LINEAR ROBUST CONTROL OF MECHANICAL MANIPULATORS*

W.-S. Lu

Department of Electrical and Computer Engineering
University of Victoria
P.O. Box 1700
Victoria, B.C., Canada V8W 2Y2

Abstract: Based on an eigen-analysis of the inertia matrix of the mechanical manipulator considered and a least-square type approximation of the Coriolis, centrifugal, and gravity terms in the manipulator's dynamics, this paper presents a linear feedback control scheme for trajectory tracking which is shown to be robust against model uncertainties. More precisely, the above mentioned eigen-analysis and least-square approximation lead to a simple yet reasonable model which in turn defines a structure of the constant feedback control. To determine the parameters of this controller, an analysis of the tracking-error dynamics is carried out using a Lyapunov approach. It turns out that by properly choosing a set of feedback gains, the tracking error can be kept to a prescribed range during the task in the presence of gravity effect and model uncertainties.

I. INTRODUCTION

Due to the complexity of the dynamics for most of industrial robots [1] [2], it is often desirable to introduce approximate models, based on which simple and possibly robust controllers may be designed. In this regard, various nonlinear and linear robust control schemes have recently been proposed by a number of researchers, see [3]-[8] among others.

Based on a simplified manipulator model, this paper presents a linear feedback control for trajectory tracking. The model is obtained using an eigen-analysis of the inertia matrix of the manipulator considered combining with a least-square type approximation of the Coriolis, centrifugal, and gravity terms, which in turn defines the structure of the control law to be developed. An analysis of the tracking error dynamics then follows using the Lyapunov approach [5], [9]-[12] in order to determine the parameters of the controller.

The proposed control scheme is shown to be robust against model uncertainties since its parameters depend only on some spectral bounds of the inertia matrix and the 2-norm bound of the lower-order terms in the "true" dynamic model of the manipulator.

II. APPROXIMATION OF THE MANIPULATOR DYNAMICS

1. The Manipulator Dynamics

The equation of motion for an n degree of freedom (d.o.f) mechanical manipulator is given by

$$\hat{H}(q)\ddot{q} + \hat{h}(q,\dot{q}) = \tau \tag{1}$$

where q is the joint displacement vector of dimension n, $\hat{H}(q)$ denotes the inertia matrix, $\hat{h}(q,\dot{q})$ includes the Coriolis, centrifugal and gravity terms and τ represents the joint forces/ torques.

When the joint actuators used are constant magnetic field DC motors and are operated in their linear ranges, by neglecting the inductance of the motor armature winding, each actuator can be modelled by the second-order equation

$$J_i\ddot{q}_i + c_i\dot{q}_i = d_i v_i - \tau_i \qquad 1 \le i \le n$$

where v_i is the input voltage of the ith actuator, J_i is the actuator inertia, and c_i, d_i are determined by the back EMF constant, the resistance of the armature winding, and J_i, etc. Thus the actuators' dynamics can be described as

$$J_a\ddot{q} + C\dot{q} = Dv - \tau \tag{2}$$

with $v = [v_1 \cdots v_n]^T$, $J_a =$ diag $\{J_1, \cdots, J_n\}$, $C =$ diag $\{c_1, \cdots, c_n\}$, and $D =$ diag $\{d_1, \cdots, d_n\}$.

By combining Eqn. (1) with Eqn. (2), the manipulator dynamics is described by

$$H(q)\ddot{q} + h(q,\dot{q}) = Dv \tag{3}$$

where

$$H(q) = \hat{H}(q) + J_a$$

and

$$h(q,\dot{q}) = \hat{h}(q) + C\dot{q}$$

2. Approximation of $H(q)$

In this subsection we present a method to determine a positive constant β such that

$$||\beta H^{-1}(q) - I|| \le \alpha < 1 \qquad \text{for all } q \epsilon R^n \tag{4}$$

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where $||\cdot||$ denotes the 2-norm and α is a constant. It is noted that the problem of approximating $H(q)$ by a constant βI in the sense of (4) has also been considered in [8] where only the upper and lower bounds of $||H^{-1}(q)||$ are used. The formula to compute such a constant approximation is, however, not valid in general as will be shown through the example given in Section II, 4. The method proposed here requires that the spectral behaviour of $H(q)$ is known in a certain sense as described below.

Let

$$\lambda_1(q) \geq \lambda_2(q) \geq \cdots \geq \lambda_n(q) > 0 \tag{5}$$

be the eigenvalues of $H(q)$, and let

$$\underline{\lambda} = \min_q \lambda_n(q) \qquad , \qquad \bar{\lambda} = \max_q \lambda_1(q) \tag{6}$$

As in practice the set of all possible configurations for a mechanical arm is always bounded, it is assumed throughout that $\underline{\lambda} > 0$ and $\bar{\lambda} < +\infty$.

Since $H(q)$ is positive definite, it follows from (5) and (6) that for any $\beta > 0$

$$\begin{aligned} ||\ \beta H^{-1}(q) - I\ || &= \max_i \left|\frac{\beta}{\lambda_i(q)} - 1\right| \\ &= \max\left\{\left|\frac{\beta}{\lambda_1(q)} - 1\right|, \left|\frac{\beta}{\lambda_n(q)} - 1\right|\right\} \\ &\leq \max\left\{\left|\frac{\beta}{\bar{\lambda}} - 1\right|, \left|\frac{\beta}{\underline{\lambda}} - 1\right|\right\} \end{aligned} \tag{7}$$

Clearly, the upper bound of $||\beta H^{-1}(q) - I||$ given by (7) is independent of joint displacement vector q and achieves its minimum if β is taken to be

$$\beta = \frac{2\bar{\lambda}\underline{\lambda}}{\bar{\lambda} + \underline{\lambda}} \tag{8}$$

which leads (7) to (4) with

$$\alpha = \frac{\bar{\lambda} - \underline{\lambda}}{\bar{\lambda} + \underline{\lambda}} < 1 \tag{9}$$

By (8), only the "largest" and the "smallest" eigenvalues of $H(q)$ are required to find such a β. In case $\bar{\lambda}$ and $\underline{\lambda}$ defined by (6) are not precisely known, a suitable β leading to inequality (4) may still be possible if a lower bound of $\lambda_n(q)$ and a upper bound of $\lambda_1(q)$ are available. Indeed, let $\underline{\lambda}_e$ and $\bar{\lambda}_e$ be the lower and upper bounds of λ_n and λ_1 respectively, i.e.

$$0 < \underline{\lambda}_e \leq \lambda_n(q), \qquad \lambda_1(q) \leq \bar{\lambda}_e < +\infty \quad \text{for all } q \epsilon R^n \tag{6'}$$

then it follows from (7) that

$$||\beta H^{-1}(q) - I|| \leq \max\left\{\left|\frac{\beta}{\bar{\lambda}_e} - 1\right|, \left|\frac{\beta}{\underline{\lambda}_e} - 1\right|\right\}$$

Therefore by taking

$$\beta = \beta_e = \frac{2\bar{\lambda}_e\underline{\lambda}_e}{\bar{\lambda}_e + \underline{\lambda}_e} \tag{10}$$

estimate (4) holds with $\alpha = \alpha_e$ where

$$\alpha_e = \frac{\bar{\lambda}_e - \underline{\lambda}_e}{\bar{\lambda}_e + \underline{\lambda}_e} < 1 \tag{11}$$

Note that, however, the incomplete knowledge of $\bar{\lambda}$ and $\underline{\lambda}$ yields a looser estimate as α_e in (11) is always not smaller than α in (9). The importance of having a tighten estimate for $||\beta H^{-1}(q) - I||$ shall become apparent when the design issues are considered in Section III.

3. Approximation of $h(q, \dot{q})$

The n-dimensional vector $h(q, \dot{q})$ includes many highly nonlinear terms representing the Coriolis, centrifugal, gravity effects of the manipulator and some electric parameters of the actuators. In order to derive a linear control scheme, it is required that $h(q, \dot{q})$ be approximated in 2-norm by a vector which is linear with respect to both q and $\dot{q}$. Namely we seek constant $n \times n$ matrices L_0, L_1 and vector $\ell_0 \epsilon R^n$ such that

$$\eta = \max_{\substack{q\epsilon D_0 \\ \dot{q}\epsilon D_1}} ||h(q, \dot{q}) - \ell(q, \dot{q})|| \tag{12}$$

is minimized, where

$$\ell(q, \dot{q}) = \ell_0 + L_0 q + L_1 \dot{q} \tag{13}$$

and $D_0 \subseteq R^n, D_1 \subseteq R^n$ are subsets of R^n representing the actual workspace of the manipulator and constraints on the joint velocities, respectively. The minmax optimization problem associated with (12) may be considered as a variation of an L^∞-optimization problem and is difficult to solve analytically. Instead we now seek a suboptional solution to the above approximation problem as follows.

Let $\{q_i, 1 \leq i \leq k\}$ and $\{p_j, 1 \leq j \leq m\}$ be the two sets of sampled values over D_0 and D_1 respectively, constant matrices $L_0, L_1 \epsilon R^{n\times n}$ and constant vector $\ell_0 \epsilon R^n$ are sought such that

$$\eta_0 = ||H_{km} - [\ell_0\ L_0\ L_1]F_{km}|| \tag{14}$$

is minimized where

$$H_{km} = [h(q_1, p_1)\ h(q_1, p_2) \cdots h(q_k, p_m)]_{n\times km}$$

$$F_{km} = [f_{11}\ f_{12} \cdots f_{\ell k}] = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ q_1 & q_1 & \cdots & q_k \\ p_1 & p_2 & \cdots & p_m \end{bmatrix}_{(2n+1)\times km}$$

This is a standard least-square problem and the solution is given by

$$\begin{aligned} [\ell_0\ L_0\ L_1] &= H_{km}F_{km}^+ = H_{km}F_{km}^T(F_{km}F_{km}^T)^{-1} \\ &= \left(\sum_{i=1}^k \sum_{j=1}^m h(q_i, p_j) f_{ij}^T\right)\left(\sum_{i=1}^k \sum_{j=1}^m f_{ij} f_{ij}^T\right)^{-1} \end{aligned} \tag{15}$$

where F_{km}^+ denotes the pseudo-inverse of F_{km}. Note that with k and m large enough, $F_{km}F_{km}^T$ is generically of full rank and therefore its inverse exists. Also note that the computation burden involved in (15) depends entirely on the number of manipulator joints as well as the number of sampling points over D_0 and D_1, and it is an off-line task.

4. An Illustrative Example

Consider the 2 degree-of-freedom planar arm shown in Fig. 1 where $\ell_1 = \ell_2 = .432$m, $m_1 = 15.91$ kg, $m_2 = 11.36$ kg. With the actuator effects included, the associated $H(q)$ and $h(q,\dot{q})$ are given by

$$H(q) = \begin{bmatrix} 3.82 + 2.12\cos\theta_2 & .71 + 1.06\cos\theta_2 \\ .71 + 1.06\cos\theta_2 & .81 \end{bmatrix} \quad (16)$$

and

$$h(q,\dot{q}) = \begin{bmatrix} -2.12\sin\theta_2(\dot{\theta}_1\dot{\theta}_2 + 0.5\dot{\theta}_2^2) + 5\dot{\theta}_1 + \\ 81.82\cos\theta_1 + 24.06\cos(\theta_1+\theta_2) \\ 1.06\sin\theta_2\dot{\theta}_1^2 + 2\dot{\theta}_2 + 24.06\cos(\theta_1+\theta_2) \end{bmatrix}$$

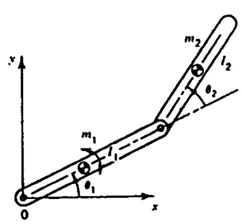

Fig. 1 The 2 d.o.f. planar arm

where $q = [\theta_1\ \theta_2]^T, 0 \le \theta_1, \theta_2 \le 2\pi$, and $|\dot{\theta}_{1,2}| \le 1$ rad./sec. Using PC-MATLAB [13] on an IBM PC/AT, it is found that $\bar{\lambda} = 6.4914$, $\underline{\lambda} = 0.2586$, therefore, $\beta = 0.4973$ and $\alpha = 0.9234$. It is noted that $1.2346 \le ||H^{-1}(q)|| \le 3.8674$, thus the formula described in [8] gives $\beta = 2.5510$ leading to $\max ||\beta H^{-1}(q) - I||$ $= 8.8657$ (or, if $\beta = \frac{1}{0.2510}$ is used, then $\max||\beta H^{-1}(q) - I|| =$ 2.8674.)

To approximate $h(q,\dot{q})$, intervals [0, 2π] for θ_1, θ_2 and [-1, 1] for $\dot{\theta}_1, \dot{\theta}_2$ are equally partitioned into 36 and 20 subintervals respectively. Computing $[\ell_0\ L_0\ L_1]$ through (15) with $k = 37^2$ and $m = 21^2$ on the same machine, the coefficients of linear function $\ell(q,\dot{q})$ are then found as

$$\ell_0 = \begin{bmatrix} 1.8877 \\ 0.3588 \end{bmatrix}, L_0 = \begin{bmatrix} 0 & 0.1086 \\ 0 & -0.1086 \end{bmatrix}, L_1 = \begin{bmatrix} 5.0 & 0 \\ 0 & 2.0 \end{bmatrix}$$

III. A LINEAR ROBUST CONTROL LAW

1. The Computed Torque Method as Applied to the Approximate Dynamics

Based on the approximation of $H(q)$ and $h(q,\dot{q})$, the approximate dynamics for the manipulator may be described as

$$\beta\ddot{q} + \ell(q,\dot{q}) = Dv \quad (17)$$

where β and $\ell(q,\dot{q})$ are given by (8) (or (10)), (13) and (15). As this simplied model is linear and time-invariant, the application of the computed torque control [14] to (17) yields a linear control scheme as follows

$$\begin{aligned} v &= D^{-1}(\beta w + \ell(q,\dot{q})) \\ &= D^{-1}(\beta w + \ell_0 + L_0 q + L_1\dot{q}) \end{aligned} \quad (18)$$

where w is the external input to be specified. The closed-loop dynamics as control (18) applied to the manipulator modelled by (3) is given by

$$\ddot{q} - H^{-1}(q)\triangle h(q,\dot{q}) = \beta H^{-1}(q)w \quad (19)$$

where

$$\triangle h(q,\dot{q}) = \ell(q,\dot{q}) - h(q,\dot{q}) \quad (20)$$

2. A Linear Compensation of the Tracking Error

Let $q_d(t)$ denote the desired joint displacement vector and define the 2n-dimensional vector $e(t)$ representing the tracking error as

$$e(t) = \begin{bmatrix} q(t) - q_d(t) \\ \dot{q}(t) - \dot{q}_d(t) \end{bmatrix}$$

By (19), the dynamics of the tracking error is described by

$$\dot{e} = \hat{A}e + B\triangle\hat{f}(e) + (B + \triangle B)w \quad (21)$$

where

$$\hat{A} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ I \end{bmatrix}, \triangle B = BE, E = \beta H^{-1}(q) - I, \quad (22)$$

and

$$\triangle\hat{f}(e) = H^{-1}(q)\triangle h(q,\dot{q}) - \ddot{q}_d$$

The input signal w in (18) can now be specified as

$$w = -[K_1\ K_2]e + u \triangleq -Ke + u \quad , \quad K_1, K_2 \epsilon R^{n\times n} \quad (23)$$

in order to stablize the linear part of dynamic equation (21), where u is the external control signal to be further specified. In so doing we obtain

$$\dot{e} = Ae + Bu + B(Eu + \triangle f(e)) \quad (24)$$

with

$$\begin{aligned} \triangle f(e) &= \triangle\hat{f}(e) - EKe \\ &= H^{-1}\triangle h - \ddot{q}_d - EKe \end{aligned} \quad (25)$$

and

$$A = \hat{A} - BK = \begin{bmatrix} 0 & I \\ -K_1 & -K_2 \end{bmatrix}$$

where K_1 and K_2 are chosen such that matrix A is stable (i.e. all eigenvalues of A are located in the open left-half complex plane).

To obtain the desired stabilizing compensation, let P be the positive definite solution of the Lyapunov equation [15]

$$A^T P + PA = -I$$

and define the Lyapunov function $V(e) = e^T Pe$. The time derivative of $V(e)$ along any solution trajectory of (23) is then given by

$$\dot{V} = -||e||^2 + 2e^T PB(u + Eu + \triangle f)$$

If we choose

$$u = -\gamma B^T Pe \tag{26}$$

then from (22) and (4) it follows that

$$\begin{aligned} \dot{V} &= -||e||^2 - 2\gamma(||B^T Pe||^2 + x^T PBEB^T Pe) + 2e^T PB\triangle f \\ &\leq -||e||^2 - 2\gamma(1-\alpha)||B^T Pe||^2 + 2e^T PB\triangle f \end{aligned}$$

Note that for any $\varepsilon > 0$

$$2e^T PB\triangle f \leq \frac{||B^T Pe||^2}{\varepsilon} + \varepsilon||\triangle f||^2$$

where, by (25), (6) and (12),

$$\begin{aligned} ||\triangle f|| &\leq ||H^{-1}||\ ||\triangle h|| + ||\ddot{q}_d|| + \alpha||K||\ ||e|| \\ &\leq \frac{\eta}{\underline{\lambda}} + ||\ddot{q}_d|| + \alpha||K||\ ||e|| \\ &= \delta + \alpha_k||e|| \end{aligned} \tag{27}$$

with

$$\delta = \frac{\eta}{\underline{\lambda}} + \max||\ddot{q}_d||, \qquad \alpha_k = \alpha||K|| \tag{28}$$

Thus

$$\dot{V} \leq -(1-2\alpha_k^2\varepsilon)||e||^2 + 2\delta^2\varepsilon - [2\gamma(1-\alpha) - \frac{1}{\varepsilon}]||B^T Pe||^2 \tag{29}$$

Given a prescribed ultimate error bound ρ, ε may be determined such that

$$-(1 - 2\alpha_k^2\varepsilon)\rho^2 + 2\delta^2\varepsilon = 0 \tag{30}$$

i.e. $\varepsilon = \rho^2/2(\delta^2 + \alpha_k^2\rho^2)$. The gain factor γ required to implement (26) can then be chosen such that

$$2\gamma(1-\alpha) - \frac{1}{\varepsilon} > 0$$

i.e.

$$\gamma > \frac{\delta^2 + \alpha_k^2\rho^2}{(1-\alpha)\rho^2} \tag{31}$$

Evidently, with γ chosen by (31), the last term in the right-hand side of (29) remain non-positive and the sum of the first terms in (29) becomes negative whenever $||e|| > \rho$. We are therefore in a position to apply the standard results from nonlinear control theory [10][12] to conclude that tracking error $e(t)$ is uniformly ultimately bounded, i.e. using linear control v characterized by (18), (23), and (26), the tracking error $||e(t)||$ will be ultimately bounded by ρ after a finite time duration.

From (18), (23) and (26), the overall linear control can be expressed in terms of a constant linear feedback control (PD control) combined with a pre-compensation term

$$v = K^* e + Q \tag{32}$$

where

$$\begin{aligned} K^* &= \beta D^{-1}(K + \gamma B^T P - L) \\ L &= [L_0\ L_1] \end{aligned} \tag{33}$$

and

$$Q = D^{-1}(\ell_0 + L_0 q_d + L_i \dot{q}_d) \tag{34}$$

3. Robustness of Linear Control (32)

It is noted that the control law given by (32) does not depend on precise knowledge of the manipulator dynamics but the spectral bounds $\bar{\lambda}, \underline{\lambda}$ of $H(q)$ and the number η defined by (12). As demonstrated in Section II, 2, when $\bar{\lambda}$ and $\underline{\lambda}$ are not precisely known due to either parameter uncertainties or unknown load, a upper bound of $\bar{\lambda}$ and a lower bound of $\underline{\lambda}$ can be used and, under such circumstances, parameters $\underline{\lambda}, \beta$ and α used in this section should be replaced by $\underline{\lambda}_e, \beta_e$ and α_e which are defined by (6′), (10)and (11), respectively.

Another term that should be considered in robustness analysis of control law (32) is $h(q, \dot{q})$. Note that the control parameters related to $h(q, \dot{q})$ are η, ℓ_0, L_0 and L_1 given by (12), (13) and (15). If $h(q, \dot{q})$ is numerically too involved, or if $h(q, \dot{q})$ is not precisely known, one may simply take $\ell(q, \dot{q}) \equiv 0$ and η then becomes a upper bound of $||h(q, \dot{q})||$. Consequently, control law (32) becomes

$$v = -K^* e \tag{32'}$$

where

$$K^* = \beta D^{-1}(K + \gamma B^T P) \tag{33'}$$

Of course, the acquisition of better control robustness demands a larger feedback gain factor γ due to larger α, α_k and δ as is seen from (28) and (31).

IV. CONCLUSIONS

A PD-like linear control for robotic manipulators is proposed based primarily on an eigen-analysis of the inertia matrix of the manipulator considered. The proposed control law is robust against model uncertainties since its parameters depend only on the lower and upper bounds of the spectrum of $H(q, \dot{q})$ and the bound of $||h(q, \dot{q})||$.

REFERENCES

[1] J.Y.S. Luh, "An anatomy of industrial robots and their controls", IEEE Trans. Automat. Contr., vol. AC-28, pp. 133-153, Feb. 1983.

[2] R.P. Paul, Robot Manipulator: Mathematics, Programming and Control, MIT Press, Cambridge, Mass., 1982.

[3] S. Arimoto and F. Miyazaki, "Stability and robustness of PID feedback control for robot manipulators of sensory capability", in Proc. 1st Int. Symp. on Robotics Research, pp. 783-799, 1983.

[4] C. Samson, "Robust nonlinear control of robotic manipulators", in Proc. 1983 IEEE CDC, 1983.

[5] M.W. Spong, J.S. Thorp, and J.M. Kleinwaks, "The control of robot manipulators with bounded input. Part II: Robustness and disturbance rejection", Proc. 23rd Conf. on Decision and Control, pp. 1047-1052, Dec. 1984.

[6] V.D. Tourassis and C.P. Neuman, "Robust nonlinear feedback control for robotic manipulators", IEE Proc. vol, 132, Pt.D, No. 4, pp. 134-143, July 1985.

[7] M.W. Spong and H. Sira-Ramirez, "Robust control design techniques for a class of nonlinear systems", Proc. 1986 American Control Conference, pp. 1515-1522, June 1986.

[8] M.W. Spong and M. Vidyasagar, "Robust linear compensator design for nonlinear robotic control", IEEE J. Robotics and Automation, vol. RA-3, pp. 345-351, Aug. 1987.

[9] S. Gutman, "Uncertain dynamical system - a Lyapunov min-max approach", IEEE Trans. Automat. Contr., vol. AC-24, pp. 437-443, June 1979.

[10] M.J. Corless and G. Leitmann, "Continuous state feedback guaranteeing uniform ultimate boundedness for uncertain dynamic systems", IEEE Trans. Automat. Contr., vol. AC-26, pp. 1139-1144, Oct. 1981.

[11] J.S. Thorp and B.R. Barmish, "On guaranteed stability of uncertain linear system via linear control", J. Optimiz. Appl., vol. 35, No. 4, pp. 559-579, Dec. 1981.

[12] B.R. Barmish, M. Corless, and G. Leitmann, "A new class of stabilizing controllers for uncertain dynamical systems", SIAM J. Contr. Optimiz., vol. 21, No. 2, pp. 246-255, 1983.

[13] PC-MATLAB, User's Guide. The MathWorks, Inc. Sherborn, MA, 1987.

[14] A.C. Bejczy, "Robot arm dynamics and control", JPL Technical Memorandum 33-369, Feb. 1974.

[15] R.V. Patel and M. Toda, "Quantative measures of robustness for multivariable systems", Proc. America Control Conference, TP8-A, San Francisco, 1985.