DESIGN OF TWO-DIMENSIONAL DIGITAL FILTERS USING SINGULAR-VALUE DECOMPOSITION AND BALANCED APPROXIMATION METHOD

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ABSTRACT

It is shown that the singular-value decomposition (SVD) can be applied along with 1-D finite-impulse response (FIR) techniques for the design of linear-phase 2-D filters with arbitrary prescribed amplitude responses which are symmetrical with respect to the origin of the $(\omega_1,\ \omega_2)$ plane. In the second half of the paper, the well known balanced approximation method is applied to linear-phase 2-D FIR filters of the type that may be obtained by using the SVD method. The method leads to economical and computationally efficient filters, usually IIR filters, which have prescribed amplitude responses and whose phase responses are approximately linear.

I. INTRODUCTION

Linear-phase two-dimensional (2-D) digital filters find many applications in image processing where phase distortion is particularly objectionable [1]. Filters of this type can be designed by using the window method [2] - [3], the McClellan transformation [4], and as was shown recently, by using the singular-value decomposition (SVD) [5] - [6]. The last method offers two important advantages. First, the design can be accomplished by designing a set of 1-D subfilters using well-known 1-D techniques and, second, the resulting 2-D filter can be fast since the 1-D subfilters form a parallel structure which allows extensive pipelining.

A limitation of the SVD method as reported in [6] is that the amplitude response of the filter is required to have quadrantal symmetry with respect to the $(\omega_1,\ \omega_2)$ plane. This limits the application of the SVD method.

In this paper, it is shown that the SVD method can be applied along with 1-D FIR filter techniques for the design of linear-phase 2-D filters with arbitrary amplitude responses which are symmetrical with respect to the origin of the $(\omega_1,\ \omega_2)$ plane.

In the second half of the paper, a model reduction technique known as balanced approximation (BA) method [7] - [9] is applied in the design of 2-D digital filters. The design starts with a linear-phase 2-D FIR filter of the type that may be obtained by using the proposed general SVD method and concludes with a lower-order 2-D digital filter, usually an infinite-impulse response (IIR) filter. The resulting 2-D filter has a specified amplitude response and its phase response is approximately linear. Furthermore, the 2-D filter obtained is more economical and computationally more efficient than the original 2-D FIR filter.

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Combining the general SVD and BA methods leads to excellent results as is demonstrated by designing a lowpass filter with a rotated elliptical passband.

II. GENERAL SVD METHOD

A. Theoretical Development

A 2-D FIR digital filter with support in the rectangle defined by $-N_i/2 \le n_i \le N_i/2$, $i = 1,\ 2$, can be characterized by the transfer function

$$H(z_1,\ z_2) = \sum_{n_1=-N_1/2}^{N_1/2} \sum_{n_2=-N_2/2}^{N_2/2} h(n_1,\ n_2) z_1^{-n_1} z_2^{-n_2} \quad (1)$$

If $h(n_1,\ n_2)$ is real and $h(n_1,\ n_2) = h(-n_1,\ -n_2)$, then the frequency response of the filter given by

$$H(e^{j\omega_1 T_1},\ e^{j\omega_2 T_2}) = X(\omega_1,\ \omega_2) \quad (2)$$

satisfies

$$X(\omega_1,\ \omega_2) = X(-\omega_1,\ -\omega_2) \quad -\pi \le \omega_1,\ \omega_2 \le \pi \quad (3)$$

A 2-D FIR filter having an arbitrary amplitude response satisfying (3) can readily be designed by using a parallel arrangement of K 2-D FIR sections each comprising two 1-D subfilters in cascade as will now be demonstrated. Such an arrangement can be represented by the transfer function

$$H(z_1,\ z_2) = \sum_{i=1}^{K} F_i(z_1) G_i(z_2) \quad (4)$$

where $F_i(z_1)$ and $G_i(z_2)$ are the transfer functions of two cascaded 1-D subfilters. If these subfilers are FIR filters with support in the rectangle defined by $-N_i/2 \le n_i \le N_i/2$, $i = 1,\ 2$, we have

$$F_i(z_1) = \sum_{n_1=-N_1/2}^{N_1/2} f_i(n_1) z_1^{-n_1},\ G_i(z_2) = \sum_{n_2=-N_2/2}^{N_2/2} g_i(n_2) z_2^{-n_2}$$

and if $F_i(z_1)$ and $G_i(z_2)$ are assumed to represent zero-phase or $\pi/2$-phase filters, then their frequency responses are $F_i(e^{j\omega_1 T_1}) = \Phi_i(\omega_1) e^{j\theta_i}$, $G_i(e^{j\omega_2 T_2}) = \Gamma_i(\omega_2) e^{j\theta_i}$. If $f_i(n_1)$ and $g_i(n_2)$ are mirror-image symmetric, then $\theta_i = 0$ and $\Phi_i(\omega_1)$ and $\Gamma_i(\omega_2)$ are real functions which are even with respect to ω_1 and ω_2, respectively; if $f_i(n_1)$ and $g_i(n_2)$ are mirror-image anti-symmetric, then $\theta_i = \pi/2$ and $\Phi_i(\omega_1)$ and $\Gamma_i(\omega_2)$ are real functions which are odd with respect to ω_1 and ω_2, respectively. Under these circumstances, a zero-phase 2-D filter is obtained as

$$H(e^{j\omega_1 T_1},\ e^{j\omega_2 T_2}) = \sum_{i=1}^{K} \pm \Phi_i(\omega_1)\ \Gamma_i(\omega_2) \quad (5)$$

where '+' corresponds to $\theta_i = 0$, and '−' corresponds to $\theta_i = \pi/2$.

Now assume that matrix $\mathbf{A} = \{a_{l,m}\}$ represents a desired arbitrary frequency response such that (3) is satisfied, i.e.

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$$X(\pi\mu_l,\ \pi\nu_m) = X(-\pi\mu_l,\ -\pi\nu_m) = a_{l,m} \tag{6}$$

where $1 \le l \le L$ and $1 \le m \le M$. The quantities μ_l, ν_m are normalized frequencies over the entire baseband. The SVD of **A** gives

$$\mathbf{A} = \sum_{i=1}^{r} \sigma_i \mathbf{u}_i \mathbf{v}_i^T = \sum_{i=1}^{r} \tilde{\mathbf{u}}_i \tilde{\mathbf{v}}_i^T \tag{7}$$

where σ_i are the singular values of **A**. A very important observation at this point is that $\tilde{\mathbf{u}}_i$, and $\tilde{\mathbf{v}}_i$ are either mirror-image symmetric or anti-symmetric simultaneously. The proof of this property is omitted here due to the space limitation.

On comparing (5) with (7), $\tilde{\mathbf{u}}_i$ and $\tilde{\mathbf{v}}_i$ may be taken to be sampled versions of the frequency responses $\Phi_i(\omega_1)$ and $\Gamma_i(\omega_2)$, respectively. By designing the 1-D FIR filters characterized by $F_i(z_1)$ and $G_i(z_2)$ $(1 \le i \le K,\ 1 \le K \le r)$ as zero-phase or $\pi/2$-phase filters and then interconnecting the filters obtained, a zero-phase 2-D filter is obtained whose amplitude response is a minimal mean-square-error approximation to the desired amplitude response [5].

The impulse response of the resulting filter is given by

$$h(n_1,\ n_2) = \sum_{i=1}^{K} f_i(n_1)\ g_i(n_2) \tag{8}$$

A 2-D causal linear-phase filter can readily be obtained by shifting the impulse response by $N_1/2$ and $N_2/2$ samples with respect to the n_1 axis and n_2 axis, respectively. This can be accomplished by multiplying $F_i(z_1)$ and $G_i(z_2)$ by $z_1^{-N_1/2}$ and $z_2^{-N_2/2}$, respectively.

As $K \to r$, the approximation error in the design of the 2-D filter is reduced to that introduced in the design of the 1-D FIR filter but the number of multiplications required in the realization becomes very large. Hence K is, in practice, chosen to be as low as possible without increasing the approximation error beyond a certain specified upper bound.

The design of the 2-D filter can be completed by using any one of the standard methods for the design of 1-D FIR filters [10].

B. Algorithm

An algorithm implementing the above design procedure is as follows:

1) Specify the desired amplitude response and thereby obtain the corresponding sampled amplitude response matrix **A**.

2) Decompose matrix **A** using (7) to get $\tilde{\mathbf{u}}_i$ and $\tilde{\mathbf{v}}_i$, where $1 \le i \le r$.

3) Obtain K $(1 \le K \le r)$ 2-D FIR filters by designing either two 1-D zero-phase or two $\pi/2$-phase FIR filters characterized by transfer functions $F_i(z_1)$ and $G_i(z_2)$ assuming sampled frequency responses $\tilde{\mathbf{u}}_i$ and $\tilde{\mathbf{v}}_i$, respectively.

4) Obtain the impulse response of the resulting zero-phase 2-D filter through (8).

5) Multiply the resulting 2-D zero-phase transfer function by $z_1^{-N_1/2}$ and $z_2^{-N_2/2}$ to obtain a causal linear-phase 2-D FIR filter.

C. Example 1

In this section, a 2-D lowpass FIR filter with a rotated elliptical passband is designed to illustrate the effectiveness of the proposed method.

The desired amplitude response of the 2-D lowpass FIR filter, shown in Fig. 1, is specified by

$$|H(e^{j\omega_1 T_1},\ e^{j\omega_2 T_2})| = \begin{cases} 1 & (\tilde{\omega}_1/\omega_{p_1})^2 + (\tilde{\omega}_2/\omega_{p_2})^2 \le 1 \\ 0 & (\tilde{\omega}_1/\omega_{a_1})^2 + (\tilde{\omega}_2/\omega_{a_2})^2 > 1 \end{cases}$$

where $\alpha = \pi/6$, $\omega_{p_1} = 0.32\pi$, $\omega_{p_2} = 0.52\pi$, $\omega_{a_1} = 0.48\pi$, $\omega_{a_2} = 0.68\pi$, and $\tilde{\omega}_1 = \omega_1 \cos\alpha + \omega_2 \sin\alpha$, $\tilde{\omega}_2 = -\omega_1 \sin\alpha + \omega_2 \cos\alpha$.

The 1-D FIR filters were designed by using the Fourier series method along with the Kaiser window function. By trial and error, it has been found that a value of 29 for both N_1 and N_2 gives satisfactory results.

There were 25 non-zero singular values resulting from the SVD of matrix **A**. With $K = 12$, the maximum passband and stopband errors are 0.0393 and 0.0255 respectively, and the amplitude response of the FIR filter designed is shown in Fig. 2.

III. APPLICATION OF BALANCED APPROXIMATION METHOD

A. Background Information

In this section, it is shown that the BA method can be put to good use in the design of 2-D digital filters as well.

As in the 1-D case, the BA method can be applied to a 2-D system by using a state-space characterization Consider Roesser's local state-space characterization given by

$$\begin{bmatrix} \mathbf{x}^h(i+1,\ j) \\ \mathbf{x}^v(i,\ j+1) \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 \\ \mathbf{A}_3 & \mathbf{A}_4 \end{bmatrix} \begin{bmatrix} \mathbf{x}^h(i,\ j) \\ \mathbf{x}^v(i,\ j) \end{bmatrix} + \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{bmatrix} u(i,j) \equiv \mathbf{A}\mathbf{x} + \mathbf{b}u$$

$$y(i,j) = \begin{bmatrix} \mathbf{c}_1 & \mathbf{c}_2 \end{bmatrix} \begin{bmatrix} \mathbf{x}^h(i,\ j) \\ \mathbf{x}^v(i,\ j) \end{bmatrix} + d\ u(i,j) \equiv \mathbf{c}\mathbf{x} + du \tag{9}$$

where $\mathbf{x}^h(i,\ j) \in R^{N_1}$ and $\mathbf{x}^v(i,\ j) \in R^{N_2}$ form the local state for the system at (i,j).

If $\tilde{\mathbf{f}}(z_1,\ z_2) = [\mathbf{I}(z_1,\ z_2) - \mathbf{A}]^{-1}\mathbf{b}$, $\tilde{\mathbf{g}}(z_1, z_2) = \mathbf{c}[\mathbf{I}(z_1,\ z_2) - \mathbf{A}]^{-1}$ where $\mathbf{I}(z_1,\ z_2) = z_1\mathbf{I}_{N_1} \oplus z_2\mathbf{I}_{N_2}$, then the generalized reachability and observability gramians of (9) [9] are defined as

$$\mathbf{K} = \frac{1}{(2\pi j)^2} \oint_{|z_1|=1} \oint_{|z_2|=1} \tilde{\mathbf{f}}(z_1,\ z_2)\tilde{\mathbf{f}}^*(z_1,\ z_2)\frac{dz_1}{z_1}\frac{dz_2}{z_2} \tag{10}$$

$$\mathbf{W} = \frac{1}{(2\pi j)^2} \oint_{|z_1|=1} \oint_{|z_2|=1} \tilde{\mathbf{g}}(z_1,\ z_2)\tilde{\mathbf{g}}^*(z_1,\ z_2)\frac{dz_1}{z_1}\frac{dz_2}{z_2} \tag{11}$$

respectively. Further denote the N_1-dimensional upper left blocks and the N_2-dimensional lower left blocks of **K** and **W** by $\mathbf{K}_{11}$, $\mathbf{K}_{22}$ and $\mathbf{W}_{11}$, $\mathbf{W}_{22}$, respectively. System (9) is said to be (locally) balanced if $\mathbf{K}_{ii} = \mathbf{W}_{ii}$, $(i = 1,\ 2)$ are diagonalized. If system (9) is not locally balanced, then a 2-D balancing transformation $\mathbf{T} = \mathbf{T}_1 \oplus \mathbf{T}_2$ can be found such that the system realization $(\mathbf{T}^{-1}\mathbf{A}\mathbf{T},\ \mathbf{T}^{-1}\mathbf{b},\ \mathbf{c}\mathbf{T},\ d)$ is locally balanced. A balancing transformation **T** can be computed by using $\mathbf{K}_{ii}$ and $\mathbf{W}_{ii}$ (i=1, 2) through any reliable algorithm for the 1-D balancing transformation [11]. Once a balanced realization $(\hat{\mathbf{A}},\ \hat{\mathbf{b}},\ \hat{\mathbf{c}})$ is found, a reduced state-space model $(\mathbf{A}_r,\ \mathbf{b}_r,\ \mathbf{c}_r)$ of order $(r_1,\ r_2)$ can be obtained by subpartitioning $\hat{\mathbf{A}}$, $\hat{\mathbf{b}}$, and $\hat{\mathbf{c}}$ as

$$\hat{\mathbf{A}} = \begin{bmatrix} \overbrace{\mathbf{A}_{1r}}^{r_1} & * & \overbrace{\mathbf{A}_{2r}}^{r_2} & * \\ * & * & * & * \\ \mathbf{A}_{3r} & * & \mathbf{A}_{4r} & * \\ * & * & * & * \end{bmatrix},\ \hat{\mathbf{b}} = \begin{bmatrix} \mathbf{b}_{1r} \\ * \\ \mathbf{b}_{2r} \\ * \end{bmatrix},\ \hat{\mathbf{c}} = \begin{bmatrix} \mathbf{c}_{1r} \\ * \\ \mathbf{c}_{2r} \\ * \end{bmatrix}^T$$

respectively, and then taking $(\mathbf{A}_r,\ \mathbf{b}_r,\ \mathbf{c}_r)$ where

$$\mathbf{A}_r = \begin{bmatrix} \mathbf{A}_{1r} & \mathbf{A}_{2r} \\ \mathbf{A}_{3r} & \mathbf{A}_{4r} \end{bmatrix},\ \mathbf{b}_r = \begin{bmatrix} \mathbf{b}_{1r} \\ \mathbf{b}_{2r} \end{bmatrix},\ \mathbf{c}_r = \begin{bmatrix} \mathbf{c}_{1r} & \mathbf{c}_{2r} \end{bmatrix} \tag{12}$$

In what follows, particular attention will be given to the application of this reduction approach to 2-D FIR filters.

B. Theoretical Development

$$\text{Let}\ \ H(z_1,\ z_2) = \sum_{n_1=0}^{N_1} \sum_{n_2=0}^{N_2} h(n_1\ n_2) z_1^{-n_1} z_2^{-n_2} \tag{13}$$

be the transfer function of a 2-D FIR filter of order $(N_1,\ N_2)$. In Roesser's local state-space characterization, (13) can be represented by (9) with

$$\mathbf{A} = \left[\begin{array}{cccccccccc} 0 & 1 & 0 & \dots & 0 & h_{1N_2} & \dots & & \dots & h_{11} \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \dots & & \dots & \vdots \\ 0 & 0 & 0 & \dots & 1 & \vdots & \dots & & \dots & \vdots \\ 0 & 0 & 0 & \dots & 0 & h_{N_1N_2} & \dots & & \dots & h_{N_11} \\ & & & & & 0 & 1 & 0 & \dots & 0 \\ & & & & & \vdots & \vdots & \vdots & \dots & \vdots \\ & & 0 & & & 0 & 0 & 0 & \dots & 1 \\ & & & & & 0 & 0 & 0 & \dots & 0 \end{array}\right],$$

$$\mathbf{b} = \left[\begin{array}{cccccc} h_{10} & \cdots & h_{N_10} & 0 & \cdots & 1 \end{array}\right]^T$$

$$\mathbf{c} = \left[\begin{array}{cccc|ccc} 1 & 0 & \dots & 0 & h_{0N_2} & \dots & h_{01} \end{array}\right] \text{ and } d = h_{00} \quad (14)$$

To compute $\mathbf{K}_{ii}$ and $\mathbf{W}_{ii}$ $(i = 1,\ 2)$, define

$$\tilde{\mathbf{A}}_1(z_2) = \mathbf{A}_1 + \mathbf{A}_2(z_2\mathbf{I} - \mathbf{A}_4)^{-1}\mathbf{A}_3$$

$$\tilde{\mathbf{b}}_1(z_2) = \mathbf{b}_1 + \mathbf{A}_2(z_2\mathbf{I} - \mathbf{A}_4)^{-1}\mathbf{b}_2$$

It is known that $\mathbf{K}_{11}$ can be found through the integral [9]

$$\mathbf{K}_{11} = \frac{1}{(2\pi j)} \oint_{|z_2|=1} \mathbf{K}_1(z_2)\frac{dz_2}{z_2} \quad (15)$$

where $\mathbf{K}_1(z_2)$ satisfies

$$\tilde{\mathbf{A}}_1(z_2)\mathbf{K}_1(z_2)\tilde{\mathbf{A}}_1^*(z_2) - \mathbf{K}_1(z_2) = -\tilde{\mathbf{b}}_1(z_2)\tilde{\mathbf{b}}_1^*(z_2)$$

For an FIR filter, $\tilde{\mathbf{A}}_1(z_2) = \mathbf{A}_1$, evaluating (15) shows that

$$\mathbf{K}_{11} = \sum_{n_1=0}^{N_1-1} \mathbf{A}_1^{n_1}\mathbf{P}(\mathbf{A}_1^T)^{n_1} \quad (16)$$

with

$$\mathbf{P} = \mathbf{H}_b\mathbf{H}_b^T \quad (17)$$

and

$$\mathbf{H}_b = \begin{bmatrix} h_{10} & h_{11} & \dots & h_{1N_2} \\ h_{20} & h_{21} & \dots & h_{2N_2} \\ \vdots & \vdots & \dots & \vdots \\ h_{N_10} & h_{N_11} & \dots & h_{N_1N_2} \end{bmatrix} \quad (18)$$

On the other hand, since both $\tilde{\mathbf{A}}_2(z_1) = \mathbf{A}_4$ and $\tilde{\mathbf{b}}_2(z_1) = \mathbf{b}_2$ are independent of z_1 and of the filter coefficients, $\mathbf{K}_{22}$ is the positive-definite solution of the Lyapunov equation

$$\mathbf{A}_4\mathbf{K}_{22}\mathbf{A}_4^T - \mathbf{K}_{22} = -\mathbf{b}_2\mathbf{b}_2^T$$

i.e.

$$\mathbf{K}_{22} = \sum_{n_2=0}^{N_2-1} \mathbf{A}_4^{n_2}\mathbf{b}_2\mathbf{b}_2^T(\mathbf{A}_4^T)^{n_2} = \mathbf{I}_{N_2} \quad (19)$$

Similarly, it can be shown that

$$\mathbf{W}_{11} = \mathbf{I}_{N_1} \quad (20)$$

$$\mathbf{W}_{22} = \sum_{n_2=0}^{N_2-1} (\mathbf{A}_4^T)^{n_2}\mathbf{Q}\mathbf{A}_4^{n_2} \quad (21)$$

with

$$\mathbf{Q} = \mathbf{H}_c^T\mathbf{H}_c \quad (22)$$

and

$$\mathbf{H}_c = \begin{bmatrix} h_{01} & h_{02} & \dots & h_{0N_2} \\ h_{11} & h_{12} & \dots & h_{1N_2} \\ \vdots & \vdots & \dots & \vdots \\ h_{N_11} & h_{N_12} & \dots & h_{N_1N_2} \end{bmatrix} \quad (23)$$

The above method usually results in an IIR design which has approximately the same frequency and space-domain responses as the original FIR filter. The reduction in the filter order achieved in (12) leads to a more economical and computationally efficient design. Furthermore, if the original FIR filter has a linear phase response, an approximately linear phase response is achieved irrespective of whether an FIR or IIR filter is obtained. Some additional attractive features of the method are as follows:

(a) If an IIR filter is obtained, the denominator of the transfer function is separable.

(b) The stability of an IIR design is guaranteed.

Property (a) follows from the fact that $\mathbf{A}_3 = 0$, which implies that $\mathbf{A}_{3r} = 0$.

C. Algorithm

An algorithm implementing the above design procedure is as follows:

1) Form $\mathbf{A}$, $\mathbf{b}$, $\mathbf{c}$ and calculate d according to (14) and obtain $\mathbf{A}_1$ - $\mathbf{A}_4$, $\mathbf{b}_1$, $\mathbf{b}_2$, $\mathbf{c}_1$ and $\mathbf{c}_2$ by subpartitioning $\mathbf{A}$, $\mathbf{b}$ and $\mathbf{c}$, respectively.

2) Obtain $\mathbf{H}_b$ and $\mathbf{H}_c$ using (18) and (23). Then compute matrices $\mathbf{P}$ and $\mathbf{Q}$ using (17) and (22), respectively.

3) Calculate $\mathbf{K}_{11}$ and $\mathbf{W}_{22}$ using equations (16) and (21), respectively. Then obtain $\mathbf{K}_{22}$ and $\mathbf{W}_{11}$ through (19) and (20).

4) From $\mathbf{K}_{ii}$ and $\mathbf{W}_{ii}$, $(i = 1,\ 2)$, compute the balancing transformations $\mathbf{T}_1$ and $\mathbf{T}_2$, respectively, by using Laub's algorithm and thereby obtain $\mathbf{T} = \mathbf{T}_1 \oplus \mathbf{T}_2$.

5) Form the balanced realization $(\hat{\mathbf{A}},\ \hat{\mathbf{b}},\ \hat{\mathbf{c}})$ with $\hat{\mathbf{A}} = \mathbf{T}^{-1}\mathbf{A}\mathbf{T}$, $\hat{\mathbf{b}} = \mathbf{T}^{-1}\mathbf{b}$ and $\hat{\mathbf{c}} = \mathbf{c}\mathbf{T}$.

6) Determine the order $(r_1,\ r_2)$ of the reduced filter by neglecting the insignificant singular values calculated using Laub's algorithm [11].

7) Obtain the reduced state-space realization $(\mathbf{A}_r,\ \mathbf{b}_r,\ \mathbf{c}_r)$ of order $(r_1,\ r_2)$ by subpartitioning $\hat{\mathbf{A}}$, $\hat{\mathbf{b}}$, and $\hat{\mathbf{c}}$ as shown in (12).

D. Example 2

In order to demonstrate the effectiveness of the BA method, it was applied to the linear-phase 2-D FIR digital filter designed for $K = 25$ in Section II - C. Following the steps of the algorithm described above, the balanced realization was first obtained. Then by examining the two sets of singular values and neglecting the insignificant ones, a reduced digital-filter realization of order $(r_1,\ r_2) = (13,\ 15)$ was obtained.

The amplitude response of the reduced filter, an IIR filter in this case, is shown in Fig. 3. The maximum passband and stopband errors are 0.0230 and 0.0202, respectively.

The group delays of the 2-D IIR filter, were calculated for $(r_1,\ r_2) = (13,\ 15)$, assuming fifty layers. Numerical results show that the linear-phase characteristic of the original FIR filter is well preserved over the entire passband of the 2-D filter. The maximum relative errors in the group delays are 1.48% in ω_1 and 1.49% in ω_2.

VI. CONCLUSIONS

In the first part of the paper, the SVD has been applied in conjunction with 1-D FIR filter techniques for the design of 2-D, causal, linear-phase FIR filters with arbitrary amplitude responses. The method is relatively simple to apply and leads to a parallel arrangement of pairs of cascade sections. This configuration is suitable for parallel processing and is amenable to VLSI implementation.

In the second part of the paper, a method for the design of 2-D digital filters based on the BA method has been developed. In this approach, the design starts with a 2-D causal, linear-phase, FIR filter of the type that can be obtained using the SVD and concludes with a design of reduced order, which is almost always an IIR design. The method leads to computationally efficient designs and preserves the linear phase of the original FIR filter irrespective of whether an FIR or an IIR design is obtained. Furthermore, the designs obtained are causal and in cases where IIR designs are obtained, stability is guaranteed. In other words, the method is highly suitable for the design of 2-D, causal, linear-phase IIR filters, which are very difficult to design by other methods.

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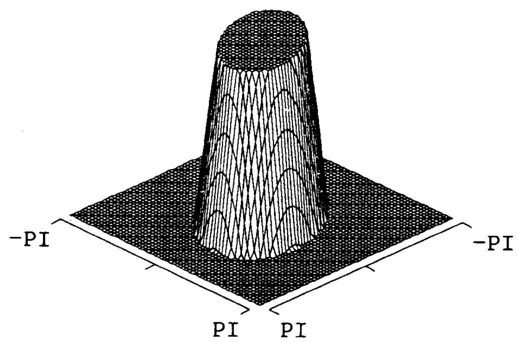

Fig. 1 Ideal amplitude response of 2-D filter with rotated elliptical passband.

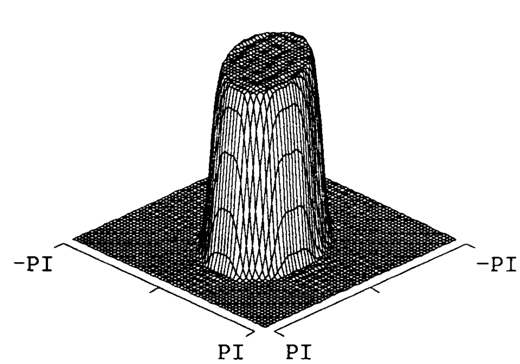

Fig. 2 Amplitude response of 2-D FIR filter obtained by using SVD method with $N_1 = N_2 = 29$ and $K = 12$.

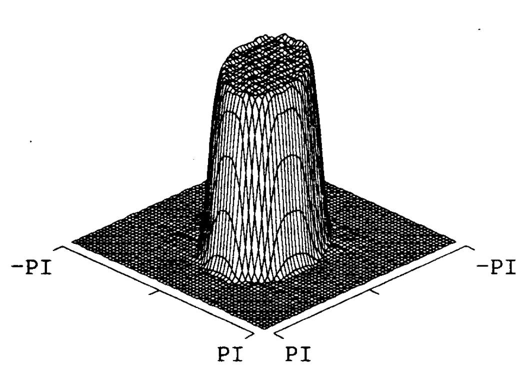

Fig. 3 Amplitude response of 2-D IIR filter obtained by using SVD and BA methods with $r_1 = 13$ and $r_2 = 15$.