AN IMPROVED WAVELET-PACKET-DIVISION MULTIPLE ACCESS SYSTEM

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ABSTRACT

An improved wavelet-packet-division multiple access (WPDMA) system is presented. In a WPDMA environment, each message signal is waveform coded onto a wavelet function to enhance the immunity of the system against interference. We propose a wavelet design method based on numerical optimization to mitigate the effects of asynchronism in the system. In brief, the algorithm consists of two parts: (a) identification of the feasible region, and (b) use of a quasi-Newton search algorithm to find the minimizer. By employing this technique, we found the optimal wavelet function which minimizes the interference due to timing errors. The advantages of the proposed WPDMA design algorithm with respect to a previously reported procedure are discussed.

I. INTRODUCTION

Wavelet-packet-division multiple access (WPDMA) is a promising new technique which provides an alternative to well known schemes such as time-division (TDMA) and frequency-division (FDMA) multiple access. A key to all multiple access methods is that various message signals (or users) share a common channel without creating unmanageable interference at the receiver end.

To provide better immunity against interference, waveform coding is usually employed in digital communication systems. The interference can be effectively reduced by maximizing the distance or, equivalently, by minimizing the crosscorrelation betweeen message waveforms. In waveform coding, the message data is transformed into a message waveform in order to make the crosscorrelation as small as possible. Orthogonal signaling is a waveform coding technique which ensures zero crosscorrelation between message waveforms. Two primary types of orthogonality signaling are widely used: FDMA and TDMA where the message waveforms are orthogonal in frequency and time respectively. However, orthogonality is not limited to either the frequency or the time domain. One can have a dual situation in which mutual orthogonality is given by different orthogonal subspaces, and self-orthogonality is achieved by translation in time. Wavelets provide this kind of orthogonality by allowing orthogonal message waveforms to overlap in the time and frequency domain [1]. Furthermore, the overlapping nature of the WPDMA system yields a capacity improvement over other multiple access techniques [2].

In this paper, we closely follow the WPDMA scheme introduced by Wong [2]. The issue of synchronization is critical in the design of any communications network. Since synchronization between the transmitter and the receiver is not always possible, the system may be subjected to detection errors. To decrease the interference caused by the timing discrepancies, an optimum wavelet design procedure method was proposed in [2]. In this work, we implement an alternative optimization technique to minimize the interference in a WPDMA system. Our approach results in further interference reduction relative to that reported in [2], and consequently leads to an improvement in bit error rate (BER) performance.

II. SYSTEM MODEL

An L-level WPDMA system can be represented by the wavelet transform tree [1] as follows:

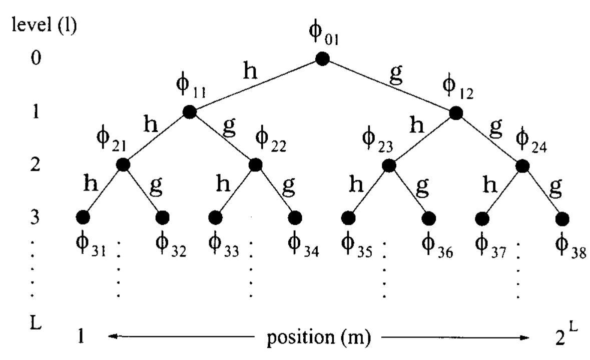

Figure 1: WPDMA tree structure

In the above diagram, the branches of the tree corre-

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spond to lowpass h and highpass g filtering followed by downsampling. Also, each node (l,m) of the tree represents a wavelet packet subspace W_{lm} whose basis is given by the wavelet packet scaling functions:

$$\{\phi_{lm}(t-nT_l)\}_{n\in Z}\,. \tag{1}$$

This tree structure yields the following relationships between the basis functions at different levels:

$$\phi_{l+1,2m-1}(t)=\sum_n h[n]\phi_{lm}(t-nT_l) \tag{2a}$$

$$\phi_{l+1,2m}(t)=\sum_n g[n]\phi_{lm}(t-nT_l) \tag{2b}$$

Furthermore, one can express a wavelet terminal function ϕ_{lm} in terms of the wavelet root function ϕ_{01} as

$$\phi_{lm}(t)=\sum_n f_{lm}[k]\phi_{01}(t-kT_0) \tag{3}$$

where $f_{lm}[n]$ is the equivalent filter generated from the combinations of $h[n]$, $g[n]$ and downsampling which lead from the root to the (l,m) terminal.

In a WPDMA environment, each message signal or bit sequence $\{b_{lm}[n]\}_{n\in Z}$, where $b_{lm}[n]\in\{-1,+1\}$, is mapped onto the wavelet basis described by (1) according to the rule: "1"$\mapsto$ ϕ_{lm} and "-1"$\mapsto$ $-\phi_{lm}$. This waveform coding technique constitutes an orthogonal antipodal signaling system. The above system has some interesting properties that should be emphasized. First, the duration of the bit interval at level l, denoted by T_l, varies as we move vertically along the tree structure. In fact, we have that $T_l=2T_{l-1}$. Secondly, the number of bases or subspaces W_{lm} required depends on the number of message signals to be sent. Moreover, it is desirable to map each sequence of bits, namely, $\{b_{l1}[n]\}_{n\in Z}$, $\{b_{l2}[n]\}_{n\in Z}$, ..., onto subspaces located on the same level in the tree, namely, $W_{l1}, W_{l2},\ldots$, since these spaces are mutually orthogonal [1].

Next, the transmitted signal can be written as

$$s(t)=\sum_{l\in L,m\in M}\sum_n b_{lm}[n]\phi_{lm}(t-nT_l) \tag{4}$$

where $(l\in L, m\in M)$ indicates the different bases utilized during the transmission. A quick inspection of (4) reveals the inherent self- and mutual orthogonality of a WPDMA scheme. That is, (a) different bits $b[n]$ are orthogonalized by shifting $\phi(t-nT)$ in time, and (b) different message signals b_{lm} are orthogonalized by using different orthogonal bases ϕ_{lm}.

An alternative expression for the transmitted signal can be easily obtained by substituting (3) into (4):

$$s(t)=\sum_n b_{01}[n]\phi_{01}(t-nT_0) \tag{5}$$

where $b_{01}[k]$ is the equivalent sequence at the root of the tree

$$b_{01}[k]=\sum_{l\in L,m\in M}\sum_n f_{lm}[k-2^l n]b_{lm}[n]\,. \tag{6}$$

It is worth noting that the above expressions for the transmitted signal reflect two modes of operation of the WPDMA system. In (4), each user is assigned a different $\phi_{lm}(t)$ to carry the message signal in a FDMA-like fashion. Alternatively, equation (5) emulates a TDMA operation in which only $\phi_{01}(t)$ is employed to carry the time-multiplexed data from different users. Indeed, a WPDMA system can be viewed as a hybrid form of FDMA and TDMA, enriched with overlapping time and frequency slots.

For the system under consideration, the optimal receiver in an AWGN channel consists of a synchronous correlator followed by a sampler [3]. However, perfect synchronization is not always possible. As a result the system may be subjected to errors, that is, interference caused by timing discrepancies between the transmitter and the receiver. Let us denote the timing discrepancy by Δ, and the transmitted signal by (5). Then, in the absence of noise, the output of the correlator at the receiver is given by

$$\hat{b}_{01}[n]=\sum_k b_{01}[k]R_\phi(kT_0-nT_0+\Delta) \tag{7}$$

where $R_\phi(\tau)=\int\phi(t)\phi(t-\tau)dt$ is the autocorrelation function of the root wavelet $\phi_{01}(t)$. After some mathematical manipulations [2], the interference term in (7) can be isolated and expressed as

$$I_{01}[n]=\Delta\sum_k(b_{01}[n+k]-b_{01}[n-k])R'_\phi(kT_0)\,. \tag{8}$$

If the message bits are independent, then the energy of the interference is directly proportional to the function

$$F=\sum_n|R'_\phi(nT_0)|^2\,. \tag{9}$$

In the next section, we develop an optimization method to minimize F in (9).

III. OPTIMAL WAVELET DESIGN

Our goal is to find a wavelet scaling function ϕ_{01} or, equivalently, a wavelet filter h such that the function F in (9) is minimized.

Let us begin by demonstrating some important results. The Z-transform of a wavelet filter h of length N can be written as

$$H(z)=\left(\frac{1+z^{-1}}{2}\right)^L B(z) \tag{10}$$

where L is the number of vanishing moments (or zeros at $\omega = \pi$) of the filter h, and $B(z)$ is a Kth-order polynomial in z^{-1}.

In addition, we are concerned with an orthogonal wavelet filter, i.e.,

$$\left|\hat{h}(\omega)\right|^2 + \left|\hat{h}(\omega+\pi)\right|^2 = 2, \ \forall\omega \tag{11}$$

where $\hat{h}$ represents the Fourier Transform of h. It can be shown [4] that the orthogonality condition in (11) holds if and only if:

$$\mathbf{Ab} = \mathbf{m} \tag{12}$$

where $\mathbf{A} \in R^{\frac{N+1}{2}\times(K+1)}$ is determined by the term $\left(\frac{1+z^{-1}}{2}\right)^L$ in (10), $\mathbf{b} \in R^{K+1}$ is related to the polynomial $B(z)$, and $\mathbf{m} = [0\cdots 0\,1]^T \in R^{\frac{K+1}{2}}$. In other words, the solution of the linear system (12) will lead to an orthogonal filter h. One can see that if the number of vanishing moments L need not be maximum, then system (12) becomes underdetermined. In fact, the number of degrees of freedom is:

$$\eta = \frac{K-L+1}{2} = \frac{N-2L}{2}. \tag{13}$$

Therefore, the solution of (12) can be parametrized as

$$\mathbf{b} = \mathbf{b}_0 + \mathbf{V}_\eta\theta \tag{14}$$

where $\mathbf{b}_0$ is the last column of the Moore-Penrose pseudo inverse of $\mathbf{A}$, $\mathbf{V}_\eta$ consists of the last η columns of $\mathbf{V}$ where $\mathbf{A} = \mathbf{USV}^T$ denotes the singular value decomposition of $\mathbf{A}$, and $\theta \in R^\eta$ is a free parameter vector. If $\eta = 0$ (L is maximum) then $\mathbf{b} = \mathbf{b}_0$ and h reduces to the known Daubechies filter [1]. The dimensionality or degrees of freedom of the parameter θ will be exploited in the search of the optimal h.

We can now proceed to formulate an optimization problem as follows:

$$\text{minimize } F = \sum_n \left|R'_\phi(nT_0)\right|^2 \tag{15}$$

$$\text{subject to } \begin{cases} \left|\hat{h}(\omega)\right|^2 + \left|\hat{h}(\omega+\pi)\right|^2 = 2, \ \forall\omega \\ \hat{h}(0) = \sqrt{2}, \\ \hat{h} \text{ has } \eta \text{ degrees of freedom.} \end{cases} \tag{16}$$

The constraints in (16) can be categorized into two groups. Constraints (a)-(b) guarantee that the wavelet filter h is orthogonal and normalized respectively. Constraint (c) sets the number of degrees of freedom η which is directly related to the number of vanishing moments L via equation (13). Finally, it should be emphasized that the objective function in (15) is nonlinear and computationally complex.

The design of a WPDMA system by optimization can be implemented in the following steps:

1. Set the filter length N and choose the number of vanishing moments L between 1 and $\frac{N}{2}$. This fixes the number of degrees of freedom η.
2. Determine the feasible region given by the constraints (16). The geometry of the region will change with η.
3. Apply the Broyden-Fletcher-Goldfarb-Shanno (BFGS) search algorithm [5] over the feasible region to find the minimizer $h^\star$ of (15). A good initial point is also required to obtain the global minimum.
4. Derive the optimal wavelet scaling function $\phi^\star_{01}$ from the optimal wavelet filter $h^\star$ through elementary wavelet theory [1].

In this algorithm we have decoupled the constraints from the objective function. This ensures that the filter will be perfectly orthogonal. Orthogonality is a necessary condition in order to have exact wavelet packet reconstruction and decomposition at the transmitter and receiver ends of the WPDMA system respectively. By contrast, in the approach proposed in [2], the constraints are incorporated into the objective function via Lagrange multipliers. Thus, perfect orthogonality might not be possible in this case.

IV. RESULTS

The performance of the proposed design method was compared with that of Wong's [2]. We set the length of the wavelet filter h to be $N = 14$. As a result, the number of vanishing moments L could vary in the range $[1, 7]$ or, equivalently, the number of degrees of freedom η could be up to 6. We tested our algorithm for $\eta = 1$, 2, 3, 4, and 5. The case $\eta = 0$ corresponds to the Daubechies wavelet filter which was also included for reference.

The interference performance is gathered in Fig. 2. It is apparent from the graph that the proposed algorithm produced significantly better results than that of [2]. The minimizer was found by applying our design algorithm with $\eta = 3$. Thus, the optimal wavelet filter has 4 vanishing moments. Most notably, the reduction in interference relative to [2] is almost 50%, and close to 70% with respect to the Daubechies filter.

The behaviour of the optimal filter is displayed in Fig. 3. First, the frequency response of the filter reflects a transfer of energy from the passband to the stopband. Secondly, the orthogonality of the filter given by (11) is analyzed. It is clear from the graph that the orthogonality equation is satisfied within 10^{-6}. This discrepancy coincides with that of the Daubechies filter. However, the filter in [2] shows a discrepancy of 6×10^{-5}, i.e., one order of magnitude larger. These findings confirm that the proposed method provides also a "more" orthogonal wavelet filter.

V. CONCLUSION

We have developed a design method to mitigate the interference caused by asynchronism in a WPDMA system. Using numerical optimization techniques, we have found the optimal wavelet function which minimizes the above interference. The performance of the optimal wavelet filter in terms of interference reduction is superior to that of a previously reported filter. Finally, the proposed algorithm ensures perfect orthogonality of the wavelet filter which is needed for an accurate wavelet reconstruction and decomposition in the system.

VI. REFERENCES

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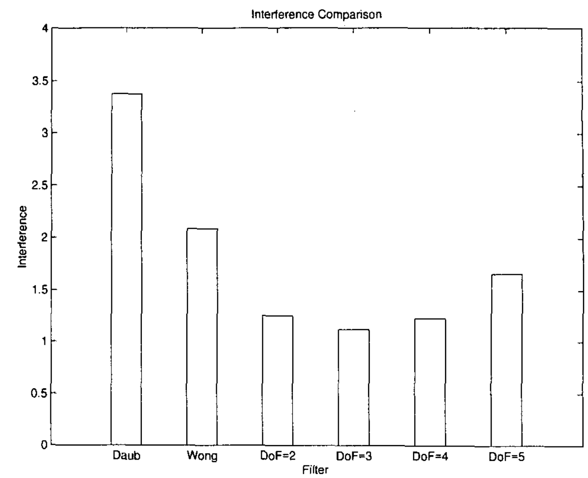

Figure 2: Interference comparison. Wavelets filters plotted from left to right are: Daubechies(14), Wong's [2], and the filters obtained using the proposed design algorithm for $\eta = 2, 3$ (optimal), 4, and 5.

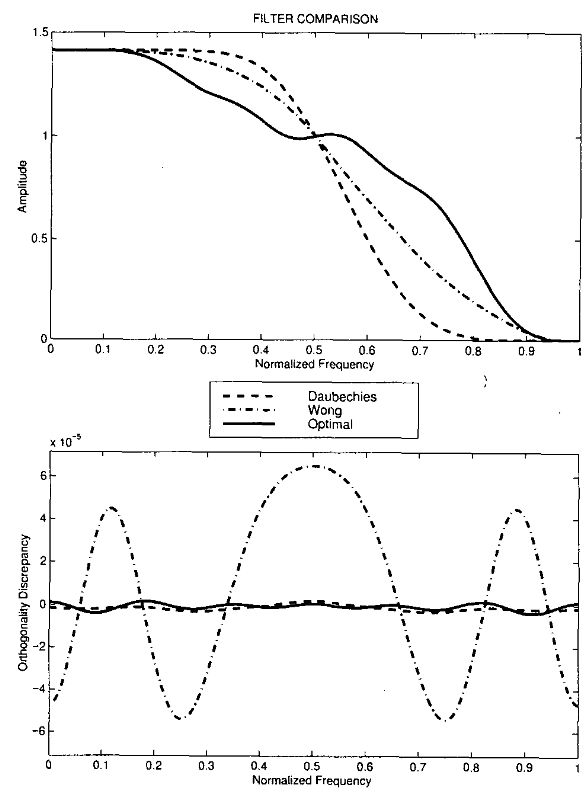

Figure 3: Frequency response and orthogonality comparison. Wavelet filters displayed are: Daubechies(14), Wong's [2], and the optimal one.