A Constant-Modulus Algorithm for Blind Multiuser Detection in DS-CDMA Systems with Antenna Array

X. M. Wang, W.-S. Lu, and A. Antoniou
Dept. of Elec. and Comp. Eng., University of Victoria
Victoria, B.C., CANADA, V8W 3P6

Abstract— **Blind multiuser detectors based on the constant modulus (CM) criterion have shown significant resistance to the spreading code estimation error in its demodulation. In this paper, we consider a CM-based blind adaptive multiuser detector which utilizes the spatial diversity provided by an antenna array to improve the demodulation performance. Our analysis and simulation results show that the introduction of an antenna array leads to a substantial improvement in the demodulation performance and, compared with the constrained minimum output energy (CMOE) space-time blind adaptive multiuser detector, the proposed detector is shown robustness to the signature code inaccuracy.**

I. Introduction

The constrained minimum output energy (CMOE) blind adaptive multiuser detector for the demodulation in DS-CDMA system was proposed in [1]. The results presented in this paper show that the signature code estimation error in the receiver leads to significant degradation in the demodulation performance. To ease this problem, a constant modulus (CM)-based blind multiuser detection was proposed in [2] and [3] and a further analysis of this method was presented in [4] and [5]. It was shown that the CM-based blind adaptive multiuser detector is robust against the estimation error of the spreading code in the demodulation. In this paper, we consider a CM-based blind adaptive multiuser detector which utilizes spatial diversity provided by an antenna array in the receiver. In each antenna of the proposed space-time detector, a chip-rate matched-filter (MF) is followed by a tap-delayed adaptive filter whose tap weights are adapted according to the CM adaptation rule. The adaptive filters in the receiver are implemented independently to suppress multiple accessinterference (MAI). Once the adaptive filters reach steady state, the demodulation decision is made by considering the weighted filtered outputs altogether. Our simulations show that the proposed CM-based blind space-time detector is robust with respect to signature code estimation error and offers a bit-error-rate (BER) performance comparable with that of the training-sequence-based minimum mean-squared-error (MMSE) space-time detector, which is considerably better than that of the CMOE blind space-time detector [6].

The rest of this paper is organized as follows. Section II describes a signal model for DS-CDMA systems. Section III first presents a review of the constant modulus algorithm (CMA) and linear constrained CM-based blind detectors. A CM-based blind adaptive space-time detector is then proposed. In section IV, the performance of the proposed detector in presence of signature code estimation error is examined through computer simulations.

II. Signal Model

For simplicity of exposition, we consider a synchronous DS-CDMA system, where K users transmit data packets through an additive white Gaussian noise (AWGN) channel. The bit interval of each user is T_b seconds and each information bit belongs to the set $\{1, -1\}$. The kth user signal is assigned a signature waveform $s_k(t)$ given by

$$s_k(t) = \sum_{i=1}^{N} (-1)^{s_k^i} p_{T_c}[t-(i-1)T_c] \quad \text{for } t \in [0,\ T_b] \tag{1}$$

where $p_{T_c}(t)$ is a chip waveform which is nonzero for $0 \le t \le T_c$ and zero outside, $\mathbf{s}_k = [s_k^1\ s_k^2\ \cdots\ s_k^N]^T$ is the binary-valued signature code vector, $N = T_b/T_c$ is the processing gain. The signature waveforms are normalized to have unit energy, i.e., $||s_k(t)||^2 = 1$, for $1 \le k \le K$. The transmitted kth user's baseband signal is given by

$$x_k(t) = \sum_{i=1}^{N} A_k b_k (-1)^{s_k^i} p_{T_c}[t-(i-1)T_c] \tag{2}$$

where b_k is the transmitted information bit, A_k is the received signal amplitude of the kth user.

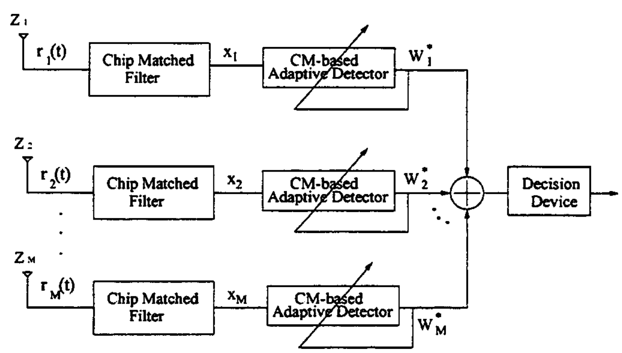

Fig. 1. CM-based multiuser detector with multiple antenna.

A diagram of the proposed space-time CM-based multiuser detector is shown in Fig. 1. In the receiver an array of M antennas is used to exploit the space diversity. The received signal from the M antennas is an M-dimensional vector $\mathbf{r}(t) = [r_1(t)\ r_2(t)\ \cdots\ r_M(t)]^T$ which can be expressed as

$$\mathbf{r}(t) = \sum_{k=1}^{K} \sum_{i=1}^{N} A_k b_k \mathbf{a}_k (-1)^{s_k^i} p_{T_c}[t-(i-1)T_c] + \mathbf{n}(t) \tag{3}$$

where $\mathbf{n}(t) = [n_1(t)\ n_2(t)\ \cdots\ n_M(t)]^T$ is a vector of complex-valued independent white Gaussian noise with

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zero mean and crosscorrelation matrix $E\left[\mathbf{n}(t)\mathbf{n}^H(t)\right] = \sigma^2\mathbf{I}$. $\mathbf{a}_k = [a_{1k}\ a_{2k}\ \cdots\ a_{Mk}]^T$ is the M-element antenna response for the kth user, which is often referred to as *spatial signature*. For a linear array structure described in Fig. 2, the mth component of this array response can be expressed as

$$a_{mk} = \exp\left[\frac{-j2\pi d}{\lambda}(m-1)cos(\alpha_{mk})\right]$$

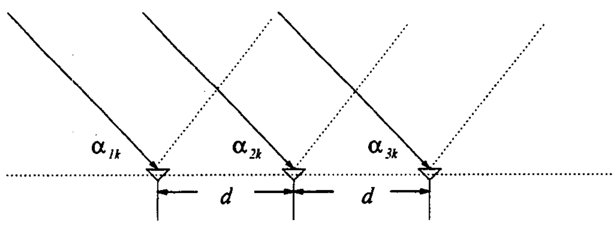

Fig. 2. Structure of three element antenna array.

where d is the distance between two elements of the antenna array, λ is the wave-length of the wireless signal and α_{mk} is the direction of arrival (DOA) of the kth user to the mth array element. If the array's inter-element distance is significantly longer than the wave length of the radio frequency (RF) signal, then the effects of the wireless channel on the signals received by different antennas are very likely different. Consequently, it is expected that the usage of multiple antennas will improve the demodulation performance, especially in terms of outage probability. On the other hand, if the array's inter-element distance is comparable with the wave length of the RF signal, then the received signals are affected almost equally. In this case, an antenna array with non-omnidirectional radiation pattern can be used to cancel the interference generated by the signals of other users whose DOAs are different from the desired one.

The demodulation begins by passing the received signals through a bank of chip-rate matched filters (MF) and chip-rate samplers. This process can be modeled as

$$\mathbf{X} = \sum_{k=1}^{K} A_k b_k \mathbf{S}_k + \mathbf{N} \tag{4}$$

where $\mathbf{X} = [\mathbf{x}_1\ \mathbf{x}_2\ \cdots\ \mathbf{x}_M]^T$ is the sampled signal after the bank of M chip-rate samplers in the array with $\mathbf{x}_m$ being the sampled signal after the sampler in the mth antenna, $\mathbf{S}_k = \mathbf{a}_k\mathbf{s}_k^T$ is a $M \times N$ space-time signature for the kth user and $\mathbf{N} = \{n_{mk}\}$ is the sampled noise $\mathbf{n}(t)$ in (3).

III. A Space-time Blind Adaptive Multiuser Detection Based on CM Criterion

A. CM Criterion and CM Approach

The constant modulus algorithm (CMA) was initially proposed in [7] and [8] for blind channel equalization. Due to its simplicity in implementation as a least-mean-square (LMS) type adaptive filter, the CMA is one of the most widely studied and implemented blind adaptive equalization algorithms in digital communication systems. In a CM-based equalizer, the CMA penalizes deviations of the blind equalization output away from the constant modulus of transmitted signal. The cost function involved is given by

$$\mathbf{J}(n) = E[||\mathbf{w}^H(n)\cdot\mathbf{x}(n)|^2 - R|^2] \tag{5a}$$

$$R = \frac{E[|\mathbf{u}(n)|^4]}{E[|\mathbf{u}(n)|^2]} \tag{5b}$$

where $\mathbf{x}(n)$ is an input signal vector for the equalizer, $\mathbf{w}(n)$ is the tap-weight vector of the adaptive filter at nth time instant, and $\mathbf{u}(n)$ is the transmitted signal which has constant modulus. The constant R is chosen such that the gradient of the cost function $\mathbf{J}(n)$ is zero when perfect equalization is achieved. Based on the stochastic gradient algorithm [10], the adaptation rule for $\mathbf{w}$ in (5) is given by

$$\hat{\mathbf{w}}(n+1) = \hat{\mathbf{w}}(n) + \mu\cdot\mathbf{x}(n)e^*(n) \tag{6a}$$

$$e(n) = \mathbf{w}^H(n)\mathbf{x}(n)(R - |\mathbf{w}^H(n)\cdot\mathbf{x}(n)|^2) \tag{6b}$$

where μ is a step size. Compared with training-sequence based equalization algorithms, the CM-based algorithm adapts the equalizer *blindly* and thus improves channel capacity.

B. Linear Constrained CM-Based Blind Adaptive Multiuser Detection

Several CM-based blind adaptive multiuser detectors, were proposed in [2] and [3]. In [3], the detection vector $\mathbf{w}_k$ is characterized as the solution of the constrained optimization problem

$$\text{minimize:}\quad \mathrm{E}[||\mathbf{w}_k^H\mathbf{x}|^2 - r|^2] \tag{7a}$$

$$\text{subject to:}\quad \mathbf{w}_k^H\mathbf{s}_k = 1 \tag{7b}$$

where $\mathbf{x}$ is the chip-rate sampled input signal to the detector, which contains MAI and additive channel noise, $\mathbf{w}_k$ is the detection vector, and $\mathbf{s}_k$ is the normalized signature vector for the kth user. The linear constraint in (7b) is introduced so that when the objective function is minimized subject to this constraint, the interference from other users will be effectively suppressed. It is worthwhile to point out a connection of the problem in (7) to the well know decorrelating detector: in a noise-free communication channel, it can be shown that minimizing the objective function in (7a) achieves perfect MAI suppression and the detector obtained is the same as the decorrelating detector.

Because of the presence of the constraint in (7), the stochastic gradient algorithm shown in (6) cannot be applied directly. To deal with this problem, reference [3] proposed a subspace type algorithm which projects the input $\mathbf{x}$ onto the null space of desired user signature vector $\mathbf{s}_1$. Although in general this algorithm works well, the computational complexity of the algorithm is rather high because of the projection operation involved.

C. An Improved Blind Adaptive CM-based Detector

A computationally more efficient blind adaptive detector can be derived based on the adaptation rule in [1] which was initially proposed for the CMOE detector. The algorithm is described in Table 1, where the detection vector is divided into two parts: the desired signature vector $\mathbf{s}_k$ and the adaptive term $\mathbf{y}(n)$ which is orthogonal to $\mathbf{s}_k$. In this normalized adaptation algorithm, the step-size μ is chosen to minimize the error signal $e = (|\mathbf{w}_k^H(n)\mathbf{x}_k(n)|^2 - 1)^2$ in each iteration. In practical implementation, the value of μ is reduced by multiplying it with a small positive constant in order to reduce the noise effect in the adaptation.

TABLE 1
SUMMARY OF THE NORMALIZED STOCASTIC GRADIENT ALGORITHM SOLVING THE PROBLEM IN (7).

Parameters:	$\mathbf{x}(n)$:	input signal
	$\mathbf{w}_k(n)$:	adaptive detector
	$\mathbf{s}_k$:	signature vector for the kth user
	μ:	step size
Initialize the algorithm by setting:		
	$n = 1,\ \ \mathbf{y}(1) = \mathbf{0}$	
At time instant $n = 1,\ 2,\ \ldots,$ compute		
	$\mathbf{w}_k(n) = \mathbf{s}_k + \mathbf{y}(n)$	
	$Z = \mathbf{w}_k^H(n)\mathbf{x}(n)$	
	$e = \lvert Z\rvert^2 - 1$	
	$Z_{mf} = \mathbf{s}^H\mathbf{x}(n)$	
	$\mathbf{x}_a = \mathbf{x} - Z_{mf}\cdot\mathbf{s}_k$	
	$\mu = 1/(\lvert e\rvert\cdot\lVert\mathbf{x}_a\rVert^2)$	
	$\mathbf{y}(n+1) = \mathbf{y}(n) - \mu\cdot e^*\ Z^*\cdot\mathbf{x}_a$	

The signal-to-interference-plus-noise ratio (SINR) defined by

$$\mathrm{SINR} = \frac{A_k^2(\mathbf{w}_k^T\mathbf{s}_k)^2}{\sigma^2\|\mathbf{w}_k\|^2 + \sum_{\substack{j=1\\ j\neq k}}^{K} A_j^2(\mathbf{w}_j^T\mathbf{s}_j)^2} \tag{8}$$

is used to measure the tracking ability of the adaptive detectors, where $\|\mathbf{w}_k\|$ is the 2-norm of $\mathbf{w}_k$. From the analysis of global convexity of the objective function in (7a) [3-5], it is known that the minimizer $\mathbf{w}_k^*$ is unique.

D. CM-Based Space-Time Blind Adaptive Multiuser Detection

In the CM-based space-time detector shown in Fig. 1, there are M CM-based adaptive detectors that independently suppress the MAI. The algorithm presented in Table 1 can be applied for filter adaptation for each element of the array. After the blind tracking period terminates, the binary demodulation decision is made by combining the outputs of the adaptive filters in the array altogether. To achieve best performance, the combiner multiplies each filtered output by the corresponding amplitude of the desired user signal [9]. This approach enables us to weight the signal by a factor proportional to the signal strength to achieve maximum combination gain. After the weighting operation is performed, the decision is made by taking the sign of the sum of the weighted outputs, i.e.,

$$\hat{b}_k = \mathrm{sgn}\left[\mathrm{Re}\left(\sum_{m=1}^{M} A_{mk}\mathbf{w}_{mk}^H\mathbf{x}_m\right)\right] \tag{9}$$

where A_{mk} is the received signal amplitude, and $\mathbf{w}_{mk}$ is the filtering vector for the kth user in the mth array element. The above signal processing is often referred to as *maximum ratio combination* (MRC).

It can be observed from (9) that the realization of MRC requires the knowledge of A_{mk} which usually is not available in the receiver. Note that as soon as the adaptive filter of each antenna begins to converge, the MAI present in the filtered outputs is considerably suppressed yet most energy of the desired signal is reserved. Therefore, the amplitude of the filtered output signal is proportional to the strength of the desired signal. Consequently, the ratio of the amplitude of the filtered outputs can be considered as an alternative to disclose the relative signal strength of the desired user. This approach is particularly effective in a CDMA system with moderate near-far problem and high signal-to-noise ration (SNR) because in such a system, the MAI residue in the filtered output is insignificant and the above ratio becomes an accurate resemble of A_{mk}. Finally we remark that the accuracy of the approach can be improved by normalizing the filtering vectors in the receiver and averaging the amplitude of the filtered output over certain time duration.

IV. Simulation Results

Computer simulations were conducted to evaluate the performance of the proposed space-time CM based blind adaptive multiuser detector and to compare it with several existing detectors. Performance was evaluated in terms of SINR in tracking period and bit-error-rate (BER). Based on (8), the SINR for space-time linear detectors is meant to be

$$\mathrm{SINR} = \frac{\sum_{m=1}^{M} A_{mk}^2(\mathbf{w}_{mk}^T\mathbf{s}_k)^2}{\sum_{m=1}^{M}\left[\sigma^2\|\mathbf{w}_{mk}\|^2 + \sum_{k=2}^{K} A_{mk}^2(\mathbf{w}_{mk}^T\mathbf{s}_k)^2\right]} \tag{10}$$

The demodulation performance of the CM-based and the CMOE space-time blind adaptive detectors [6] with respect to signature estimation error was also investigated in our simulations.

We considered an eight-user DS-CDMA system where each interference signal was 6dB stronger than the desired user. The user signatures used were the Gold codes of length 15. The DOAs of six users are evenly distributed from [20° to 145°], and the estimation error of the desired user signature was modeled as a vector of independent and identically distributed (iid) complex Gaussian variables with zero mean and a variance of 0.05.

The SINR in the tracking period of the MMSE, CMOE and CM-based space-time detectors was plotted in Fig. 3, where the receivers were equipped with a 3-element antenna array with an inter-element spacing of $d = 2\lambda$. The SNRs of desired user signal for the three antennas were 10, 10.5 and 11dB.

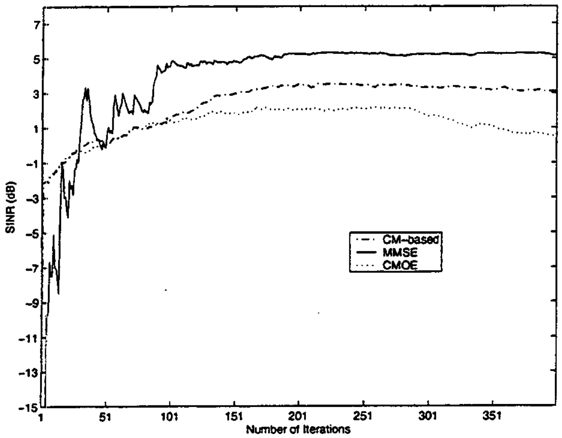

Fig. 3. SINR in the tracking period of three space-time detectors with a 3-element array.

From Fig. 3 it is observed that as soon as the adaptive algorithms begin to converge, the CM-based space-time detector demonstrates a higher SINR compared with that of the CMOE detector. Due to the presence of signature estimation error, however, both the CM-based and CMOE blind adaptive detector offer lower SINR than that of the training-sequence-based MMSE detector.

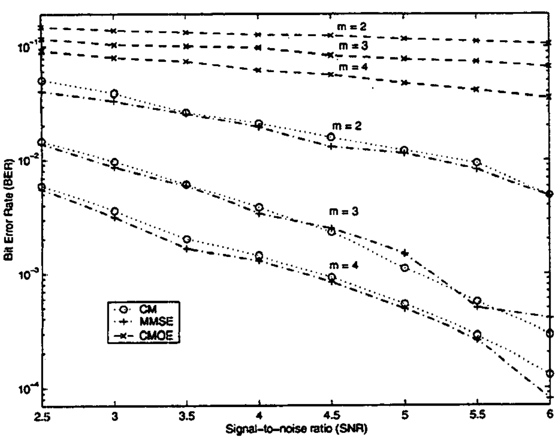

Fig. 4. BER of eight-user CDMA system versus SNR of desired user with m antennas.

Next, the demodulation performance of the CM-based, CMOE, and training-sequence-based MMSE detectors were evaluated versus the number of antennas used, which varies from 2 to 4 with an inter-element spacing of $d = 4\lambda$ and the relative signal strength for the desired user of different antennas were set to 1, 1.1, 1.2, 0.8, respectively. The detection vectors of the three detectors were obtained after 500 tracking iterations and the MRC weights were obtained by averaging the amplitude of filtered output over 100 symbols. The obtained BER of three space-time detectors were plotted in Fig. 4.

It is observed from Fig. 4 that the CM-based space-time detector demonstrates practically the same performance as that of the training-sequence-based MMSE detector. It is also observed that compared with the CMOE detector, the proposed CM-based blind detector shows a much improved BER performance.

V. Conclusions

In this paper, the CM based space-time multiuser detector has been investigated. From the analysis and simulation results presented, we have seen that the CM based space-time multiuser detector offers a significant capacity gain with the presence of estimation error of signature for desired user.

Acknowledgement

The authors are grateful to PMC-Sierra, Micronet, NCE Program, and the Natural Sciences and Engineering Research Council of Canada for supporting this work.

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