# A Comparative Evaluation of Adaptive, Robust and Classical Feedback Controllers used in Unconstrained Trajectory Tracking for Robot Manipulators

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## Abstract

Recently, much research has taken place to advance the state of the art in unconstrained motion control of robot manipulators. However, the comparative evaluation of classical, robust and adaptive control schemes is not yet complete. Many computer simulation results are available; however, they are often misleading because they do not adequately reflect the effects of unmodeled dynamics. The unmodeled dynamics often total to more than half of the required control torque. In order to evaluate the efectiveness of the controllers properly, experimentation is required. Only then can conclusions be drawn on the relative merit of the controllers.

In this paper the results of an extensive comparative evaluation of recent and classical trajectory tracking controllers as applied to the RM-501 Mitsubishi robot will be given . The controllers evaluated are a **P**roportional-**D**erivative (**PD**) controller, a **M**odel **R**eference **A**daptive **C**ontroller (**MRAC**) which includes actuator and friction modelling, a robust **PD**-like **A**pproximate **D**ynamics (**AD**) controller, and a **C**omputed **T**orque (**CT**) controller.

## 1. Introduction

The dynamic equation of a "mechanically perfect manipulator" with instantaneous torque actuation is given by

$$M(q)\ddot{q} + C(q,\dot{q}) + g(q) = \tau \tag{1}$$

If all physical parameters were precisely known then the **CT** controller

$$\begin{aligned} \tau &:= M(q)v + C(q,\dot{q})\dot{q} + g(q) \\ v &= \ddot{q}_d - K_d(\dot{q}-\dot{q}_d) - K_p(q-q_d) \end{aligned} \tag{2}$$

would be ideal. Application of (2) to the manipulator's dynamic equation (1) yields the error dynamics

$$\ddot{\tilde{q}} + K_d\dot{\tilde{q}} + K_p\tilde{q} = 0$$

where $\tilde{q} = q - q_d$. Critically damped response is achieved with $K_d = 2K_p^{1/2}$. Unfortunately, the "mechanically perfect manipulator" does not exist. Indeed, imprecision in the estimation of the manipulator's mass and inertial properties, uncertainty in the physical parameters of a possibly large and geometrically complex load and lack of an adequate model which predicts the behavior of friction make the control problem difficult. Consequently, better controllers which are based on more complete manipulator models must be designed and tested. The controllers tested in this paper are the **PD**, **CT**, **AD** and **MRAC** manipulator trajectory tracking controllers.

## 2 The Control Algorithms

### 2.1 Proportional - Derivative Control

The control torque generated by the **PD** controller is given by the vector equation

$$\eta := -K_d\dot{\tilde{q}} - K_p\tilde{q} \tag{3}$$

where $\tilde{q} = q - q_d$ is the position tracking error and $K_p > 0$ and $K_d > 0$ are the position and velocity gain matrices respectively. The Lyapunov approach will be used for stability analysis of a manipulator with dynamic equation

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + K_\omega\dot{q} = \eta \tag{4}$$

where the gravitational and friction terms have been neglected for purpose of analysis [1]. In addition $q_d = constant$ must hold to guarantee Lyapunov stability. Consider the Lyapunov function candidate

$$V = \frac{1}{2}[\dot{q}^T H(q)\dot{q} + \tilde{q}^T K_p\tilde{q}] \tag{5}$$

which is a measure of the energy present in the closed loop system. The total energy of the system is always positive since $H(q)$ and $K_p$ are positive definite matrices. To prove stability, the energy function (5) should always be decreasing until the desired position is achieved (*ie.* $q_d = q$).

The first derivative of (5) with respect to time with control law (3) included is

$$\dot{V} = -\dot{q}^T(K_d + K_\omega)\dot{q}$$

which is always negative since $K_d + K_\omega > 0$. To prove the only possible zero energy configuration is the desired position $q_d = q$, we use Lasalle's theorem. Set $\dot{q} = 0$ in (4) and solve for joint acceleration

$$\ddot{q} = \begin{cases} H^{-1}(-C\dot{q} - K_\omega\dot{q} + \eta) & , \dot{q} \neq 0 \\ H^{-1}\eta = -H^{-1}K_p\tilde{q} & , \dot{q} = 0 \end{cases} \tag{6}$$

Equation (6) shows that the acceleration of the manipulator's joints is non-zero for all positions and velocities except the desired position $\tilde{q} = 0$. Thus Lyapunov stability has been proven for the PD controller as applied to a manipulator with a fixed goal position and no gravity term. Obviously, a better controller is required for trajectory tracking.

### 2.2 Approximate Dynamics Control

The **AD** controller uses a similar algebraic structure to the **CT** controller. It is given here.

$$\begin{aligned} \eta &:= H_c v + h_c \\ v &= \ddot{q}_d - K_d\dot{\tilde{q}} - K_p\tilde{q} \\ h_c &= K_\omega\dot{q} + B_c[\dot{q}\dot{q}] + D_c[\dot{q}^2] + g_c + f_c \end{aligned}$$

where $H_c$ and $D_c$ are *(nxn)* matrices with constant elements, $B_c$ is a constant matrix with dimension *nxn(n-1)/2*, $g_c$ and $f_c$ are vectors with constant elements of dimension *n*. The last term $K_\omega$ is an *(nxn)* positive definite diagonal matrix representing actuator dynamic parameters.

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By solving the optimization problem

$$J_1 = \min_{H_c} \max_{q\in\Omega_d} \left\| H^{-1}(q)H_c - I \right\|_2$$

a good constant matrix approximation to the inertial matrix can be found which guarantees the closed loop stability of this controller [2].

Let's consider one possible design of $H_c$. When the approximation to the inertia matrix is given by $H_c = \beta I$, $\beta > 0$. The optimization problem reduces to

$$J_2 = \min_{\beta} \max_{q\in\Omega_d} \left\| H^{-1}(q)\beta - I \right\|_2$$

Since the inertial matrix $H(q)$ is positive definite, a transformation $T$ can always be found such that

$$THT^{-1} = \begin{bmatrix} \lambda_1(q) & & 0 \\ & \ddots & \\ 0 & & \lambda_n(q) \end{bmatrix}$$

which implies

$$\left\| \beta H^{-1} - I \right\|_2 = \left\| \begin{bmatrix} \frac{\beta}{\lambda_1(q)} - 1 & & 0 \\ & \ddots & \\ 0 & & \frac{\beta}{\lambda_n(q)} - 1 \end{bmatrix} \right\|_2$$

consequently

$$\left\| \beta H^{-1} - I \right\|_2 = \max\left\{ \left| \frac{\beta}{\lambda_1} - 1 \right|, \ldots, \left| \frac{\beta}{\lambda_n} - 1 \right| \right\}$$

whose solution is given by

$$\beta = \frac{2\underline{\lambda}\overline{\lambda}}{\underline{\lambda} + \overline{\lambda}}$$

It has been proven [2] that selection of $\beta$ in this way and the assignment $h_c$ guarantees Lyapunov stability for trajectory tracking.

### 2.3 Model Reference Adaptive Control

A recent result in model reference adaptive control has been presented by Slotine and Li [4]. Their approach does not linearize and decouple the nonlinear dynamics of the manipulator as does Craig's method [5]. It is globally stable, however, and does not require joint acceleration information nor inversion of the inertial matrix.

Consider the robot manipulator with dynamic equation

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + \Omega\dot{q} + f(\dot{q}) + g(q) = \eta$$

where the term representing friction $f(\dot{q})$ is a smooth and differentiable function. Define the reference velocity by

$$\dot{q}_r = \dot{q}_d - \Lambda\tilde{q}$$

where $\tilde{q} = q - q_d$ is the position tracking error and $\Lambda > 0$ is a diagonal proportional gain matrix. Also, define the performance index as

$$s = \dot{q} - \dot{q}_r = \dot{\tilde{q}} + \Lambda\tilde{q}$$

Since the physical parameters of a manipulator are usually poorly known, a manipulator model with parameter estimates is required

$$\hat{H}(q)\ddot{q} + \hat{C}(q,\dot{q})\dot{q} + \hat{\Omega}\dot{q} + \hat{f}(\dot{q}) + \hat{g}(q) = \hat{\eta}$$

Subtracting the manipulator model from the manipulator model with estimated parameters yields

$$\hat{\eta} - \eta = \tilde{H}(q)\ddot{q}_r + \tilde{C}(q,\dot{q})\dot{q}_r + \tilde{\Omega}\dot{q}_r + \tilde{f}(\dot{q}) + \tilde{g}(q) \quad (7)$$

Due to linearity in the model's unknown dynamic parameters, equation (7) can be reorganized to give

$$\tilde{H}(q)\ddot{q}_r + \tilde{C}(q,\dot{q})\dot{q}_r + \tilde{\Omega}\dot{q}_r + \tilde{f}(\dot{q}) + \tilde{g}(q) = Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\tilde{a}$$

where $Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)$ is an $(nxr)$ matrix of known functions known as the regressor and $\tilde{a} = \hat{a} - a$ is the $(rx1)$ parameter estimation error vector.

Consider the Lyapunov Function candidate

$$V(t) = \frac{1}{2}[s^T H s + \tilde{a}^T \Gamma^{-1} \tilde{a}] \quad (8)$$

where $\Gamma > 0$ is a diagonal matrix. This function is always positive due to the positive definiteness of $H$ and $\Gamma$. To show that the second stability condition, the derivative of (8) must be negative.

$$\dot{V} = s^T\left(H\dot{s} + \tfrac{1}{2}\dot{H}s\right) + \tilde{a}^T\Gamma^{-1}\dot{\hat{a}}$$

Given the control structure

$$\eta := \hat{H}\ddot{q}_r + \hat{C}\dot{q}_r + \hat{\Omega}\dot{q}_r + \hat{K}_c \arctan(\chi) + \hat{g} - K_D s$$

it can be shown that

$$\dot{V} = -s^T(\Omega + K_D)s + \tilde{a}^T\left(Y^T s + \Gamma^{-1}\dot{\hat{a}}\right) \quad (9)$$

Since $\Omega + K_D$ is positive definite, the first term of (9) is always equal to or less than zero. Consequently, to insure Lyapunov stability, the second term can be set to zero. The solution for $\dot{\hat{a}}$ is then given by

$$\dot{\hat{a}} = \Gamma Y^T s$$

The model reference adaptive controller according to Slotine and Li is therefore given by

$$\eta := Y\hat{a} - K_D s$$
$$\hat{a}(t) = \hat{a}(t_0) - \int_{t_0}^{t} \Gamma Y^T s dt$$

This control structure is superior to the others for several reasons. First, it has been proven to be Lyapunov stable in trajectories. It is also capable of compensating for the effects of friction where friction is modelled as a smooth differentiable function. Furthermore, it has the capability to adapt to changing loads which may be of complex geometric shape.

## 3. Physical Manipulator Experiments

The dynamic equation of motion for the first two links of the RM-501 Mitsubishi robot is given by

$$\begin{bmatrix}\eta_1\\ \eta_2\end{bmatrix}=\begin{bmatrix}a_1+a_2c_2^2 & 0\\ 0 & a_3\end{bmatrix}\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix}+\begin{bmatrix}a_4s_2c_2\dot{\theta}_2 & a_5s_2c_2\dot{\theta}_1\\ a_6s_2c_2\dot{\theta}_1 & 0\end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}$$
$$+\begin{bmatrix}a_8 & 0\\ 0 & a_9\end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}+\begin{bmatrix}f(\dot{\theta}_1)\\ f(\dot{\theta}_2)\end{bmatrix}$$

where $\eta_1$ and $\eta_2$ are the applied joint torques.

### 3.1 PD Control

The control input generated by the PD controller is given by

$$\eta_1 := -k_{p1}(\theta_1-\theta_{1d})-k_{d1}\dot{\theta}_1$$
$$\eta_2 := -k_{p2}(\theta_2-\theta_{2d})-k_{d2}\dot{\theta}_2$$

where the velocity gain has been chosen as $k_{di}=\sqrt{k_{pi}}$. The performance of the **PD** controller is tabulated in Fig. 1 for the tracking of the fifth order polynomial trajectory

$$\theta_{1d}(t)=\begin{cases}-4.01+1.75t^2-1.01t^3+.202t^4-.0134t^5, & 0\le t<6\\ -4.01+1.75\tau^2-1.01\tau^3+.202\tau^4-.0134\tau^5, & 6\le t\le 12\end{cases}$$

$$\theta_{2d}(t)=\begin{cases}1.57+1.35t^2-.75t^3+.143t^4-.009t^5, & 0\le t<6\\ 1.57+1.35\tau^2-.75\tau^3+.143\tau^4-.009\tau^5, & 6\le t\le 12\end{cases}$$

where $\tau=12-t$. The performance indices are defined by

$$e_p=\sqrt{\frac{1}{nN}\sum_{i=1}^{n}\sum_{k=1}^{N}\left(\theta_{id}(k)-\theta_i(k)\right)^2}$$

$$e_v=\sqrt{\frac{1}{nN}\sum_{i=1}^{n}\sum_{k=1}^{N}\left(\dot{\theta}_{id}(k)-\dot{\theta}_i(k)\right)^2}$$

$$e_h=\sqrt{\frac{1}{nN}\sum_{i=1}^{n}\sum_{k=1}^{N}v_i^2}$$

where $e_p$ is the position tracking performance index, $e_v$ is the velocity tracking performance index $e_h$ is the energy performance index and $k$ is the number of sample points points.

| **Controller:** Proportional-Derivative<br>**Trajectory:** Fifth Order<br>**Proportional gain:** 100 | | | |
|---|---|---|---|
| Sampling Period (msec) | Pos. Error (deg) | Vel. Error (deg/sec) | Energy Perf. (Volts) |
| 6 | 3.143 | 26.48 | 14.85 |
| 8 | 3.529 | 28.69 | 15.25 |
| 12 | 3.910 | 29.94 | 15.63 |
| 24 | 4.378 | 30.39 | 15.65 |

**Fig. 1**

Furthermore, Fig. 6 shows the result of the PD controller in velocity tracking.

### 3.2 CT Control

The computed torque control equations as applied to the first two links of the RM-501 are given by

$$\eta_1 := (1.88-1.24c_2^2)v_1-11.4s_2c_2\dot{\theta}_1\dot{\theta}_2+18.7\dot{\theta}_1+5.79\arctan(3\dot{\theta}_1)$$
$$\eta_2 := .196v_2+.100s_2c_2\dot{\theta}_1^{\,2}+1.84c_2+8.65\dot{\theta}_2+5.91\arctan(3\dot{\theta}_2)$$
$$v_1 := \ddot{\theta}_{1d}-k_{p1}(\theta_1-\theta_{1d})-k_{d1}(\dot{\theta}_1-\dot{\theta}_{1d})$$
$$v_2 := \ddot{\theta}_{2d}-k_{p2}(\theta_2-\theta_{2d})-k_{d2}(\dot{\theta}_2-\dot{\theta}_{2d})$$

The dynamic parameters which are used in the **CT** control scheme were estimated by the adaptive component of the **MRAC** controller. The values of the estimated parameters are

| | |
|---|---|
| $\hat{a}_1=1.884$ | $\hat{a}_7=1.836$ |
| $\hat{a}_2=-1.236$ | $\hat{a}_8=18.66$ |
| $\hat{a}_3=0.1959$ | $\hat{a}_9=8.650$ |
| $\hat{a}_4=-5.983$ | $\hat{a}_{10}=5.792$ |
| $\hat{a}_5=-5.462$ | $\hat{a}_{11}=5.910$ |
| $\hat{a}_6=0.100$ | |

**Fig. 2**

The performance measure of the **CT** controller in following the specified fifth order trajectory is given in Fig. 3. Note that the **CT** controller improves with smaller sampling period.

| **Controller:** Computed Torque<br>**Trajectory:** Fifth Order<br>**Proportional gain:** 100 | | | |
|---|---|---|---|
| Sampling Period (msec) | Pos. Error (deg) | Vel. Error (deg/sec) | Energy Perf. (Volts) |
| 8 | 1.850 | 6.480 | 12.10 |
| 12 | 1.939 | 6.404 | 12.17 |
| 24 | 2.154 | 6.683 | 12.39 |

**Fig. 3**

The velocity tracking characteristics are shown graphically in Fig. 7.

### 3.3 AD Control

The **AD** controller implemented for our tests is the simplest form of the controller where the approximation to the inertia matrix is given by $H_c=\beta I$ and the other terms are reduced to $h_c=0$. The **AD** control equations are given by

$$\eta_1 := 2.82v_1+h_{c1}$$
$$\eta_2 := 2.82v_2+h_{c2}$$
$$v_1 := \ddot{\theta}_{1d}-k_{p1}(\theta_1-\theta_{1d})-k_{d1}(\dot{\theta}_1-\dot{\theta}_{1d})$$
$$v_2 := \ddot{\theta}_{2d}-k_{p2}(\theta_2-\theta_{2d})-k_{d2}(\dot{\theta}_2-\dot{\theta}_{2d})$$

where $h_{c1}=h_{c2}=0.$

The measured performance for the **AD** controller along the fifth order trajectory is given in Fig. 4 and the velocity tracking graph is given in Fig. 8.

| **Controller:** Approximate-Dynamics<br>**Trajectory:** Fifth Order<br>**Proportional gain:** 100 | | | |
|---|---|---|---|
| SamplingPeriod (msec) | Pos. Error (deg) | Vel.Error (deg/sec) | Energy Perf. (Volts) |
| 6 | 2.792 | 3.841 | 10.61 |
| 8 | 2.787 | 4.608 | 11.19 |
| 12 | 2.969 | 9.391 | 13.81 |
| 24 | 3.229 | 9.826 | 14.35 |

**Fig. 4**

### 3.4 MRAC Control

The control equations for the **MRAC** controller are

$$\eta_1 := (\hat{a}_1+\hat{a}_2c_2)\ddot{\theta}_{1r}+\hat{a}_4s_2c_2\dot{\theta}_2\dot{\theta}_{1r}+\hat{a}_5s_2c_2\dot{\theta}_1\dot{\theta}_{2r}+\hat{a}_8\dot{\theta}_1+\hat{f}_1(\dot{\theta}_1)-k_{d1}s_1$$

$$\eta_2 := \hat{a}_3\ddot{\theta}_{2r} + \hat{a}_6 s_2 c_2 \dot{\theta}_1 \dot{\theta}_{1r} + \hat{a}_7 c_2 + \hat{a}_9 \dot{\theta}_2 + \hat{f}_2(\dot{\theta}_2) - k_{d2} s_2$$

$$\dot{\theta}_{1r} = \dot{\theta}_{1d} - \lambda_1(\theta_1 - \theta_{1d})$$

$$\dot{\theta}_{2r} = \dot{\theta}_{2d} - \lambda_2(\theta_2 - \theta_{2d})$$

$$s_1 = \dot{\theta}_1 - \dot{\theta}_{1r}$$

$$s_2 = \dot{\theta}_2 - \dot{\theta}_{2r}$$

The adaptive part of this controller is given by

$$\hat{a}_i(t) = \hat{a}_i(0) - \gamma_i \int_0^t \left( \sum_{j=1}^{n} y_{ji} \right) (\dot{\theta}_i - \dot{\theta}_{ir}) dt, \qquad i = 1, \ldots, r$$

The performance of the method is given in Fig. 5 and its skill in velocity tracking of fifth order polynomial trajectories is portrayed in Fig. 9. Note that the trajectory tracking capability of this controller outperforms the capabilities of the other controllers easily. This is visually observed by comparing the wave forms in Figures 6, 7, 8 and 9. Also, note that position tracking using the **MRAC** control is twice better than any of the other controllers and that velocity tracking is three times better than any of the other controllers. This is evidenced in the tables of Figures 1,3,4 and 5.

**Controller:** MRAC with Arctan. Friction Model
**Trajectory:** Fifth Order
**Proportional gain:** 100

| Sampling Period (msec) | Pos. Error (deg) | Vel. Error (deg/sec) | Energy Perf. (Volts) |
|---|---|---|---|
| 12 | 0.876 | 2.317 | 11.69 |
| 24 | 0.904 | 2.266 | 12.04 |

**Fig. 5**

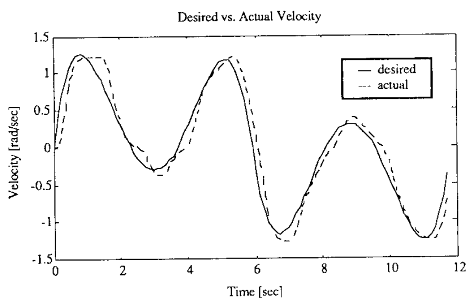


**Fig. 6:** PD Control; Desired vs. Actual Velocity

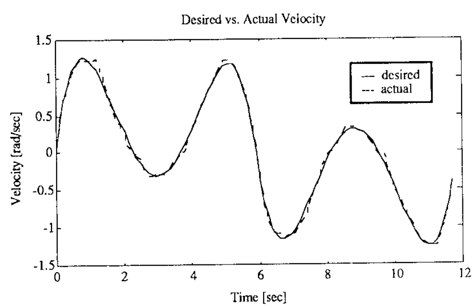


**Fig. 7:** CT Control; Desired vs. Actual Velocity

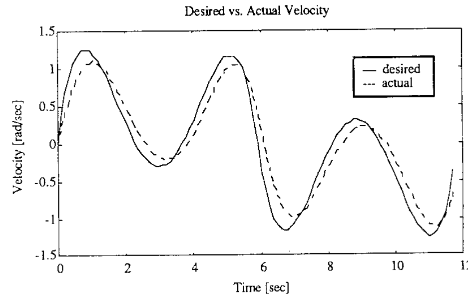


**Fig. 8:** AD Control; Desired vs. Actual Velocity

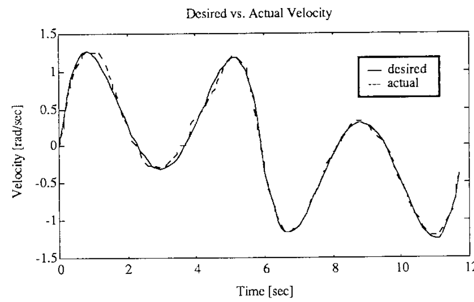


**Fig. 9:** MRAC ; Desired vs. Actual Velocity

## 4. Conclusions

In this paper we have shown a comparison of four different controllers used in the tracking a demanding fifth order polynomial trajectory. The results of the evaluation were very conclusive. It was found that the adaptive controller outperforms the other controllers hands down. Good performance was also achieved using the computed torque method. The proportional-derivative controller was found to perform poorly for velocity tracking. The AD controller gave fair overall performance. The adaptive algorithm is of course computationally demanding. However, with the new floating point digital signal processing products available today this issue is no longer a limiting factor. Indeed, we were able to implement a full blown model reference adaptive controller for the first two links of the RM-501 using an IBM/AT with floating point unit. Update rates faster than 12 msec were executed.

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