An Advanced Software Tool for the Design of Two–Dimensional Digital Filters

H. Luo, W.-S. Lu and A. Antoniou

Department of Electrical and Computer Engineering,
University of Victoria,
P.O. Box 3035
Victoria, B.C., Canada V8W 3P6

Abstract

An advanced software tool that can be used in conjunction with MATLAB for the design of 2-D recursive and nonrecursive digital filters of many types is described. The basic theory that supports the proposed software tool is the singular-value decomposition (SVD) of nonnegative real matrices satisfying certain symmetry conditions, and the balance approximation. Prescribed specifications, such as the maximum passband and stopband ripples, the order of the 2-D digital filter, or linear phase response, can be easily obtained. The tool should be useful to engineers and researchers who are involved with the processing of 2-D data.

1 Introduction

Two-dimensional (2-D) digital filters are used extensively in many areas ranging from the digital processing of satellite photographs, radar maps, and medical X-ray images to the planning of power systems. Hence the analysis, design, and realization of these systems have received considerable attention during the last two decades. As a result, many algorithms for the design and realization of 2-D digital filters have been described in the literature.

The singular-value decomposition (SVD) is a numerically reliable matrix decomposition method [1]. It has found applications in the design of 2-D digital filters whose frequency response satisfy certain symmetry conditions [2]-[5]. The filter obtained is always stable and is in the form of a parallel structure which allows extensive pipelining [5]. This paper describes an advanced software tool that can be used in conjunction with MATLAB for the design of 2-D recursive and nonrecursive digital filters of various types.

2 Basic Design Functions

The basic design functions are listed in Table I. The command for designing a 2-D, highpass, circularly symmetric, recursive digital filter with specified maximum passband and stopband ripples is

$$[me_{pd}, me_{sd}, sn_1, sn_2, sn_n, d_1, d_2, N_l, N_u] \\ = lpiir2d(\omega_p, \omega_s, r_p, r_s, a, k_2, k_0)$$

where ω_p, ω_s are normalized passband and stopband frequency edges, respectively; $0 < \omega_s < \omega_p < 1.0$, with 1.0 corresponding to the normalized Nyquist frequency; r_p and r_s are the required maximum passband and stopband ripples; a is the number of amplitude response samples in the frequency range $0 \le \omega \le 1.0$. By setting $k_2 = 1$ or 2, the Kaiser or Hamming window is used for 1-D FIR filter design. By setting $k_0 = 1$, the amplitude response and the corresponding contour of the resulting filter can be plotted on the screen. The above design command is written as a MATLAB function that returns the maximum design errors over the passband and stopband, i.e. me_{pd} and me_{sd}, the order of the filter (sn_1, sn_2), the number of sections sn_n, the coefficient vectors of two separable denominator polynomials $D_1(z_1)$ and $D_2(z_2)$, i.e. d_1 and d_2; and the coefficient matrices N_l, N_u of the numerator polynomial, which form an LU decomposition of the coefficient matrix of the numerator polynomial. N_l and N_u, in conjunction with d_1 and d_2, lead to an economical realization of the filter designed [3].

The command for designing a 2-D, highpass, circularly symmetric, IIR filter with specified FIR order (n_1, n_2) is

$$[me_{pd}, me_{sd}, sn_1, sn_2, sn_n, d_1, d_2, N_l, N_u] \\ = hpiir2df(\omega_s, \omega_p, n_1, n_2, a, k_2, k_0)$$

3 Auxiliary Functions

Each design function listed in Table I involves a number of steps, each of which requires a large number of basic algebraic operations. Some of these operations are used many times in performing one design function. For this reason, the software package also provides auxiliary functions that can be used for the efficient implementation of the design functions. These functions are listed in Table II.

Table I: 2-D Digital-Filter Design Functions

Filter type	Function with specified ripples	Function with specified order
lowpass CS* FIR	lpfir2d	lpfir2df
highpass CS FIR	hpfir2d	hpfir2df
bandpass CS FIR	bpcsfir2d	bpcsfir2df
bandstop CS FIR	bscsfir2d	bscsfir2df
bandpass fan FIR	bpfanfir2d	bpfanfir2df
bandstop fan FIR	bsfanfir2d	bsfanfir2df
user-defined FIR	udfir2d	udfir2df
lowpass CS IIR	lpiir2d	lpiir2df
highpass CS IIR	hpiir2d	hpiir2df
bandpass CS IIR	bpcsiir2d	bpcsiir2df
bandstop CS IIR	bscsiir2d	bscsiir2df
bandpass fan IIR	bpfaniir2d	bpfaniir2df
bandstop fan IIR	bsfaniir2d	bsfaniir2df
user-defined IIR	udiir2d	udiir2df

CS*: Circularly Symmetric.

0-7803-0593-0/92 $3.00 1992 IEEE

Table II: 2-D Auxiliary Design Functions

Function	Design Purpose
fir2d	FIR filter with specified maximum ripples
fir2df	FIR filter with specified order
iir2d	IIR filter with specified maximum ripples
iir2df	IIR filter with specified FIR order

3.1 Design function *fir2d*

The steps involved in the implementation of the design function *fir2d* are as follows:

Step 1: Compute the SVD of the sampled amplitude response matrix of the desired filter, i.e.

$$A \;=\; USV^T = \sum_{i=1}^{r_a} s_i u_i v_i^T = \sum_{i=1}^{r_a} \tilde{u}_i \tilde{v}_i^T \tag{1}$$

where $A \in R^{a_1 \times a_2}, U \in R^{a_1 \times a_1}, S \in R^{a_1 \times a_2}$, and $V \in R^{a_2 \times a_2}$; r_a is the rank of A, $s_1 \geq s_2 \geq ... \geq s_{r_a} > 0$ are the singular vales of A, u_i, v_i are the singular vectors of A, and $\tilde{u}_i = s_i^{1/2} u_i, \tilde{v}_i = s_i^{1/2} v_i$ are the weighted singular vectors of A.

Step 2: Design two sets of 1-D FIR digital filters. As each singular vector, $(\tilde{u}_i, \tilde{v}_i)$ for $i = 1, 2, ..., r_a$, is either mirror-image symmetric or anti-symmetric with respect to its midpoint [4], only half of each singular vector, i.e. $(\tilde{u}_i \in R^{\frac{a_1+1}{2} \times 1}, \tilde{v}_i \in R^{\frac{a_2+1}{2} \times 1})$, is needed for the design. Here, the window method [6] is used with the Kaiser or Hamming window to generate transfer functions $F_i(z_1)$ and $G_i(z_2)$ that approximate the singular vectors $\tilde{u}_i$ and $\tilde{v}_i$, respectively.

Step 3: Form the coefficient matrix of a 2-D FIR filter. A 2-D FIR filter having an amplitude response satisfying

$$H(\omega_1, \omega_2) = H(-\omega_1, -\omega_2) \tag{2}$$

can readily be designed by using a parallel arrangement of r_a 2-D FIR sections each comprising two 1-D subfilters. The transfer function of the 2-D filter is given by

$$H(z_1, z_2) = \sum_{i=1}^{r_a} F_i(z_1) G_i(z_2) = Z_1 C Z_2^T \tag{3}$$

where

$$F_i(z_1) = \sum_{j_1=0}^{n_1} f_i(j_1)\, z_1^{-j_1}, \quad G_i(z_2) = \sum_{j_2=0}^{n_2} g_i(j_2)\, z_2^{-j_2} \tag{4}$$

with $Z_1 = [1, z_1^{-1}, \cdots, z_1^{-n_1}]$, $Z_2 = [1, z_2^{-1}, \cdots, z_2^{-n_2}]$; n_1, n_2 are the orders of 1-D filters F_i and G_i, respectively; f_i and g_i are the coefficients of the 1-D filters F_i and G_i, respectively; and $C(j_1, j_2) = \sum_{i=1}^{r_a} f_i(j_1) g_i(j_2)$ is the (j_1, j_2) element of the coefficient matrix C of the 2-D filter.

Step 4: Determine the order of the filter. The design error matrices over the passband and stopband are determined as

$$E_p = |A_p - |H(z_1, z_2)| \cdot * A_p|, \quad E_s = A_s \cdot * |H(z_1, z_2)| \tag{5}$$

where A_p, A_s are the amplitude response matrices over the passband and stopband, respectively; and $\cdot *$ denotes the entrywise product. The maximum design errors over the passband and stopband, i.e. me_p and me_s, are calculated as the maximum elements of the corresponding matrices. The order of the 2-D filter is increased, and steps 2, 3, and 4 are repeated until the inequalities $me_p < r_p$ and $me_s < r_s$ are satisfied, where r_p and r_s are the required maximum ripples over the passband and stopband. In this way the order of the 2-D FIR filter (n_1, n_2) is determined.

Step 5: Decompose the coefficient matrix C of the filter by using the SVD as

$$C = U_c S_c V_c^T = \sum_{i=1}^{n_c} s_{ci}\, u_{ci}\, v_{ci}^T = \sum_{i=1}^{n_c} \tilde{u}_{ci}\, \tilde{v}_{ci}^T \tag{6}$$

Step 6: Reduce the number of the sections of the 2-D FIR filter. The number of the sections sn_c, i.e. the number of nonzero singular values to be used, s_{ci} for $(1 \leq i \leq sn_c \leq n_c)$, is determined by employing a trial-and-error approach to assure that the maximum errors of the filter with reduced sections does not exceed the specified values.

Step 7: Perform the LUD realization of C. The LU decomposition of C is $C = N_l N_u$. Since both N_l and N_u contain a large number of zero entries [3], extra savings can be obtained by the LUD realization given by

$$H_c(z_1, z_2) = Z_1\, N_l\, N_u\, Z_2^T = \sum_{i=1}^{sn_c} N_{li}(z_1)\, N_{ui}(z_2) \tag{7}$$

3.2 Design function *fir2df*

The steps involved in the implementation of the design function *fir2df* are the same as those for *fir2d* except for steps 4 and 6. In this case, the order of the filter (n_1, n_2) is specified. The number of sections sn_c is determined by employing a trial-and-error approach to assure that the maximum errors of the filter with reduced sections does not exceed the maximum design errors me_p and me_s determined in step 4.

3.3 Design function *iir2d*

Steps 1-4: Steps 1 through 4 are the same as those for the design function *fir2d*.

Step 5: Represent the FIR filter obtained in step 4 by Roesser's local state-space model (A_s, b_s, c_s, d), i.e.

$$\begin{bmatrix} x^h(i+1, j) \\ x^v(i, j+1) \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} x^h(i, j) \\ x^v(i, j) \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u(i, j),$$
$$y(i, j) = [c_1 \;\; c_2] \begin{bmatrix} x^h(i, j) \\ x^v(i, j) \end{bmatrix} + du(i, j) \tag{8}$$

where

$$A_1 = \begin{bmatrix} 0 & I_{n_1-1} \\ 0 & 0 \end{bmatrix}, \quad A_4 = \begin{bmatrix} 0 & I_{n_2-1} \\ 0 & 0 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} C(2, n_2+1) & \dots & C(2, 2) \\ \dots & \dots & \dots \\ C(n_1+1, n_2+1) & \dots & C(n_1+1, 2) \end{bmatrix},$$
$$A_3 = 0, \quad b_1 = [\, C(2,1) \;\; \dots \;\; C(n_1+1, 1)\,]^T,$$
$$b_2 = [\,0 \;\; \dots \;\; 0 \;\; 1\,]^T, \quad c_1 = [\,1 \;\; 0 \;\; \dots \;\; 0\,],$$
$$c_2 = [\, C(1, n_2+1) \;\; \dots \;\; C(1, 2)\,], \quad d = C(1, 1)$$

Step 6: Obtain the balanced approximation of the filter. First, the upper left blocks (K_{11}, W_{11}) and the lower left blocks (K_{22}, W_{22}) of the generalized controllability gramian K and observability gramian W are computed [4] as

$$K_{11} = \sum_{i=0}^{n_1-1} A_1^i P_b P_b^T (A_1^T)^i, \quad K_{22} = I_{n_2},$$

$$W_{11} = I_{n_1}, \quad W_{22} = \sum_{i=0}^{n_2-1} (A_4^T)^i Q_c^T Q_c A_4^i \tag{9}$$

$$P_b = \begin{bmatrix} C(2,1) & \dots & C(2, n_2+1) \\ \dots & \dots & \dots \\ C(n_1+1, 1) & \dots & C(n_1+1, n_2+1) \end{bmatrix},$$

$$Q_c = \begin{bmatrix} C(1, n_2+1) & \dots & C(1,2) \\ \dots & \dots & \dots \\ C(n_1+1, n_2+1) & \dots & C(n_1+1, 2) \end{bmatrix}$$

Then, the Cholesky factors L_i of K_{ii}, and the SVD of the matrices $L_i W_{ii} L_i^T$ are obtained to form the transformation matrices T_i $(i = 1,2)$ and $T = T_1 \oplus T_2$ [7] as

$$K_{ii} = L_i L_i^T, \quad L_i W_{ii} L_i^T = U_i S_i V_i^T, \quad T_i = L_i U_i S_i^{-1/2} \tag{10}$$

Finally, the locally balanced realization and the corresponding controllability and observability gramians are obtained [8] as $(A_b, b_b, c_b, d) = (T^{-1}A_s T, T^{-1} b_s, c_s T, d)$ and

$$K_{bi} = T_i^{-1} K_{ii} T_i^{-T}, \quad W_{bi} = T_i^T W_{ii} T_i \tag{11}$$

$$K_{bi} = W_{bi} = diag(\sigma_{i1}, ..., \sigma_{in_i}), \ (i = 1,2) \tag{12}$$

Step 7: Reduce the order of the IIR filter. A reduced state-space model (A_r, b_r, c_r, d) can be obtained by partitioning A_b, b_b, and c_b as

$$A_b = \left[\begin{array}{c|c} A_{b1} & A_{b2} \\ \hline A_{b3} & A_{b4} \end{array}\right] = \left[\begin{array}{cc|cc} A_{1r} & A_{12} & A_{2r} & A_{22} \\ A_{13} & A_{14} & A_{23} & A_{24} \\ \hline A_{3r} & A_{32} & A_{4r} & A_{42} \\ A_{33} & A_{34} & A_{43} & A_{44} \end{array}\right],$$

$$b_b = [\, b_{b1}^T \mid b_{b2}^T \,]^T = [\, b_{1r}^T \ b_{12}^T \mid b_{2r}^T \ b_{22}^T \,]^T,$$

$$c_b = [c_{b1} \mid c_{b2}] = [c_{1r} \ c_{12} \mid c_{2r} \ c_{22}] \tag{13}$$

and letting

$$A_r = \begin{bmatrix} A_{1r} & A_{2r} \\ A_{3r} & A_{4r} \end{bmatrix}, \quad b_r = \begin{bmatrix} b_{1r} \\ b_{2r} \end{bmatrix}, \quad c_r = [\ c_{1r} \ \ c_{2r}\ ]$$

The order of the filter (sn_1, sn_2) is determined by employing a trial-and-error approach to assure that the maximum errors of the reduced state-space model do not exceed the required maximum ripples. The resulting filter is a stable, low-order, recursive 2-D filter with separable denominator.

As the approximate error in the phase angle is bounded by the sum of the neglected Hankel singular values $(\sigma_{1i_1}, \sigma_{2i_2})$ for $(sn_1 \le i_1 \le n_1, sn_2 \le i_2 \le n_2)$, the phase response of the resulting filter is approximately linear over passband [4].

Step 8: Form the coefficient matrix of the filter. The coefficient matrix of the numerator polynomial $N(z_1, z_2)$ and the coefficient vectors of the separable denominator polynomials $D_1(z_1)$ and $D_2(z_2)$ can be obtained by manipulating the amplitude response of the filter as

$$\begin{aligned} H_r(z_1,z_2) &= c_r[I_r(z_1,z_2) - A_r]^{-1} b_r + d \\ &= [c_{1r} \ \ c_{2r}] \begin{bmatrix} M_1^{-1} & M_1^{-1} A_{2r} M_2^{-1} \\ 0 & M_2^{-1} \end{bmatrix} \begin{bmatrix} b_{1r} \\ b_{2r} \end{bmatrix} + d \\ &= c_{1r} M_1^{-1} b_{1r} + \sum_{l=1}^{r_k} [c_{1r} M_1^{-1} \tilde{u}_{2l}][\tilde{v}_{2l}^T M_2^{-1} b_{2r}] \\ &\quad + c_{2r} M_2^{-1} b_{2r} + d \\ &= \frac{1}{\det M_1 \det M_2} [(\det M_3 - \det M_1)\det M_2 \\ &\quad + \sum_{l=1}^{r_k} (\det M_{41} - \det M_1)(\det M_{42} - \det M_2) \\ &\quad + \det M_1 (\det M_5 - \det M_2)] + d \end{aligned} \tag{14}$$

where

$$M_1 = z_1 I - A_{1r}, \quad M_2 = z_2 I - A_{4r},$$

$$M_3 = z_1 I - A_{1r} + b_{1r} c_{1r}, \quad M_{41} = z_1 I - A_{1r} + \tilde{u}_{2l} c_{1r},$$

$$M_{42} = z_2 I - A_{4r} + b_{2r} \tilde{v}_{2l}, \quad M_5 = z_2 I - A_{4r} + b_{2r} c_{2r},$$

$$A_{2r} = \sum_{l=1}^{r_k} s_{2l} u_{2l} v_{2l}^T = \sum_{l=1}^{r_k} \tilde{u}_{2l} \tilde{v}_{2l}^T \tag{15}$$

Step 9: Decompose the coefficient matrix of the numerator polynomial by using the SVD as

$$N_{um} = U_{nu} S_{nu} V_{nu}^T = \sum_{i=1}^{n_{nu}} s_{nui}\, u_{nui}\, v_{nui}^T = \sum_{i=1}^{n_{nu}} \tilde{u}_{nui}\, \tilde{v}_{nui}^T \tag{16}$$

Steps 10-11: Steps 10 and 11 are the same as the last two steps for *fir2d*.

3.4 Design function *iir2df*

The steps involved in the implementation of design function *iir2df* are the same as those for *iir2d* except for steps 4, 7 and 10. The order of the 2-D FIR filter (n_1, n_2) is specified in step 4. The order of the 2-D IIR filter (sn_1, sn_2) in step 7 and the number of sections of the IIR filter sn_n in step 10 are determined by employing a trial-and-error approach to assure that the maximum errors of the IIR filter with the reduced number of the sections does not exceed the maximum design errors me_p and me_s determined in step 4.

4 Design Examples

The software tool described above was tested by designing a variety of 2-D FIR and IIR filters. Presented below are three design examples.

The first example is an IIR, circularly symmetric, highpass filter with normalized stopband edge $\omega_s = 0.4$ and passband edge $\omega_p = 0.6$. The maximum passband and stopband ripples are required to be less than 0.05. The command *hpiir2d* was used to carry out the design, yielding a stable IIR filter of order (25, 28) whose amplitude response is shown in Fig. 1.

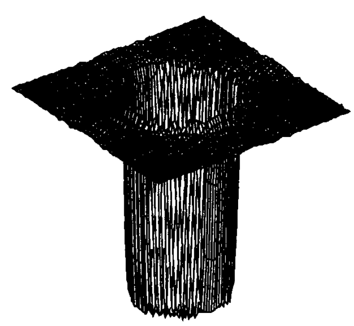

Fig.1 Amplitude response of the highpass filter.

The second example is an IIR, fan, bandstop filter of order (35, 35) that approximates the desired amplitude response

$$|H(\omega_1, \omega_2)| = \begin{cases} 0, & (\omega_2 \leq \omega_k\, \omega_1 + \omega_{ne}\pi) \\ 1, & (\omega_2 \geq \omega_k\, \omega_1 + \omega_{po}\pi) \end{cases}$$

where the normalized positive and negative transitions are $\omega_{po} = 0.11, \omega_{ne} = -0.02$, respectively; the slope of the passband transition is $\omega_k = 0.6$. The command *bsfaniir2df* was used to perform the design. The amplitude response of the filter designed is shown in Fig. 2.

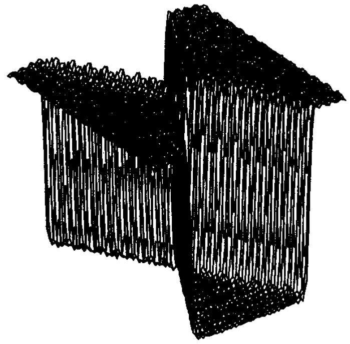

Fig.2 Amplitude response of the fan filter.

Finally, the command *udiir2df* was used to design an IIR filter of order (30, 30) that approximates the desired amplitude response shown in Fig. 3(a). This amplitude response is associated with a 2-D regularization filter which was found useful in restoration of degraded images [9]. The amplitude response of the IIR 2-D filter design is shown in Fig. 3(b).

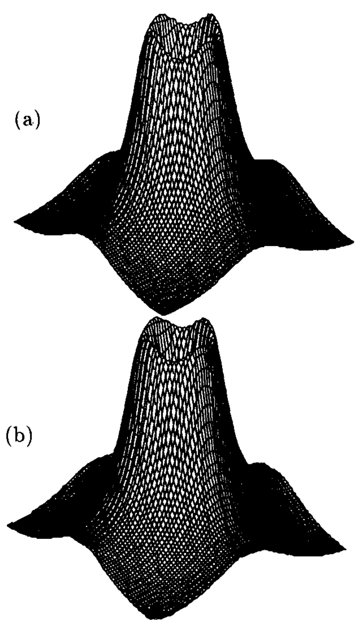

Fig.3 Amplitude response of (a) the desired filter (b) the designed filter.

5 Conclusions

A software tool that can be used to design filters for a large range of applications has been described. Prescribed specifications, such as the maximum passband and stopband ripples or the order of the 2-D digital filter, can easily be obtained. As the design approach starts with a linear-phase 2-D FIR filter, whose design is accomplished by designing two sets of 1-D subfilters, the phase response of the resulting IIR filter is approximately linear. The procedures used in the software tool are flexible, efficient, and robust, and can be easily adapted by the user. The tool should prove useful to engineers and researchers who are involved with the processing of 2-D data.

References

[1] G.-H. Golub and C.-F. Van Loan, *Matrix Computation*, 2nd ed., Baltimore, MD: The Johns Hopkins University Press, 1989.

[2] A. Antoniou and W.-S. Lu, "Design of two-dimensional digital filters by using the singular-value decomposition", *IEEE Trans. Circuits Syst.*, vol. CAS-34, pp. 1191-1198, Oct. 1987.

[3] W.-S. Lu, H.-P. Wang, and A. Antoniou, "Design of two- dimensional FIR digital filters by using the singular-value decomposition", *IEEE Trans. Circuits Syst.*, vol. CAS-37, pp. 35-46, Jan. 1990.

[4] W.-S. Lu, H.-P. Wang, and A. Antoniou, "Design of 2-D digital filters using the singular-value decomposition and balanced approximation method", *IEEE Trans. Signal Processing*, vol. 39, pp. 2253-2262, Oct. 1991.

[5] W.-S. Lu and A. Antoniou, "Design of 2-D digital filters with arbitrary amplitude and phase responses by using the singular value decomposition", *Proc. IEEE Int. Symp. on Circuits Syst.*, pp. 618-621, June 1991.

[6] H.-P. Wang, "Design of two-dimensional digital filters using singular-value decomposition", Ph.D. Thesis, Dept. of Electrical and Computer Engineering, University of Victoria, Nov. 1991.

[7] W.-S. Lu, E.-B. Lee, and Q.-T. Zhang, "Balanced approximation of two-dimensional and delay-differential systems", *Int. J. Control*, vol. 46, pp. 2199-2218, Dec. 1987.

[8] M. Kawamata and T. Higuchi, "Synthesis of 2-D separable denominator digital filters with minimum roundoff noise and no overflow oscillations", *IEEE Trans. Circuits and Systems*, vol. CAS-33, pp. 365-372, April 1986.

[9] M. Unser, A. Aldroubi, and M. Eden, "Recursive regularization filters: Design, properties, and applications", *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 13, pp. 272-277, March 1991.