$$b(k) = \frac{1}{k^2} sin(k\omega_c) - \frac{\omega_c}{k} cos(k\omega_c), \quad 1 \le k \le (N-1)/2$$

is a column vector. The impulse response $h(k)$ of the filter designed can now be obtained using (4) and (7). It is interesting to note that in the limiting case $\omega_c \to \pi$, it can be shown that

$$c(k) = \frac{2}{k}(-1)^{k+1}, \quad k = 1, 2, \cdots, (N-1)/2$$

and, therefore, the impulse response of the "all-pass" differentiator is given by

$$h(k) = \frac{(-1)^{k+1}}{\frac{N-1}{2} - k}, \qquad k = 0, 1, \cdots, (N-3)/2$$

For $N = 3$, we have $h(0) = 1, h(1) = 0, h(2) = -1$ and the differentiator has transfer function $H(z) = 1 - z^{-2}$ which is the conventional gradient edge detector. This shows that the commonly used edge detector is indeed an optimal one but it has no ability to attenuate high-frequency noise. Comparison of the proposed algorithm with the Sobel operator is shown in Fig. 1. Fig. 1(a) is a noise contaminated image with SNR = 10 dB. The edge maps by Sobel operator and the proposed algorithm ($N = 5$, $\omega_c = 1.5$) are shown in Fig. 1(b) and (c), respectively.

3. 2-D SECOND-ORDER FIR EDGE DETECTORS

The conventional gradient edge detectors usually offer satisfactory results for detecting edges with abrupt change of light intensity such as step edges. For the regions where light intensity changes slowly, it is more appropriate to use the second-order edge detectors [1]. A commonly used second-order differential operator is the Laplacian operator $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ which is a generalization of the 1-D operator $\frac{d^2}{dx^2}$. As a second-order differential operator, the Laplacian operator ∇^2 is more sensitive to noise than the first-order differential operators. Moreover, the use of a threshold to transform the resulting image after the Laplacian to a binary image often leads to double edges. A better utilization of the Laplacian is therefore to obtain its zero-crossings to detect the edge locations. As ∇^2 is very sensitive to noise, it is a common practise to apply a lowpass filter to the noise contaminated image before the Laplacian is used. The Gaussian filters have been proposed as a noise removal mechanism [2], and the zero-crossing type edge detector thus formed is called a Laplacian of Gaussian.

By the continuous-time differentiation rule for the Fourier transform of ∇^2, the frequency response of ∇^2 is given by

$$\nabla^2(\omega_1, \omega_2) = \mathcal{F}[\nabla^2] = -(\omega_1^2 + \omega_2^2), \quad |\omega_1| \le \pi, |\omega_2| \le \pi$$

We propose to design a 2-D FIR filter that it preserves the edge detection power of ∇^2 and at the same time it lessens the noise sensitivity. An easy and natural way of doing this is to design a linear filter that approximates $\nabla^2(\omega_1, \omega_2)$ in the low-frequency region and has small gain in the high-frequency region. Hence the desired amplitude response of the filter is given by

$$I(\omega_1, \omega_2) = \begin{cases} \omega_1^2 + \omega_2^2, & |\omega_1| \le \omega_{c_1}, |\omega_2| \le \omega_{c_2} \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

where ω_{c_1} and ω_{c_2} are pre-given cutoff frequencies. For convenience, here we assume $\omega_{c_1} = \omega_{c_2} = \omega_c$. Since $I(\omega_1, \omega_2)$ is circularly symmetric, it follows that an FIR transfer function $H(z_1, z_2)$ can be designed efficiently using the combined singular-value decomposition (SVD) and balance approximation (BA) method such that the frequency response of $H(z_1,z_2)$ approximates $I(\omega_1,\omega_2)$ [5][6]. An experimental comparison of the proposed filter with the Laplacian of Gaussian is shown in Fig. 2. The image 'Text' in Fig. 2(a) is contaminated by pseudo white Gaussian noise with SNR = 15 dB. Fig. 2(b) is the zero-crossings found using the Laplacian of Gaussian; the support of the Laplacian is 5×5 with standard derivation $\sigma = 2$ pixels. The zero-crossings found using a 3×3 FIR filter is shown in Fig. 2(c). It is noted that the performance of the Gaussian filter depends significantly on the value of the standard derivation and the size of the region of the support chosen. In order to obtain accurate zero-crossings using the Laplacian of Gaussian method, a number of trials have to be processed experimentally. If an $N \times N$ Gaussian filter is resulted from K trials and if a 3×3 Laplacian is used, the number of multiplications need for the realization of the Laplacian of Gaussian is $K(N^2+9)$, whereas the number of multiplications need for the realization of an $M \times M$ FIR edge detector is M^2. Experimentally, we found that $M = 3$ or 5 is satisfactory. In the above example, the processing time for our proposed filter ($M = 3$) is only $9/(34K)$ of the processing time required by the Laplacian of Gaussian.

Alternatively, an FIR filter $H(z_1,z_2)$ can be designed by minimizing the objective function

$$\int_{-\omega_c}^{\omega_c}\int_{-\omega_c}^{\omega_c} [|H(\omega_1,\omega_2)| - I(\omega_1,\omega_2)]^{2p} d\omega_1 d\omega_2 \tag{9}$$

where the support of H is assumed to be a square matrix of odd dimension. Due to the symmetric property of $I(\omega_1,\omega_2)$, the number of design variables is significantly reduced by imposing that

$$\begin{aligned} h(k,l) &= h(k,N_2-l) = h(N_1-k,l) \\ &= h(N_1-k,N_2-l) = h(l,k) \end{aligned}$$

and the minimization can be carried out by standard unconstrained optimization algorithms such as the Broyden-Fletcher-Goldfarb-Shannon (BFGS) method. This quasi-Newton algorithm requires only the gradient of the objective function and converges to a local minimum fairly quickly. Result of this design is presented in Fig. 3. The original image and its noise degraded version with SNR = 12 dB are shown in Fig. 3(a) and (b) respectively. The edge map of Fig. 3(b) found using a 5×5 2-D FIR filter designed by BFGS method is shown in Fig. 3(c).

4. 2-D FIRST-ORDER FIR EDGE DETECTORS

Typically the 1-D first-order edge detectors are used twice for a given image — one vertically and one horizontally. The edge map is then formed by adding two intermediate images together. From a 2-D signal processing point of view, this corresponds to a 2-D FIR filter of the form

$$H(z_1,z_2) = \mathbf{z_1}^T \mathbf{C} \mathbf{z_2} \tag{10}$$

with $\mathbf{z_i} = [1 \; z_i^{-1} \; \cdots \; z_i^{-(N-1)}]^T$, $i = 1,2$, and

$$\mathbf{C} = \left[\begin{array}{ccccccc} & \mathbf{0} & & \begin{array}{c} h(0) \\ \vdots \\ h((N-3)/2) \end{array} & & \mathbf{0} & \\ h(0) & \cdots & h((N-3)/2) & 0 & -h((N-3/2)) & \cdots & -h(0) \\ & \mathbf{0} & & \begin{array}{c} -h((N-3)/2) \\ \vdots \\ -h(0) \end{array} & & \mathbf{0} & \end{array}\right] \tag{11}$$

Note that the frequency response of the edge detector described above is $j(\omega_1+\omega_2)$. Therefore we propose to design an FIR 2-D filter of the form (10) where coefficient matrix $\mathbf{C}$ is given by (11) with the zero blocks replaced by non-zero parameters to be determined in the design so that the amplitude response approximates

$$I(\omega_1,\omega_2) = \begin{cases} |\omega_1+\omega_2|, & |\omega_1| \le \omega_c, |\omega_2| \le \omega_c \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

Fig. 4 shows the edge map of a noise-contaminated image using a 5×5 FIR 2-D first-order edge detector designed by the above approach.

5. CONCLUSIONS

Image edge detection problems are studied by taking the issue of sensitivity-to-noise into account and efficient 1-D and 2-D linear filtering algorithms for detecting edges of noise image are presented. The proposed first-order FIR differentiators, as compared with conventional gradient operators proposed by Prewitt, Sobel, Isotropic, etc., are much less sensitive to the noise; and the proposed second-order FIR differentiators are computationally more efficient than the popular Laplacian of Gaussian method. The edge maps of noisy images obtained from the proposed first-order FIR edge detectors (1-D or 2-D) are compared with those obtained from the commonly used Sobel operator. The comparison shows that the former is much better than the later. The comparison between the proposed second-order FIR edge detector and the Laplacian of Gaussian leads to a similar conclusion. In addition, less computation time is necessary for the implementation of the proposed second-order FIR edge detector than that of Laplacian of Gaussian.

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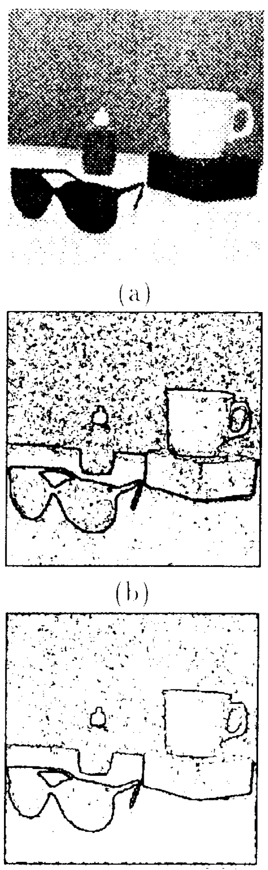
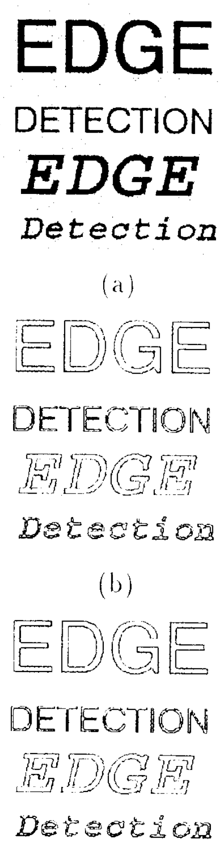


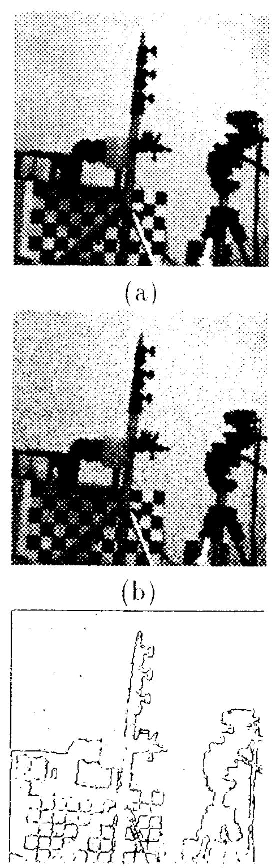
(a) (b) (c)

Figure 1



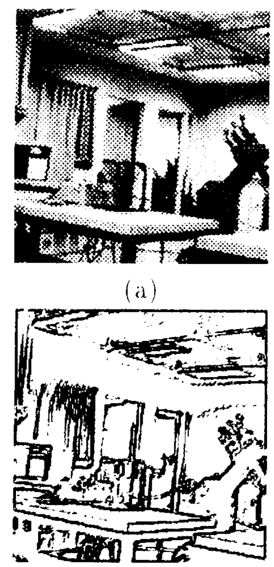
(a) (b) (c)

Figure 2



(a) (b) (c)

Figure 3



(a) (b)

Figure 4