Blind Adaptive Multiuser Detection Using a Vector Constant-Modulus Approach

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Abstract

A vector constant-modulus (VCM) criterion based space-time blind adaptive detector that utilizes spatial diversity provided by an antenna array to suppress multiple-access-interference in direct-sequence code-division multiple-access (DS-CDMA) systems is proposed. The proposed detector is characterized in terms of the solution of a constrained optimization problem whose objective function is formed based on the VCM criterion. Our simulation results show that the demodulation performance of the proposed detector is close to that of several existing space-time detectors such as the constrained minimum-output-energy and the constant-modulus based detectors. However, the number of coefficients involved in the proposed adaptive detector is reduced significantly and, therefore, the convergence rate is increased.

1. Introduction

Multiple-access interference (MAI) due to signature nonorthogonality constitutes a major factor that limits the performance of direct-sequence code-division multiple- access (DS-CDMA) systems. Many signal-processing techniques have been proposed to suppress MAI, most of which fall into two categories: multiuser detection algorithms [1] and space-time-processing based techniques [2]. The class of space-time blind adaptive detectors combines the above two techniques together and offers improved demodulation performance while requiring almost no additional information other than that required by the traditional matched-filter (MF) detector. As shown in [3] and [4] where the constrained minimum-optput-energy (CMOE) and the constant-modulus (CM) based detectors were proposed, these detectors typically involve a large number of taps in the adaptation, which leads to slow convergence, especially in CDMA systems with severe near-far problems.

In this paper, we propose a novel space-time blind adaptive approach based on the *vector constant-modulus* (VCM) criterion. In the proposed approach, the detector is characterized in terms of the solution of a constrained optimization problem whose objective function is formed based on the VCM criterion. Our simulation results show that the demodulation performance of the proposed detector is comparable with that of several existing space-time detectors such as the CMOE and the CM based detectors. However, the number of coefficients involved in the VCM based detector is reduced and, consequently, the convergence rate is increased.

2. Signal Model

Consider a K-user synchronous DS-CDMA system. The signature waveform of user k is given as

$$s_k(t) = \sum_{n=1}^{N} c_k^n p_c[t-(n-1)T_c] \tag{1}$$

where T_c is the chip interval, T_b is the bit interval, $N = T_b/T_c$ is the processing gain, $\mathbf{s}_k = [c_k^1\ c_k^2\ \cdots\ c_k^N]$ is the spreading code of the kth user, and $p_c(t)$ is the chip waveform that assumes nonzero value only on interval $[0, T_c]$. Typically $s_k(t)$ is normalized to unit energy, i.e., $\int_0^{T_b} s_k^2(t)\, dt = 1$. The transmitted baseband signal for the kth user on one symbol interval is expressed as

$$x_k(t) = \sum_{n=1}^{N} A_k b_k c_k^n p_c[t-(n-1)T_c] \tag{2}$$

where A_k is the amplitude and $b_k \in \{1, -1\}$ is the information bit of the kth user signal.

Assuming that an antenna array of M elements is used at the receiver, the received signal from the M-element array constitutes a vector $\mathbf{y}(t) = [y_1(t)\, y_2(t)\ \cdots\ y_M(t)]^T$ which can be expressed as

$$\mathbf{y}(t) = \sum_{k=1}^{K}\sum_{n=1}^{N} A_k b_k \mathbf{a}_k c_k^n p_c[t-(n-1)T_c] + \mathbf{n}(t) \tag{3}$$

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In (3), $\mathbf{a}_k = [a_{1k}\ a_{2k}\ \cdots\ a_{Mk}]^T$ is a complex-valued vector with a_{mk} representing the response of the mth element to the kth user signal and $\mathbf{n}(t) = [n_1(t)\ n_2(t)\ \cdots\ n_M(t)]^T$ is a vector of a complex-valued independent white Gaussian noise process with zero mean and cross-correlation matrix $\mathcal{E}\left[\mathbf{n}(t)\mathbf{n}^H(t)\right] = \sigma^2\mathbf{I}$, where $\mathcal{E}(\cdot)$ denotes the expectation operation.

The demodulation begins when the received signal passes through a bank of chip-rate matched filters (MF) and chip-rate samplers that yield

$$\mathbf{Y} = \sum_{k=1}^{K} A_k b_k \mathbf{S}_k + \mathbf{N} \tag{4}$$

where $\mathbf{S}_k = \mathbf{s}_k\mathbf{a}_k^H$ with $\mathbf{s}_k = [c_k^1\ c_k^2\ \cdots\ c_k^N]^T$ being the space-time signature of the kth user signal, $\mathbf{N} = [\mathbf{n}_1\ \mathbf{n}_2\ \cdots\ \mathbf{n}_M]$ with $\mathbf{n}_m$ being the complex-valued zero-mean Gaussian noise vector received from the mth array element, and $\mathbf{Y} = [\mathbf{y}_1\ \mathbf{y}_2\ \cdots\ \mathbf{y}_M]$ is the sampled signal matrix with $\mathbf{y}_m = \sum_{k=1}^{K} A_k b_k \mathbf{s}_k a_{mk}^* + \mathbf{n}_m$ being the output of the mth sampler.

3. VCM Based Space-Time Adaptive Detector

3.1 VCM Based Space-Time Detector

The VCM criterion was introduced recently as an extension of the well-known CM criterion for the equalization of nearly Gaussian distributed source data [5][6]. As stated in [7], the VCM criterion can be expressed as a composite criterion combining the CM objective function and a penalty term involving cross-correlations of the equalizer output.

If we denote the source data and the channel impulse response at the nth time interval as $\mathbf{a}_n$ and $\mathbf{h}_n$, respectively, then the channel output can be expressed as $\mathbf{y}_n = \mathbf{a}_n * \mathbf{h}_n$ where '$*$' denotes discrete convolution. If $\mathbf{c}_n = [c_n^1\ c_n^2\ \cdots c_n^{L_c}]^T$ is the equalizer vector of length L_c, then the equalized output of a VCM based detector, $\mathbf{z}_n$, is formed as the convolution of $\mathbf{y}_n$ and $\mathbf{c}_n$, i.e., $\mathbf{z}_n = \mathbf{y}_n * \mathbf{c}_n = \mathbf{c}_n^H\mathbf{Y}_n$, where

$$\mathbf{Y}_n = \begin{bmatrix} y_n & y_{n-1} & \cdots & y_{n-N+1} \\ y_{n-1} & y_{n-2} & \cdots & y_{n-N} \\ \cdots & \cdots & \cdots & \cdots \\ y_{n-L_c+1} & y_{n-L_c} & \cdots & y_{n-L_c-N+2} \end{bmatrix} \tag{5}$$

The equalizer vector $\mathbf{c}$ is determined by minimizing the objective function

$$\mathcal{J}_c(\mathbf{c}) = \mathcal{E}[||\mathbf{c}^H\mathbf{Y}_n||^2 - \alpha]^2 \tag{6}$$

where $\alpha = \mathcal{E}\{\mathbf{a}^4\}/\mathcal{E}\{\mathbf{a}^2\}$ is a constant for the source data modulus. The major difference between VCM based and CM based equalizers is that the former involves a vector which is a collection of the equalizer's outputs whose modulus is forced to be a constant while the latter only involves one output of the equalizer, whose modulus is forced to be a constant.

We now consider a VCM based blind adaptive space-time detector where the detector coefficients, $\mathbf{w}_k$, for the kth user signal demodulation are determined as the solution of the constrained optimization problem

$$\text{minimize}\quad \mathcal{E}[||\mathbf{w}_k^H\mathbf{Y}||^2 - \alpha_k]^2 \tag{7a}$$

$$\text{subject to}:\quad \mathbf{w}_k^H\mathbf{s}_k = 1 \tag{7b}$$

In (7), $\mathbf{Y}$ is as defined in (4) and $\alpha_k = A_k^2|\mathbf{a}_k|^2$ is the constant modulus of the source data of user k. Once the detector coefficients $\mathbf{w}_k$ are obtained, the demodulation output can be obtained as

$$\hat{b}_k = \text{sign}[\text{Re}(\mathbf{w}_k^H\mathbf{Y}\mathbf{a}_k)] \tag{8}$$

where sign$(\cdot)$ and Re$(\cdot)$ denote the sign and real part of the quantity involved, respectively. Using the expression of $\mathbf{Y}$ in (4), it can be shown that the constraint in (7b) assures that the desired signal is detected while the effect of MAI is suppressed.

3.2 Adaptive Implementation

A more practical way to implement the VCM based detector characterized by the solution of (7) is to develop an adaptation algorithm. For instance, one can use a normalized stochastic gradient update formula to update the detector's coefficient vector $\mathbf{w}_k$, where $\mathbf{w}_k$ is divided into a fixed part and a variable part. A step-by-step description of the adaptation algorithm is given in Table 1.

In this algorithm, step-size μ_n is chosen to minimize the objective function $\mathcal{J}(\mu_n) = [||\mathbf{w}_k^H(n+1)\mathbf{Y}(n)||^2 - \alpha_k]^2$. In a practical implementation, the value of μ_n is multiplied by a small positive scalar to control the convergence behaviour of the algorithm.

3.3 Convergence of the CMA Based Detector

In order to investigate the convergence of the proposed algorithm, we evaluate the Hessian matrix of the objective function. To this end, we write the objective function in (7) as

$$\begin{aligned} \mathcal{J}(\mathbf{w}_k) &= \mathcal{E}(||\mathbf{w}_k^H\mathbf{Y}||^2 - \alpha_k)^2 \\ &= \mathcal{E}(\sum_{m=1}^{M}|\mathbf{w}_k^H\mathbf{y}_m|^2 - \alpha_k)^2 \\ &= \mathcal{E}[\mathbf{w}_k^H(\sum_{m=1}^{M}\mathbf{y}_m\mathbf{y}_m^H)\mathbf{w}_k - \alpha_k]^2 \end{aligned}$$

Table 1: Normalized adaptation algorithm for the problem in (7).

Parameters: $\mathbf{Y}(n)$:	input signal
$\mathbf{w}_k(n)$:	detection vector
$\mathbf{s}_k$:	kth user signature vector
μ_n:	step size

Initialize the algorithm with:

$$n = 1, \ \mathbf{x}(1) = \mathbf{0}$$

At time instant $n = 1, 2, \ldots,$ compute

$$\mathbf{w}_k(n) = \mathbf{s}_k + \mathbf{x}(n)$$
$$\mathbf{Z} = \mathbf{Y}^H(n)\mathbf{w}_k(n)$$
$$e = ||\mathbf{Z}||^2 - \alpha_k$$
$$\mathbf{Z}_m = \mathbf{Y}(n)\mathbf{Z}$$
$$\mathbf{Z}_g = \mathbf{Z}_m - (\mathbf{s}_k^T\mathbf{Z}_m)\mathbf{s}_k$$
$$\mu_n = \text{abs}\left(|\mathbf{Z}_m^H\mathbf{Z}_g|/(e \cdot ||\mathbf{Z}_g^H\mathbf{Y}(n)||^2)\right)$$
$$\mathbf{x}(n+1) = \mathbf{x}(n) - \mu_n \cdot e\,\mathbf{Z}_g$$

$$\begin{aligned} &= \mathcal{E}(\mathbf{w}_k^H\hat{\mathbf{Y}}\mathbf{w}_k - \alpha_k)^2 \\ &= \mathcal{E}[\hat{\mathcal{J}}(\mathbf{w})_k] \end{aligned} \tag{9}$$

where $\mathbf{y}_m$ is the mth column vector of $\mathbf{Y}$, $\hat{\mathbf{Y}} = \sum_{m=1}^{M} \mathbf{y}_m\mathbf{y}_m^H$, and $\hat{\mathcal{J}}(\mathbf{w})_k = (\mathbf{w}_k^H\hat{\mathbf{Y}}\mathbf{w}_k - \alpha_k)^2$. If we use $\nabla^2\hat{\mathcal{J}}(\mathbf{w}_k)$ to denote the Hessian matrix of $\hat{\mathcal{J}}(\mathbf{w}_k)$, then it follows that

$$\begin{aligned} \nabla^2\hat{\mathcal{J}}(\mathbf{w}_k) &= \nabla^2[\mathcal{E}(\mathbf{w}_k^H\hat{\mathbf{Y}}\mathbf{w}_k - \alpha_k)^2] \\ &= \mathbf{H}_1 + \mathbf{H}_2 \end{aligned} \tag{10}$$

where

$$\begin{aligned} \mathbf{H}_1 &= 4\mathcal{E}\left[\left(\mathbf{w}_k^H\hat{\mathbf{Y}}\mathbf{w}_k - \alpha_k\right)\hat{\mathbf{Y}}\right] \\ \mathbf{H}_2 &= 8\mathcal{E}\left(\hat{\mathbf{Y}}^H\mathbf{w}_k\mathbf{w}_k^H\hat{\mathbf{Y}}\right) \end{aligned}$$

For any nonzero complex-valued column vector $\mathbf{x}$, it is evident that

$$\begin{aligned} \mathbf{x}^H\mathbf{H}_2\mathbf{x} &= \mathcal{E}[\mathbf{x}^H\mathbf{Y}^H\mathbf{w}_k\mathbf{w}_k^H\mathbf{Y}\mathbf{x}] \\ &= \mathcal{E}[||\mathbf{w}_k^H\mathbf{Y}\mathbf{x}||^2] \geq 0 \end{aligned}$$

which implies that matrix $\mathbf{H}_2$ is positive semidefinite. Furthermore, it can be shown that if the inequality

$$4M(M+2)\sigma^4 - \sum_{k=1}^{K}\sum_{\substack{l=1 \\ l\neq k}}^{K}\left|\sum_{m=1}^{M} A_k^2 a_{mk}a_{ml}^*\right|^2 > 0 \tag{11}$$

holds, then $\mathbf{H}_1$ becomes positive definite and hence the Hessian matrix of the objective function is positive definite. Note that for systems where the channel noise is sufficiently large (i.e., large noise variance σ^2), the first term in (11) becomes large and so (11) can be readily satisfied. In other words, the objective function is very likely convex when the channel noise is not negligible. In such a case, the convergence of the proposed adaptation algorithm is guaranteed to converge to the global minimizer.

4. Computer Simulations

Computer simulations were conducted to evaluate the performance of the proposed VCM based detectors. To compare the tracking capability of various adaptive detectors, the signal-to-interference-plus-noise ratio (SINR) in the tracking period, which is defined as

$$\text{SINR} = \frac{||\mathbf{w}_k^H\mathbf{s}_k||^2A_k^2}{\sigma^2||\mathbf{w}_k||^2 + \sum_{\substack{i=1 \\ i\neq k}}^{K} A_k^2||\mathbf{w}_k^H\mathbf{s}_k||^2} \tag{12}$$

was used as a performance measure.

An 8-user DS-CDMA system with Gold signatures of length 15 was considered. It was assumed that the directions of arrival (DOA) of the 8 users were uniformly distributed from 36° to 144° and the receivers were used with an antenna array whose inter-element distance was half of the wavelength of the wireless signal. Using the plane-waveform model [1], the array response a_{mk} was expressed as

$$a_{mk} = \exp[-j\pi(m-1)\cos(\alpha_k)]$$

where α_k denotes the DOA of the kth user signal. The SNR of the desired user signal was assumed to be 12 dB. The SINR of the MMSE, CMOE, CM based and VCM based adaptive detectors as applied to the above system are plotted in Figs. 1 and 2, where in Fig. 1 the number of antennas, m, was fixed at 3, and the two cases for MAI being 30 dB and 60 dB stronger relative to the desired user signal, are shown in parts (a) and (b), respectively. In Fig. 2, the MAI was fixed at 40 dB stronger relative to the desired user signal and the two cases with m being 3 and 6 are shown in parts (a) and (b), respectively.

It is observed that the VCM based detector has a high tracking speed compared with the other three detectors regardless of the MAI presented. In addition, it is noted that when the number of antennas increases, the tracking speeds of the VCM based adaptive detector and other adaptive detectors differ even more significantly.

Next the noise effect in the detectors was investigated. For a fair comparison, the step-size for each detector was rescaled such that the tracking speeds of all the detectors were almost the same. Fig. 3 shows the after-convergence SINR for the three blind detectors, namely, the CMOE, CM based, and VCM based detectors where the two cases with $m = 3$ and $m = 6$ are shown separately. It is observed that owing to the increasing number of coefficients involved, when the number of antennas increases,

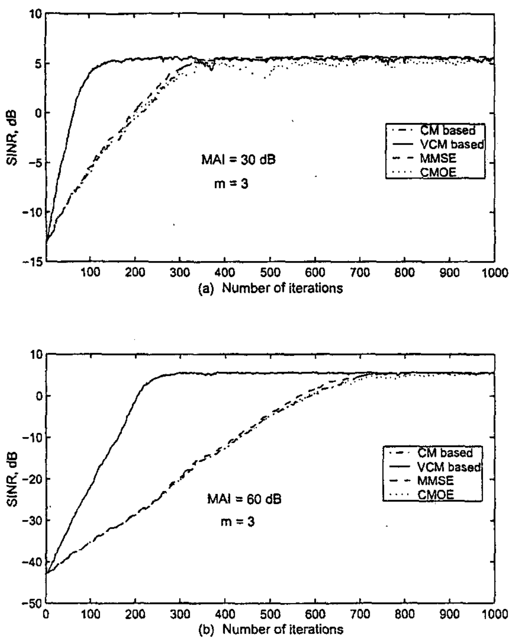

Figure 1: SINR in the tracking period of three space-time detectors. (a) Each interference signal was 30 dB stronger relative to the desired signal; (b) each interference signal was 60 dB stronger relative to the desired one. Receivers used 3-element arrays.

the noise effect of the CMOE and CM based detectors increases significantly. On the other hand, the noise effect of the VCM based detector remains almost the same for both cases.

Finally the demodulation performance of the detectors was investigated. In our simulations, each interference signal was set at 60 dB stronger relative to the desired one and the number of antennas m at the receiver varied from 2 to 4. The bit-error-rates (BER) obtained is plotted in Fig. 4. It is observed that the error probability of the VCM based detector approaches that of the training-sequence based MMSE detector, and is lower than that of the other two blind detectors when the number of antennas used is large.

5. Conclusions

A VCM based space-time blind adaptive multiuser detector was proposed. In the new approach, the detector is characterized in terms of the solution of a constrained optimization problem whose objective function is constructed based on the VCM criterion. Our simulation results show that the demodulation performance of the proposed detector is comparable with that of several existing space-time detectors such as the CMOE and the CM based detectors. However, the number of coefficients involved in the VCM based detector is reduced significantly and, therefore, the convergence rate of proposed detector is increased.

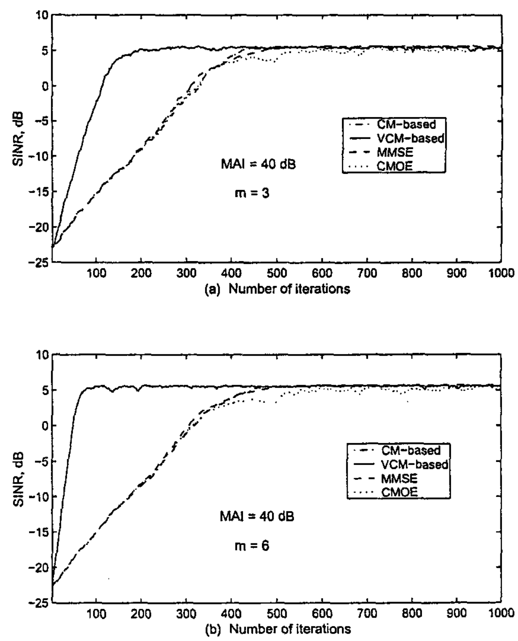

Figure 2: SINR in the tracking period of three space-time detectors. (a) Receivers used 3-element arrays; (b) receivers used 6-element arrays. In both cases, each interference signal was 40 dB stronger relative to the desired one.

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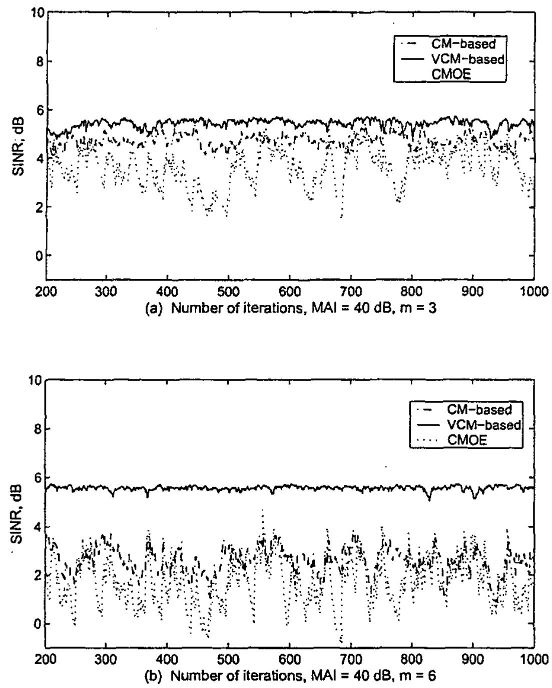

Figure 3: Noise effect of blind adaptive detectors. (a) Receivers used 3-element arrays; (b) receivers used 6-element arrays. In both cases, each interference signal was 40 dB stronger relative to the desired one.

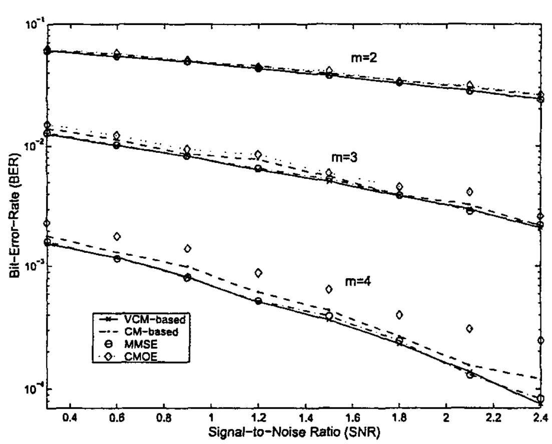

Figure 4: Bit-error-rate of the detectors.

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