Application of Space-Time Block Coding in DS-CDMA Communication Systems

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Abstract— **The basic theory of space-time block coding (STBC) is reviewed and its potential applications in DS-CDMA communication systems in a number of system configurations are investigated. Simulation results show that significant bit-error-rate (BER) performance improvement can be achieved by integrating STBC into DS-CDMA systems.**

Index Items—**Space-time code, space-time block code, DS-CDMA**

I. Introduction

In wireless communication systems, antenna arrays are used primarily to achieve spatial diversity [1]. An effective way to increase the data rate further in wireless systems is to employ coding techniques that take advantage of multiple antennas. Space-time coding techniques that can be used to introduce temporal and spatial correlation in signals received by different antennas have been proposed recently in [2]-[4].

In this paper, we review one of the space-time coding techniques, namely, space-time block coding (STBC) and investigate its potential applications in DS-CDMA communication systems in a number of system configurations. We observe that significant performance improvement can be achieved by integrating STBC into a DS-CDMA system.

The paper is organized as follows. In Section II, the basic principles of STBC are reviewed. In Section III, the system models of DS-CDMA communication systems with STBC are described and simulation results that compare the bit-error-rate (BER) performance of CDMA channels with and without STBC are presented. Conclusions are drawn in Section IV.

II. Space - Time Block Coding Basics

A. *STBC for Systems with One Receive Antenna*

A baseband representation for STBC with two transmit antennas and one receive antenna is shown in Fig. 1 where each two consecutive input symbols of the STBC encoder are grouped together. The two symbols $\{c_1, c_2\}$ are transmitted simultaneously from the two antennas A_1 and A_2 at a given symbol period. In the next symbol period, the signals transmitted from antennas A_1 and A_2 are $-c_2^*$ and c_1^*. The same transmission pattern is repeated for transmitting subsequent pairs of symbols. Since the coding method used involves *two* sets of *two*-dimensional vectors, the codes so generated will be referred to as 2×2 STBC. Let h_1 and h_2 represent the one-tap impulse responses from antennas A_1 and A_2 to the receive antenna, respectively. Throughout the paper it is assumed that the channels involved in the communication are time invariant over any two consecutive symbol periods, i.e., $h_i(nT) = h_i[(n+1)T]$ for $i = 1, 2$. It is also assumed that the envelopes of the channel impulse responses are independently Rayleigh-distributed and the average signal-to-noise ratio (SNR) per bit for the various propagation paths are identical. According to [2], in macrocell urban environment, in order to provide sufficient decorrelation between the signals transmitted from any two transmit antennas, a seperation between the two antennas of the order of ten wavelengths at the base station and of the order of three wavelengths at the remote units is needed. If we denote the signals received in two consecutive symbol periods as r_1 and r_2, then we have

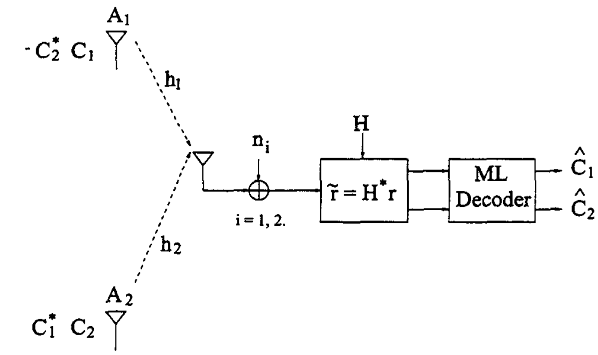

Fig. 1. STBC system with 2 transmit antennas and 1 recieve antanna.

$$r_1 = h_1 c_1 + h_2 c_2 + n_1 \quad (1)$$
$$r_2 = -h_1 c_2^* + h_2 c_1^* + n_2 \quad (2)$$

where n_1 and n_2 represent additive white Guassian noise (AWGN) [4] and are modeled as independent identically distributed (i.i.d) complex Gaussian random variables with zero mean and power spectral density $N_0/2$ per dimension. Now let the received signal vector, the code symbol vector, and the noise vector be $\mathbf{r} = [r_1 \ \ r_2^*]^T$, $\mathbf{c} = [c_1 \ \ c_2]^T$, and $\mathbf{n} = [n_1 \ \ n_2]^T$, respectively. Eqs. (1) and (2) can be written in matrix form as

$$\mathbf{r} = \mathbf{H}\mathbf{c} + \mathbf{n} \quad (3)$$

where the channel matrix $\mathbf{H}$ is given by

$$\mathbf{H} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}$$

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and $\mathbf{n}$ is a complex random vector with zero mean and covariance $N_0\mathbf{I}$. If we define C as the code set for all possible symbol vectors $\mathbf{c}$ and assume that any symbol pair is equally likely to occur, then the optimum maximum-likelihood (ML) decoder can be characterized [4] by

$$\mathbf{c} = \arg\left(\min_{\hat{\mathbf{c}}\in C} \|\mathbf{r} - \mathbf{H}\hat{\mathbf{c}}\|^2\right) \tag{4}$$

The ML decoding rule in (4) can be further simplified by realizing that the column vectors of the channel matrix $\mathbf{H}$ are orthogonal and, therefore, $\mathbf{H}^*\mathbf{H} = \rho\mathbf{I}$ where $\mathbf{H}^*$ denotes the complex conjugate transpose of $\mathbf{H}$ and $\rho = |h_1|^2 + |h_2|^2$. By multiplying (3) by matrix $\mathbf{H}^*$, we obtain

$$\tilde{\mathbf{r}} = \mathbf{H}^*\mathbf{r} = \rho\mathbf{c} + \tilde{\mathbf{n}} \tag{5}$$

where $\tilde{\mathbf{n}} = \mathbf{H}^*\mathbf{n}$. Using (5), the ML decoding rule for the 2×2 STBC can be reduced into two seperate and simple decoding rules for c_1 and c_2, namely,

$$c_1 = \arg\left(\min_{\hat{c}_1\in C_s} |\tilde{r}_1 - \rho\hat{c}_1|^2\right) \tag{6}$$

$$c_2 = \arg\left(\min_{\hat{c}_2\in C_s} |\tilde{r}_2 - \rho\hat{c}_2|^2\right) \tag{7}$$

where C_s denotes the code set for all possible symbols. Evidently, finding c_1 and c_2 using (6) and (7) requires much reduced computational complexity compared with that required by (4). Further analysis and calculation show that, at the input point of the ML decoder, the SNR is given by

$$\begin{aligned}\gamma_b &= \frac{\rho E_b}{N_0} \\ &= \frac{|h_1|^2 E_b}{N_0} + \frac{|h_2|^2 E_b}{N_0} \\ &= \gamma_1 + \gamma_2\end{aligned} \tag{8}$$

where E_b is the energy and γ_i is the SNR per bit for the ith propagation path with $E(\gamma_i) = \bar{\gamma}_c$ for $i = 1, 2$, where E($\cdot$) denotes expectation. It follows from (8) and chapter 14 of [5] that the performance of the above system is identical with that of a system that has one transmit antenna and two receive antennas and uses the maximal-ratio-combining (MRC) technique. Theoretical analysis and numerical simulations of the relation between the system performance and the correlation between the channels can be found in [6]. In particular, the performance of a communication system with 2×2 STBC gradually degrades as the correlation between the channels increases. In what follows, we examine the performance of the system in Fig. 1. In this case, γ_b in (8) is a chi-square distributed random variable with 4 degrees of freedom [5] and probability density function

$$f(\gamma_b) = \frac{\gamma_b}{\bar{\gamma}_c^2} e^{-\gamma_b/\bar{\gamma}_c} \tag{9}$$

Since the conditional probability of the error for coherent detection of quadratic phase-shift keying (QPSK) is given by

$$P_b(\gamma_b) = Q\left(\sqrt{\gamma_b}\right) \tag{10}$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2}\, dt \tag{11}$$

we obtain the average $P_b(\gamma_b)$ over the fading using (9) as

$$\begin{aligned}P_e &= \int_0^\infty P_b(\gamma_b) f(\gamma_b)\, d\gamma_b \\ &= \int_0^\infty Q\left(\sqrt{\gamma_b}\right) \frac{\gamma_b}{\bar{\gamma}_c^2} e^{-\gamma_b/\bar{\gamma}_c}\, d\gamma_b \\ &= \left[\frac{1}{2}(1-\mu)\right]^2 \sum_{k=0}^{1} \binom{1+k}{k} \left[\frac{1}{2}(1+\mu)\right]^k\end{aligned} \tag{12}$$

where $\mu = \sqrt{\bar{\gamma}_c/(2+\bar{\gamma}_c)}$. For large $\bar{\gamma}_c$, (12) can be approximated as

$$P_e \approx 3\left(\frac{1}{2\bar{\gamma}_c}\right)^2 \tag{13}$$

We observe from (13) that the error probability decreases inversely with respect to the square of the average SNR for each propagation path. Thus, a diversity gain of order 2 is achieved by integrating a 2×2 STBC into the basic communication system.

B. STBC for Systems with Multiple Receive Antennas

For a system with 2 transmit antennas and M receive antennas, the signal vector $\mathbf{r}_m$ at the mth receive antenna is given by

$$\mathbf{r}_m = \mathbf{H}_m\mathbf{c} + \mathbf{n}_m \tag{14}$$

where $\mathbf{n}_m$ is the noise vector for the channel between the transmit antennas and the mth receive antenna, and

$$\mathbf{H}_m = \begin{bmatrix} h_{1m} & h_{2m} \\ h_{2m}^* & -h_{1m}^* \end{bmatrix}$$

is the channel matrix for the same channel with h_{im} denoting the channel impulse response between the ith transmit antenna and the mth receive antenna, for $i = 1, 2$. The optimum ML decoding rule in this case becomes

$$\mathbf{c} = \arg\left(\min_{\hat{\mathbf{c}}\in C} \sum_{m=1}^{M} \|\mathbf{r}_m - \mathbf{H}_m\hat{\mathbf{c}}\|^2\right) \tag{15}$$

As before, by pre-multiplying (14) by $\mathbf{H}_m^*$, we obtain

$$\tilde{\mathbf{r}}_m = \mathbf{H}_m^*\mathbf{r}_m = \rho_m\mathbf{c} + \tilde{\mathbf{n}}_m \tag{16}$$

where $\rho_m = |h_{1m}|^2 + |h_{2m}|^2$ and $\tilde{\mathbf{n}}_m = \mathbf{H}_m^*\mathbf{n}_m$. Consequently, the objective function in (15) can be reduced to

$$\begin{aligned}\sum_{m=1}^{M} \|\tilde{\mathbf{r}}_m - \rho_m\hat{\mathbf{c}}\|^2 &= \sum_{m=1}^{M} (|\tilde{r}_{1m} - \rho_m\hat{c}_1|^2 \\ &\quad + |\tilde{r}_{2m} - \rho_m\hat{c}_2|^2)\end{aligned} \tag{17}$$

and the ML decoding rule in (15) can be replaced by the following two seperate and simple decoding rules:

$$c_1 = \arg\left(\min_{\hat{c}_1 \in C_s} \sum_{m=1}^{M} |\tilde{r}_{1m} - \rho_m \hat{c}_1|^2\right) \quad (18)$$

$$c_2 = \arg\left(\min_{\hat{c}_2 \in C_s} \sum_{m=1}^{M} |\tilde{r}_{2m} - \rho_m \hat{c}_2|^2\right) \quad (19)$$

By using an argument similiar to that for (8) and assuming that the channel impulse responses for each propagation path are mutually independent and with equal averages, it can be shown that the diversity order, in this case, is $2M$.

C. An Illustrative Example

Two system configurations were used in our simulations. One of them had two transmit antennas and one receive antenna, and the other was equipped with two antennas each for the transmitter and receiver. Each system used a 2×2 STBC for performance improvement. These system configurations are illustrated in Figs. 2 and 3, respectively. The systems work as follows. First, the information source bits are mapped into normalized QPSK constellation points such that the average energy of the constellation is half and, consequently, the total energy from the two transmit antennas is the same as that with one transmit antenna. Next, the QPSK symbols are encoded by the STBC and transmitted from two transmit antennas.

The wireless channels used in the simulation were aussumed to be invariant over a frame of length 100 bits. The channel impulse responses were modeled as samples of independent complex Guassian random variables with variance 0.5 in each dimension. At the receiver, the decoding procedures followed the descriptions in Section II. The BER performance of the systems with and without STBC are plotted in Fig. 4. It is observed that for the system shown in Fig. 3 where two receive antennas were employed, a diversity gain of 10.5 dB can be achieved at a BER = 10^{-3}. From Fig. 4 it can also be observed that larger diversity gains can be achieved for the systems whose target transmission BER is smaller.

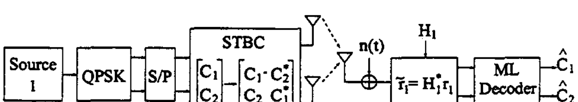

Fig. 2. A system with 2 × 2 STBC using 1 receive antenna.

III. Integration of STBC into CDMA Channels

In this section, we consider the application of STBC in DS-CDMA multiuser communication systems and present some simulation results for the performance of DS-CDMA channels with STBC.

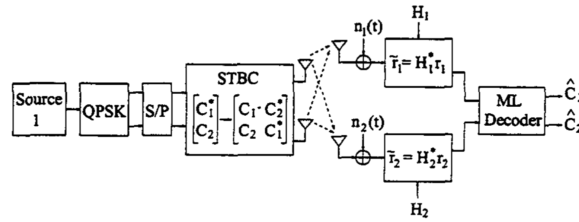

Fig. 3. A system with 2 × 2 STBC using 2 receive antennas.

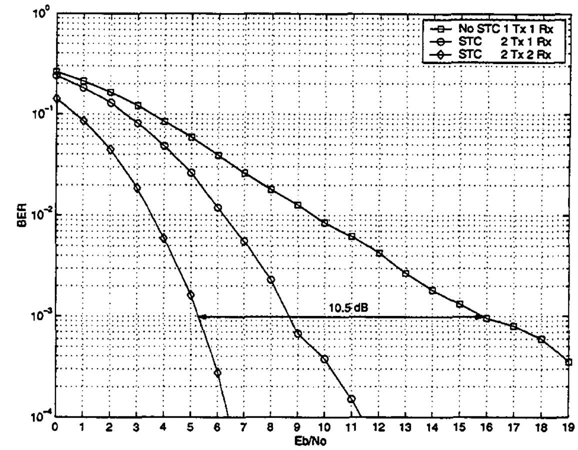

Fig. 4. Performance comparison between communication systems with and without STBC.

A. Downlink Channel

First, we consider the downlink channel of a multiuser DS-CDMA system with STBC. The transmitter and receiver models with two transmit antennas at the base station and one receive antenna at the remote unit are illustrated in Figs. 5 and 6, respectively. At the transmitter, the information bit sequence of each user is first mapped into QPSK symbols. Then, these QPSK symbols are input to the STBC encoder.

In our simulations, the output of each STBC encoder was spread by a Gold sequence of length-15. The spread signals from different users for the same transmit antenna were summed up before they were transmitted from antennas A_1 and A_2 at the base station, respectively. A matched filter was used in each remote unit to decorrelate the received signal. The output of the matched filter was then fed into the STBC decoder. Through the same decoding procedures described in Section II, the optimal decisions of the transmitted symbols can be obtained at the output of the STBC decoder. The performance improvement by integrating STBC into the downlink CDMA channels is illustrated in Fig. 7.

B. Uplink Channel

Next, we consider the uplink channel of a multiuser CDMA system with STBC. The transmitter and receiver

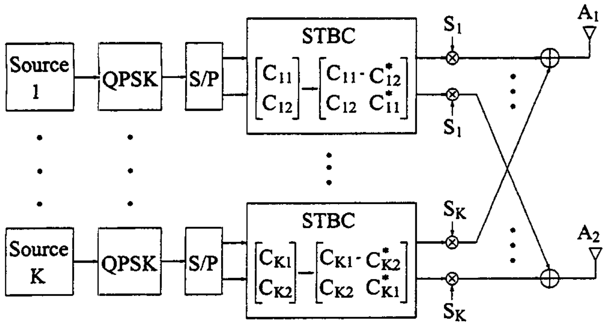

Fig. 5. Downlink STBC CDMA transmitter with 2 transmit antennas.

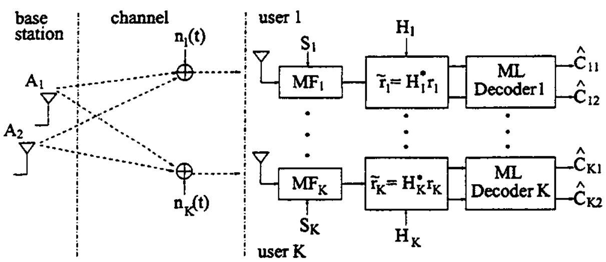

Fig. 6. Downlink STBC CDMA receiver with 1 receive antenna.

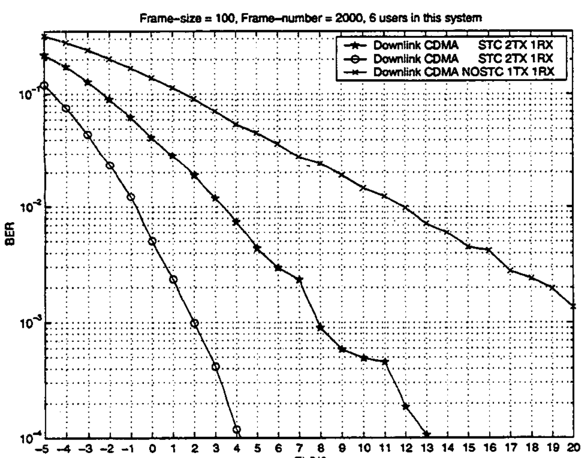

Fig. 7. Performance comparison between DS-CDMA downlink systems with and without STBC.

models for the uplink STBC CDMA system are similiar to the downlink STBC CDMA channel. The performance improvement by integrating STBC into the uplink CDMA channels is illustrated in Fig. 8.

IV. Conclusions

We have investigated some potential applications of STBC in DS-CDMA communication systems in a number of

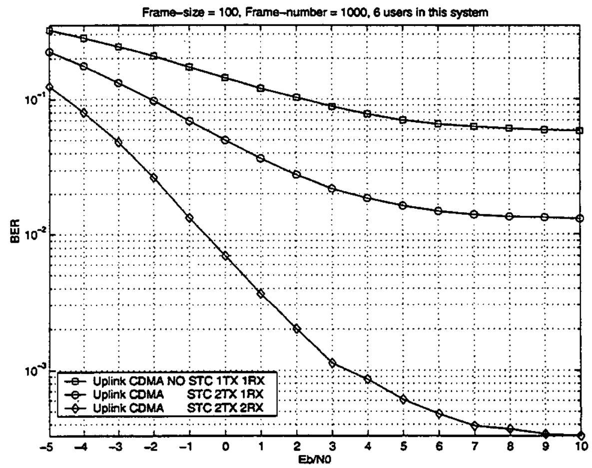

Fig. 8. Performance comparison between DS-CDMA uplink systems with and without STBC.

system configurations. Particular attention has been paid to the downlink channel of a multiuser DS-CDMA system with two transmit antennas and one receive antenna. From the analysis and simulation results presented, we have shown that a significant BER performance improvement can be achieved by integrating STBC into DS-CDMA communicatin systems.

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