Design and Application of an Optimum Orthogonal Wavelet Filter for Echo Cancellation

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Abstract— Filter banks used for subband decomposition in echo cancellation need to be perfect recontruction filter banks, and to achieve better performance of the echo canceller, the filters need to be perfect half-band filters. A method of designing filter banks for use in echo cancellation is proposed. This method is based on linear parametrization of half-band filters. The main advantage of using this method is the orthogonality of the resultant filter banks. Experimental results based on simulated echo are given and discussed.

I. Introduction

Echoes are a serious problem in data transmission and acoustics since they affect the performance of the receivers. The usual approach for cancelling echoes is to estimate the echo using the transmitted signal $x[n]$ and subtract it from the received signal $s[n]$. Adaptive filters have been successfully used to model the impulse response of the echo path. The filters are adapted so that the transfer function of the adaptive filter $\hat{W}$ approximates the transfer function of the echo path W. One of the main concerns about the performance of the adaptive filter is its rate of convergence which is dependant on the length of the adaptive filter. When the echo path has a long impulse response, a high-order adaptive filter is required and this reduces the rate of convergence. Also the computational complexity required for each iteration greatly increases with the filter length. One way to solve this problem is to divide the received signal into subbands using filter banks and have an adaptive filter in each subband to cancel the echo. Due to the downsampling of the signal, the length of the signal in each subband is reduced thus reducing the length of the adaptive filter required. This results in an improved rate of convergence as well as reduced computational complexity.

Wavelet filters are an ideal choice for the design of the filter banks because of the orthogonality property. Orthogonality is a desirable characteristic since it results in a smaller number of computations for the synthesis filters and it also results in perfect-reconstruction filter banks. In a perfect-reconstruction filter bank, the aliasing caused at the analysis filter bank is cancelled at the synthesis filter bank [1].

II. Echo Cancellation Using Filter Banks

A system that can be used for echo cancellation is shown in the Fig. 1. The configuration shown here is the same as that used for system identification since echo cancellation is a specific case of system identification where we estimate the echo path transfer function. W represents the echo path transfer function and $s[n]$ is the echo produced due to $x[n]$. $\hat{W}$ is the adaptive filter which simulates the echo path W.

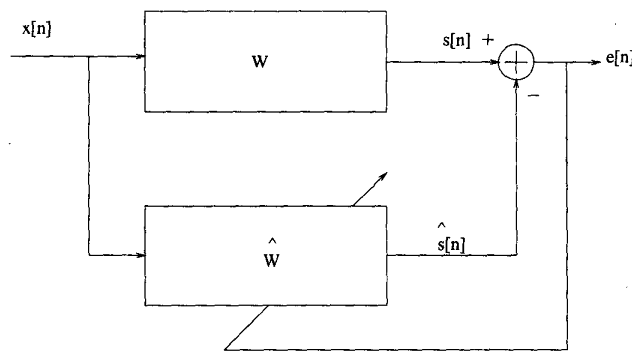

Fig. 1. General scheme for echo cancellation.

In subband adaptive filtering, the input signal and the output signal are split into adjacent frequency subbands by analysis filter banks; then each subband signal is subsampled, and the adaptive filtering algorithm is applied to each of these signals. The identified impulse response in each subband is the system impulse response filtered by the corresponding subband filter. Also, the adaptation step size, i.e. the size of increment or decrement in filter coefficients, can be matched to the energy of the signal in the subbands which results in a better rate of convergence [2].

III. Design of the Filter Banks

Due to the subsampling process, aliased versions of the input may appear at the output, especially when a critically sampled scheme is used [3]. Solutions to this problem have been proposed in [4], where the lowpass filter used in the filter bank is optimized to have most of its energy in the interval $[-\pi/2, \pi/2]$. The approach used in [4] can be outlined as follows.

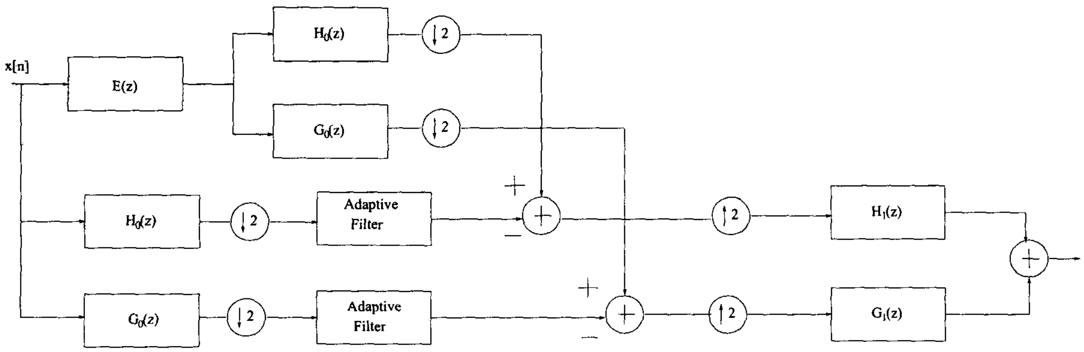

Fig. 2. Echo cancellation using filter banks: A two-band decomposition scheme.

Let $H_0(z)$ and $G_0(z)$ be the lowpass and highpass transfer functions, respectively. $H_0(e^{j\omega T})$ and $G_0(e^{j\omega T})$ are designed to approximate the desired frequency reponses,

$$H_0(e^{j\omega T}) = 0; \quad |\omega| \geq \frac{\pi}{2} \tag{1}$$

and

$$G_0(e^{j\omega T}) = 0, \quad |\omega| \leq \frac{\pi}{2} \tag{2}$$

where T is assumed to be unity.

Also let $\{h_n\}$ and $\{g_n\}$ be the lowpass and highpass FIR filter coefficients in the discrete domain, respectively. The following conditions are tabulated as the conditions for optimization.

- $\{h_n\}$ are chosen such that (1) is satisfied.
- $\{g_n\}$ are chosen such that (2) is satisfied.
- $\{h_n\}$ satisfies the bi-orthogonal condition

$$\sum_n h_{n-2l} h_n = \delta_l \tag{3}$$

where δ_l is the Kronecker delta function.

Based on the above optimization criterion, the problem was converted into the maximization of a penalty function, i.e.

$$\underset{h}{\text{maximize}} \left[\frac{\mathbf{h^T E h}}{\mathbf{h^T h}} + \sum_{l=1}^{\frac{L-1}{2}} \kappa_l \frac{\left(\sum_{n=2l}^{L-1} \mathbf{h}_{n-2l} \mathbf{h}_n\right)^2}{\mathbf{h^T h}} \right] \tag{4}$$

where $\mathbf{h} = [h_0 \; h_1 \; \cdots \; h_{L-1}]^T$, $\mathbf{E} = [\frac{\sin(n-m)\pi/2}{(n-m)\pi}]_{L\times L}$, and κ_l are the Lagrange multipliers for $l = 0, \cdots, (L-1)/2$. Note that the above function does not impose the strict condition of orthogonality and hence the resultant lowpass filter coefficient vectors obtained are not orthogonal.

IV. Design of Optimized Orthogonal Wavelets

Reference [5] describes a method of optimizing the filter bank without loosing the orthogonality property. Since the filters used in a two-band filter bank are half-band filters, their transfer function can be expressed as

$$P(z) = H_0(z) H_0(z^{-1}) \tag{5}$$

and $P(z)$ satisfies the condition

$$P(z) + P(-z) = 2, \quad \forall z \tag{6}$$

When $H_0(z)$ has at least L zeros at $\omega = \pi$, then it takes the form

$$H_0(z) = \left(\frac{1+z^{-1}}{2}\right)^L B_1(z) \tag{7}$$

where $B_1(z)$ is a Kth-order polynomial in z^{-1}. $H_0(z)$ characterizes an orthogonal, lowpass, analysis wavelet filter, whose wavelet has at least L vanishing moments. In this case, $P(z)$ takes the form

$$P(z) = \left(\frac{z^{-1}}{4} + \frac{1}{2} + \frac{z}{2}\right)^L B(z) \tag{8}$$

with

$$B(z) = \sum_{k=-K}^{K} \hat{b}_k z^k, \quad b_{2K-k} = b_k \tag{9}$$

To have (5) and (7) satisfied, $B(z)$ should be factorizable as

$$B(z) = B_1(z) B_1(z^{-1}) \tag{10}$$

This allows us to write $B(z)$ as

$$B(z) = z^{-k} \sum_{k=0}^{2K} b_k z^k, \quad b_{2K-k} = b_k \tag{11}$$

and

$$\left(\frac{z^{-1}}{4} + \frac{l}{2} + \frac{z}{4}\right)^L = z^{-L} \sum_{l=0}^{2L} a_l z_l, \quad a_{2L-l} = a_l \tag{12}$$

Reference [5] goes on to show that the condition for $P(z)$ to satisfy (6) is

$$\sum_{\substack{l+k=s \\ l\geq 0, g\geq 0}} a_l b_k = \begin{cases} 1, & s = N \\ 0, & s = 1, 3, \ldots, N-2 \end{cases} \tag{13}$$

where $N = K + L$ is assumed to be odd. We can now write (13) as

$$\mathbf{Ab} = \mathbf{m} \tag{14}$$

where $\mathbf{A}$ is an $(N+1)/2 \times (k+1)$ matrix determined by the a_l's, $\mathbf{b}$ is a column vector of b_k's of length $K+1$ and $\mathbf{m} = [0 \cdots 1]^{\mathrm{T}}$ with length $(N+1)/2$. Integer $\eta = (K-L+1)/2$ is the number of degrees of freedom of the above equation and it depends on the number of vanishing moments we would like the wavelet to have. Assuming that $\eta \geq 1$, the system of linear equations in (26) has solutions of the type

$$\mathbf{b} = \mathbf{b_0} + \mathbf{V}_\eta \mathbf{\Phi} \tag{15}$$

Now, the frequency response of the half-band filter can be expressed as

$$P(e^{j\omega}) = 2^{-L}(1+\cos\omega)^L \mathbf{h_1}(\omega)^{\mathrm{T}}(\mathbf{b_0} + \mathbf{V}_\eta \mathbf{\Phi}) \tag{16}$$

where

$$\mathbf{h_1}(\omega) = [1 \; 2\cos\omega \; \cdots \; 2\cos \mathrm{K}\omega]^{\mathrm{T}} \tag{17}$$

We can take out the constant component of $P(e^{j\omega})$ and write

$$P(e^{j\omega}) = P_0(e^{j\omega}) + \mathbf{h}^{\mathrm{T}}(\omega)\mathbf{\Phi} \tag{18}$$

where

$$P_0(e^{j\omega}) = 2^{-L}(1+\cos\omega)^L \mathbf{h_1}(\omega)\mathbf{b_0} \tag{19}$$

and

$$\mathbf{h}(\omega) = 2^{-L}(1+\cos\omega)^L \mathbf{V}_\eta^{\mathrm{T}} \mathbf{h_1}(\omega) \tag{20}$$

A. Design of Orthogonal Wavelet Filter for Echo Cancellation

The condition for factorization of $B(z)$ to exist is that $P(e^{j\omega}) \geq 0$ for $\omega \in [0, \pi]$. The discretised version of this condition is given in [5] as

$$\mathbf{D\Phi} \geq \mathbf{p} + \epsilon\mathbf{e} \tag{21}$$

where

$$\mathbf{D} = \begin{bmatrix} \mathbf{h_1^T}(\omega_1) \\ \mathbf{h_1^T}(\omega_2) \\ \vdots \\ \mathbf{h_1^T}(\omega_M) \end{bmatrix} \mathbf{V}_\eta, \; \mathbf{p} = \begin{bmatrix} \mathbf{h_1^T}(\omega_1) \\ \mathbf{h_1^T}(\omega_2) \\ \vdots \\ \mathbf{h_1^T}(\omega_M) \end{bmatrix} \mathbf{b_0} \tag{22}$$

In (22) $\{\omega_i, \; 0 \leq i \leq M-1\}$ are the frequencies on $[0, \pi]$ at which the continous-time equation is evaluated. The term ϵ is a small positive scalar and $\mathbf{e} = [1\,1 \cdots 1]^{\mathrm{T}}$

As explained in section III, we need to maximize the quantity

$$E = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} |H_0(e^{j\omega})|^2 d\omega \tag{23}$$

From (5), we have

$$E = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} P(e^{j\omega})d\omega = \mathbf{c^T\Phi} + \lambda \tag{24}$$

where

$$\mathbf{c} = -\frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} \mathbf{h}(\omega)\mathrm{d}\omega \tag{25}$$

Therefore, the optimization problem can be formulated as the linear programming (LP) problem

$$\text{minimize } \mathbf{c^T\Phi} \tag{26}$$

subject to

$$\mathbf{D\Phi} \geq \mathbf{p} + \epsilon\mathbf{e} \tag{27}$$

Once the minimizer $\mathbf{\Phi}$ to the LP problem is found, $\mathbf{b}$ can be calculated using (15). Then using (11), $B(z)$ can be constructed and $B_1(z)$ can be obtained using spectral factorization, hence $H_0(z)$ can be deduced.

B. Modified Objective Function

In order to ensure that most of the energy is concentrated in $[-\pi/2, \pi/2]$, we have previously tried to maximize the quantity

$$\int_0^{\pi/2} |H_0(e^{j\omega})|^2 d\omega \tag{28}$$

The desired energy concentration in $[-\pi/2, \pi/2]$ may be further enhanced by modifying (23) to

$$E = \int_0^{\pi/2} |H_0(e^{j\omega})|^2 d\omega - \alpha\int_{\pi/2}^{\pi} |H_0(e^{j\omega})|^2 d\omega \tag{29}$$

where α is a weight assigned based on the importance of the second term. It is easy to see that α will be meaningful if it takes values from 0 to 1. Hence we could further optimize the half-band filter by finding the value of α that gives the maximum energy concentration in $[-\pi/2, \pi/2]$ and calculating the half-band filter coefficients corresponding to that α.

A good performance measure for the optimized filter is the fraction of energy content in $[-\pi/2, \pi/2]$, which can be calculated as

$$E_f = \frac{\int_{-\pi/2}^{\pi/2} |H_0(e^{j\omega})|^2 d\omega}{\int_{-\pi}^{\pi} |H_0(e^{j\omega})|^2 d\omega} \tag{30}$$

Optimizing a filter of order 20 with 9 degrees of freedom, which is a filter whose wavelet has only one vanishing moment, we obtain an E_f of 0.9699, which is very close to the value of 0.974 obtained using the method described in [4]. This is a small difference given that the resultant filter in this method is orthogonal. Furthermore, the method used for optimization is linear programming which is much faster than unconstrained maximization on the penalty function described in [4]. The half-band filter coefficients are given in Table I.

Fig. 3 shows the amplitude responses of the optimized half-band filter of length 20 and the Daubechies orthogonal wavelet filter of the same order.

V. Simulations

For the purpose of simulation, the echo $s[n]$ was generated using an FIR filter W of length 20. The transmitted signal $x[n]$ was generated as a random binary signal and

TABLE I
FILTER COEFFICIENTS

n	Designed in [4]	Optimum orthogonal
0	-0.0160	0.00278766
1	0.0367	-0.00810576
2	-0.0142	0.00076205
3	-0.0387	0.02097110
4	0.0309	-0.01573315
5	0.0411	-0.02924597
6	-0.0445	0.03446170
7	-0.0435	0.03726315
8	0.0588	-0.05770701
9	0.0598	-0.04798513
10	-0.0762	0.08744309
11	-0.0829	0.06629413
12	0.0970	-0.12529291
13	0.1304	-0.10885254
14	-0.1202	0.17666459
15	-0.2418	0.22383742
16	0.1101	-0.21986416
17	0.5976	-0.65945582
18	0.6492	-0.59173515
19	0.2829	-0.20293388

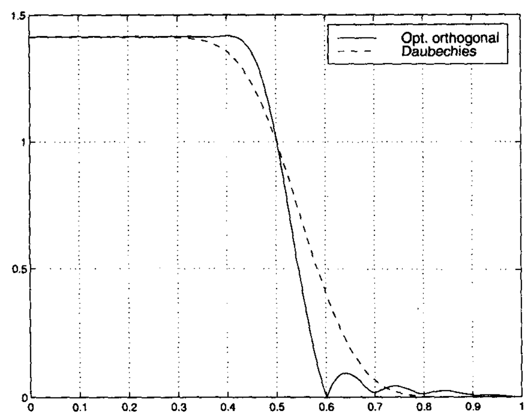

Fig. 3. Amplitude responses of optimized orthogonal filter of length 20 and Daubechies orthogonal wavelet filter.

the received signal was composed of a sinusoid $y[n]$ and the echo $s[n]$. A two-band decomposition, as described in [3], was used without the cross adaptive filters. A commonly used measure of performance of the echo canceller is the residual echo power which is calculated as

$$P_{res} = \frac{||e[n] - y[n]||}{||y[n]||} \qquad (31)$$

where $||x||$ denotes the Euclidean norm of the signal x.

Table II compares the residual echo power for three different filters, namely, the Daubechies orthogonal wavelet filter, the filter given in [4], and the optimum orthogonal filter obtained with the proposed method.

TABLE II
RESIDUAL ECHO POWER.

Daubechies'	Wavelet in [4]	Opt. Orthogonal
-15.37 dB	-17.90 dB	-18.20 dB

We see that when the optimum orthogonal wavelet filter is used, the residual echo power is smaller than that of the wavelet derived in [4]. This is because of the orthogonality of the resultant filters which results in the perfect reconstruction property.

VI. CONCLUSIONS

A half-band filter of order N with maximum energy content in $[-\pi/2, \pi/2]$ is obtained by trading the vanishing moments of the Daubechies wavelet of the same order. The optimization method used is linear programming, which is computationally faster compared to other optimization techniques. The objective function used was the energy content of the filter in $[-\pi/2, \pi/2]$ which resulted in an energy ratio E_f which is quite close to the value obtained in [4]. A further increase in E_f was obtained when the objective function was modified with weightage given to the energy content in $[\pi/2, \pi]$. The eventual resultant half-band filter has a better performance than the non-orthogonal filter.

ACKNOWLEDGMENT

The authors would like thank Micronet, Networks of Centres of Excellence Program, and PMC-Sierra Inc. for supporting this research.

REFERENCES

[1] G. Strang and T. Nguyen, *Wavelets and Filter Banks*, Wellesley, MA: Wellesley-Cambridge Press, 1996.

[2] S. Hosur and A.H. Tewfik, "Wavelet transform domain adaptive filtering", *IEEE Trans. on Signal Processing*, vol. 45, pp. 617-630, March 1997.

[3] A. Gilloire and M. Vetterli, "Adaptive filtering in subbands with critical sampling: analysis, experiments, and applications to acoustic echo cancellation", *IEEE Trans. on Signal Processing*, vol.40, pp. 1862-1875, August 1992.

[4] Q. Jin, K.M. Wong and Z.Q. Luo, "Design of an optimum wavelet for cancellation of long echoes in telephony", *Proc. IEEE-SP Time-Frequency and Time-Scale Analysis*, pp. 488-491 October 1994.

[5] W.-S. Lu and A. Antoniou, "Optimized orthogonal and biorthogonal wavelets using linear parameterization of halfband filters", *Proc. ISCAS* '98. vol. 5, pp. 98-101, May 1998.

[6] S. J. Orfanidis, *Optimum Signal Processing*, New York, Macmillan, 1988.