DESIGN AND IMPLEMENTATION OF LOW-POWER IIR DIGITAL FILTER SYSTEMS

S. Saab, W.-S. Lu, and A. Antoniou

Department of Electrical and Computer Engineering
University of Victoria
PO Box 3055 Stn CSC, Victoria, BC V8W 3P6, Canada

ABSTRACT

A method for the design of linear-phase IIR digital filters for low-power applications is proposed. In this method, the digital filter is implemented as a cascade arrangement of 2nd-order sections. Each section is designed through optimization techniques so that all sections in cascade satisfy as far as possible the overall required specifications. This process is repeated until a multisection filter is obtained which satisfies the required specifications under the most critical circumstances imposed by the application at hand. The minimum number of sections required to process a particular input signal can then by switched on through the use of a simple adaptation mechanism and, in this way, the power consumption can be minimized. This design structure is achieved by formulating the design of the $k-1$ sections as constraints. As an example, a low-power filter system is compared with a fixed-order linear-phase IIR filter of the same performance. It is shown that a power reduction of at least 10% can be achieved.

1. INTRODUCTION

Digital filters are commonly designed to assure a desirable performance for all possible signals that may be encountered. However, for many DSP applications the worst-case conditions are rarely present, which leads to wasted power and computational resources. In other circumstances, electrical systems such as portable wireless systems are dependent on power availability whereas systems such as multitasking systems are dependent on the availability of computational resources. Recent studies have shown that these dependencies can be alleviated by dynamically adjusting the complexity of the algorithms used according to predetermined criteria and the changing signal characteristics [1]–[5].

In [5], a sequential design of IIR digital filters for low-power DSP applications was proposed. In this paper, a variation of this method is described whereby a filter system is designed as a collection of 2nd-order cascaded sections that can be turned off when they are not needed so as to conserve power and/or reduce the computational complexity. The problem is set up as a sequential quadratic function whereby each new section is designed assuming that the previous sections are available while imposing constraints to assure a stable section that satisfies the prescribed specifications. An example signal is constructed from two speech signals modulated at different carrier frequencies, and an algorithm proposed in [2] is implemented as the decision device to turn off the appropriate sections. The method was used to design a low-power filter system which is compared with a fixed-order linear-phase IIR filter that has the same performance.

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2. PROBLEM FORMULATION

Consider an IIR digital filter represented by

$$H(z) = \frac{B(z)}{A(z)} = \prod_{i=1}^{K} \frac{b_i(z)}{a_i(z)}$$

with

$$\frac{b_i(z)}{a_i(z)} = \frac{b_{i_0} + b_{i_1} z^{-1} + b_{i_2} z^{-2}}{1 + a_{i_1} z^{-1} + a_{i_2} z^{-2}}$$

The frequency response of the filter can be examined by performing the substitution $z = e^{j\omega T}$ where ω is the frequency of the signal in rad/s, and T is the sampling period, which will be assumed to be 1 s throughout.

When the kth stage is to be designed, it is assumed that the sections represented by $b_i(z)/a_i(z), i = 1, \ldots, k-1$ have been designed and one seeks a stable 2nd-order section with a transfer function $b_k(z)/a_k(z)$ such that the objective function

$$\Psi(\mathbf{x}) = \int_{-\pi}^{\pi} W_k(\omega) |e(\omega, \mathbf{x})|^2 d\omega \qquad (1)$$

is minimized where $W_k(\omega)$ is a weighting function,

$$e(\omega, \mathbf{x}) = H_k(e^{j\omega}) - D_k(\omega)$$

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$$H_k(z) = H_{k-1}(z)\frac{b_k(z)}{a_k(z)}$$
$$\mathbf{x} = \begin{bmatrix}\mathbf{b}_k^{\mathrm{T}} & \mathbf{a}_k^{\mathrm{T}} & \tau_k\end{bmatrix}^{\mathrm{T}}$$
$$\mathbf{b}_k = \begin{bmatrix}b_{k_0} & b_{k_1} & b_{k_2}\end{bmatrix}^{\mathrm{T}}$$
$$\mathbf{a}_k = \begin{bmatrix}a_{k_1} & a_{k_2}\end{bmatrix}^{\mathrm{T}}$$

and

$$D_k(\omega) = M(\omega)e^{-j\omega\tau_k}$$

where $M(\omega)$ is the desired amplitude response, and τ_k is the desired passband delay.

The design method can be better described by considering the case where $D_k(\omega)$ represents a lowpass frequency response

$$D_k(\omega) = \begin{cases} e^{-j\omega\tau_k} & \omega \in [0,\ \omega_p] \\ 0 & \omega \in [\omega_a,\ \pi] \end{cases}$$

The objective function in this case is given by

$$\Psi(\mathbf{x}) = \int_0^{\omega_p} \left| H_{k-1}(e^{j\omega})\frac{b_k(e^{j\omega})}{a_k(e^{j\omega})} - e^{-j\tau_k\omega} \right|^2 d\omega + w\int_{\omega_a}^{\pi} \left| H_{k-1}(e^{j\omega})\frac{b_k(e^{j\omega})}{a_k(e^{j\omega})} \right|^2 d\omega \qquad (2)$$

where the weighting function assumes the form

$$W_k(\omega) = \begin{cases} 1 & \omega \in [0,\ \omega_p] \\ w & \omega \in [\omega_a,\ \pi] \\ 0 & \text{elsewhere} \end{cases}$$

It is known that if the scalar weight w for the stopband is sufficiently large, say $w \geq 1000$, and if a sufficient number of constraints on the filter's frequency response in the passband are imposed, an IIR filter with near-equiripple passband and least-squares stopband can be designed [6][7]. These constraints can be expressed as

$$\left| H_{k-1}(e^{j\omega})\frac{b_k(e^{j\omega})}{a_k(e^{j\omega})} - e^{-j\tau_k\omega} \right| \leq \delta_p \qquad (3)$$

for $\omega \in \{\omega_1, \ldots, \omega_N\} \subset [0,\ \omega_p]$ where δ_p denotes the passband ripple. Additional constraints on $a_k(z)$ must be imposed to assure the filter's stability. For the 2nd-order section, these constraints are simple and linear and are given by

$$\begin{aligned} a_{i2} &< 1 \\ a_{i1} - a_{i2} &< 1 \\ a_{i1} + a_{i2} &> -1 \end{aligned} \qquad (4)$$

(see p. 524 of [8]).

The design problem at hand can now be formulated as the constrained optimization problem

$$\text{minimize } \Psi(\mathbf{x}) \text{ in (2)}$$
$$\text{subject to the constraints in (3) and (4)}$$

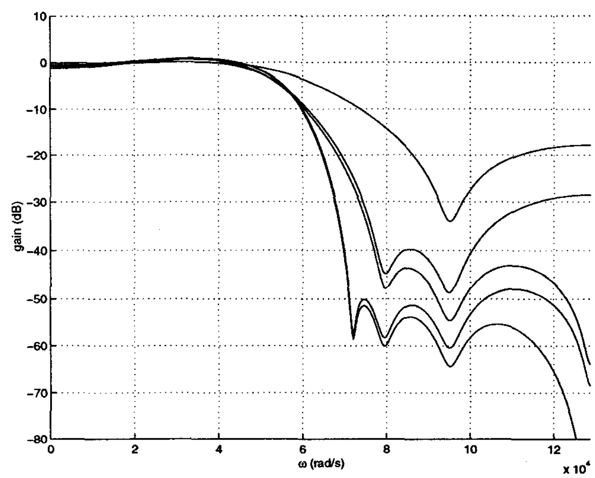

Figure 1: Amplitude responses of one, two, three, four, and five 2nd-order filters in cascade.

Evidently, both the objective function in (2) and the constraints in (3) are highly nonlinear. For a satisfactory design, the number of frequencies involved in (3) is typically in the range of 20 to 40. So if one takes $N = 30$, then the optimization problem has six parameters and involves 33 constraints. The sequential quadratic programming (SQP) method (see Ch. 12 of [9]) has been used to solve the optimization problem.

3. OPTIMIZATION PROCEDURE

In an SQP algorithm, the objective function is typically a quadratic approximation of the Lagrangian of the original problem, which is minimized subject to a set of linear constraints obtained by a linear approximation of the constraints in the original problem. The solution of the quadratic programming (QP) problem is then used as the point at which an updated quadratic approximation of the Lagrangian as well as updated linear approximations of the constraints are obtained. In this way, one solves a sequence of QP problems formulated using the method in [5], and the solution of these subproblems is shown to converge to a solution of the original problem under fairly mild conditions.

4. DESIGN EXAMPLE

The above method was used to design a five-section low-power IIR filter system with $\omega_p = 4.83 \times 10^4$ rad/s, $\omega_a = 7.08 \times 10^4$ rad/s, and $\omega_s = 2.57 \times 10^5$ rad/s. The amplitude responses obtained for one, two, three, four, and five sections in cascade are plotted in Fig. 1.

5. LOW-POWER ADAPTATION MECHANISM DESIGN

Once the sections have been designed, a low-power adaptation mechanism can be used to decide the minimum number of sections required to provide a satisfactory performance. One such mechanism [2] approximates the input and output power spectrums in consecutive windowed intervals as

$$\hat{P}_x = \frac{1}{L}\sum_{k=0}^{L-1} x^2[n-k]$$

$$\hat{P}_y[N_0] = \frac{1}{L}\sum_{k=0}^{L-1} y^2[n-k]$$

where L is the interval window, x is the input signal, y is the output signal, N_0 is the number of sections that are on, and n is the last sample of the window. It can then be shown that the number of sections (N) required to achieve a predetermined output SNR tolerance ($\mathrm{OSNR_{tol}}$) should satisfy [2]

$$P_y[N_0] - (P_x - P_y[N_0])P_h[N_0] \geq \mathrm{OSNR_{tol}}(P_h[N])(P_x - P_y[N_0])$$

where

$$\mathrm{OSNR}[N_0] \equiv \frac{P_y^{PB}[N_0]}{P_y^{SB}[N_0]}$$

$$P_y^{PB}[N_0] = \frac{1}{2\pi}\int_0^{\omega_p} S_x(e^{j\omega})|H_{N_0}(e^{j\omega})|^2 d\omega$$

$$P_Y^{SB}[N_0] = = \frac{1}{2\pi}\int_{\omega_a}^{0.5\omega_s} S_x(e^{j\omega})|H_{N_0}(e^{j\omega})|^2 d\omega$$

$$P_h[N_0] = \frac{\int_{\omega_a}^{0.5\omega_s} |H_{N_0}(e^{j\omega})|^2 d\omega}{\int_0^{\omega_p}[1-|H_{N_0}(e^{j\omega})|^2]d\omega}$$

$S_x(e^{j\omega})$ is the input power spectrum and $H_{N_0}(e^{j\omega})$ is the frequency response of N_0 cascaded sections.

By creating a lookup table of values $\mathrm{OSNR_{tol}}(P_h[N])$ for $N = \{1 \ldots K\}$, it can be shown that an accurate estimate of the number of sections required can be calculated using less power than that required by a single 2nd-order recursive digital filter.

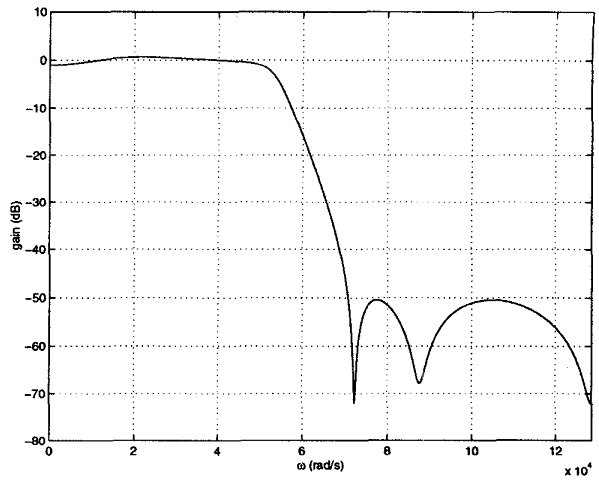

Figure 2: Reference 6th-order lowpass filter.

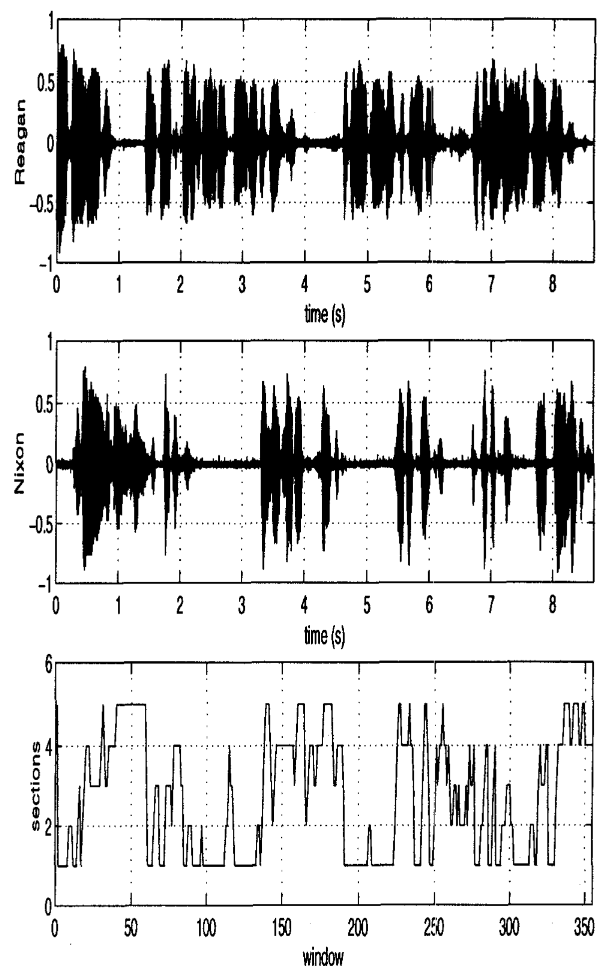

Figure 3: Number of sections required in each window.

6. LOW-POWER IMPLEMENTATION SIMULATION

Using input speech samples of Ronald Reagan and Richard Nixon modulated at 3.1×10^4 and 9.4×10^4 rad/s, respectively, with a spread of 5.1×10^4 rad/s each, the multisection low-power filter characterized by Fig. 1 was compared with a fixed 6th-order linear-phase recursive digital filter designed using the method in [10]. The amplitude response of the fixed 6th-order filter is shown in Fig. 2. Both filter systems filtered out the Nixon signal. Fig. 3 displays the two speech samples along with the number of sections used within each interval window of 1000 samples. The minimum ratio of the output passband to stopband power spectrum was set to

$$\frac{\omega_p}{0.5\omega_s - \omega_a} 10^{0.1A_a}$$

where A_a is the desired stopband attenuation in dB. The average number of sections used was 2.7. This corresponds to a 10.4% power reduction compared to the power required by the fixed filter, which satisfied the same OSNR requirements. In addition, typical communication channels have periods of no signal. This would greatly increase the percentage of power reduction.

7. CONCLUSION

A method for the design of linear-phase IIR digital filters for low-power applications has been proposed. The problem is set up as a sequential quadratic function whereby each new section is designed, assuming that the previous sections are available, while imposing constraints to assure a stable section that satisfies the prescribed specifications. This allows for quick design time using a quadratic programming algorithm. The resulting designs exhibit the desirable steep transition response inherent in IIR digital filters. Using the resulting filter system with a simple adaptive mechanism to determine the required number of sections to power up, savings of 10% compared to a fixed order linear-phase IIR digital filter were measured.

8. REFERENCES

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