WAVELET-PACKET-BASED SIGNATURES FOR ASYNCHRONOUS CDMA SYSTEMS

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ABSTRACT

A wavelet-packet-based family of user signatures for asynchronous CDMA systems is investigated. These signatures are of non-binary nature and are generated by the application of the inverse wavelet packet transform. Through numerical optimization, we obtain the wavelet filter which minimizes the crosscorrelation among signatures for different user configuration systems. The improvement in performance is especially notable in the oversaturated system scenario. Finally, we comment briefly on the topics of bandwidth efficiency, user privacy, and implementation.

I. INTRODUCTION

Code-division multiple access (CDMA) has become the desired scheme for third generation wireless communications. It is well known that the "Achilles heel" of CDMA is multiuser interference. In a direct-sequence (DS) CDMA environment, each user is assigned a specific signature or sequence. Crosscorrelation between user signatures creates multiuser interference, thereby degrading the performance of the system.

Though signature correlation may be one of the primary concerns in a CDMA system, another desirable feature is the optimal deployment of bandwidth. A measure of bandwidth efficiency can be given by the number of users sharing the available bandwidth. We know that the maximum number of independent signals of duration T and bandwidth B is upper bounded by $2BT$ [1]. This will be the case for wavelet-packet-based signatures. Note that for binary sequences this limit is reduced to the spreading factor N [2]. Consequently, a set of wavelet-packet-based signatures is well suited to an oversaturated CDMA system, that is, a system in which the number of users K is larger than N. In other words, the proposed non-binary signatures could increase the capacity of the system by enabling more users to share the same bandwidth.

The employment of wavelet-packet-based signatures is not new. Several testimonies of this can be found in the literature [3] following the work initiated by Learned [4]. It should be emphasized, however, that the above studies rely strongly on the assumption that the system is synchronous. By contrast, in this paper we move away from this restriction, and allow the users to be asynchronous, that is, their time epochs need not be aligned. Asynchronism is, in some cases, unavoidable and often preferred in some practical situations.

Although this is not the central matter of this paper, suffice it to say that the implementation of wavelet-packet-based signatures is both feasible and simple in practice. Another issue beyond the scope of this work is that of user privacy. Nevertheless, it should be noted that the level of security associated with wavelet-packet-based signatures is superior than that of linear binary ones, as a result of the non-binary nature of the former.

II. SYSTEM MODEL

Let us consider an asynchronous DS/CDMA system with multiuser detection. In this environment, the received signal can be expressed as

$$r(t) = \sum_{i=0}^{M-1} \sum_{k=1}^{K} b_k(i) s_k\left(t - iT - \tau_k\right) + n(t) \qquad (1)$$

where $b_k(i)$ is the bit transmitted by user k during time period i, $b_k(i) \in \{+1, -1\}$, $s_k(t)$ is the signature waveform of user k, $n(t)$ is white Gaussian noise with power spectrum $2\sigma^2$, M is the number of bits transmitted, K is the total number of users, and τ_k is the delay suffered by user k with $0 \leq \tau_1 \leq \ldots \leq \tau_K \leq 1$. A direct-sequence signature waveform $s_k(t)$ is given by

$$s_k(t) = \sum_{i=1}^{N} x_i \, \varphi(t - (i-1)T_c) \qquad (2)$$

where $x = (x_1, x_2, \ldots, x_N)$ is the signature sequence, N is the spreading factor, $\varphi(t)$ is the chip waveform, and T_c is the chip period.

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After passing the received signal (1) through a bank of matched filters, the discrete-time output for user k becomes

$$\begin{aligned} y_k[i] &= b_k[i] + \sum_{j<k} b_j[i+1]\rho_{kj} + \sum_{j<k} b_j[i]\rho_{jk} \\ &+ \sum_{j>k} b_j[i]\rho_{kj} + \sum_{j>k} b_j[i-1]\rho_{jk} + n_k[i] \end{aligned} \tag{3}$$

where ρ_{kj} denotes the crosscorrelation between users signatures k and j. For $k < j$, $\tau = \tau_j - \tau_k \geq 0$, we have

$$\rho_{kj}(\tau) = \int_{\tau}^{T} s_k(t)s_j(t-\tau)\,dt \tag{4a}$$

$$\rho_{jk}(\tau) = \int_{0}^{\tau} s_k(t)s_j(t+T-\tau)\,dt\,. \tag{4b}$$

It can be shown [5] that equation (4) is equivalent to the aperiodic crosscorrelation

$$C_{X,Y}(d) = \begin{cases} \sum_{i=1}^{N-d} x_i y_{i+d} & \text{if } 0 \leq d \leq N-1, \\ \sum_{i=1}^{N-d} x_{i-d} y_i & \text{if } 1-N \leq d \leq 0, \\ 0 & \text{if } d \geq N, \end{cases} \tag{5}$$

where $\mathbf{x} = (x_1, x_2, \ldots, x_N)$ and $\mathbf{y} = (y_1, y_2, \ldots, y_N)$ are the signature sequences associated with users k and j respectively, and d is the time delay between those users.

A quick inspection of (3) confirms the need for user signatures with minimal crosscorrelation.

III. WAVELET-PACKET-BASED SIGNATURES

The wavelet packet transform (WPT) of a discrete signal $\mathbf{v}$ of length $N = 2^L$, $L \in \mathbf{Z}^+$, can be represented by the following tree diagram:

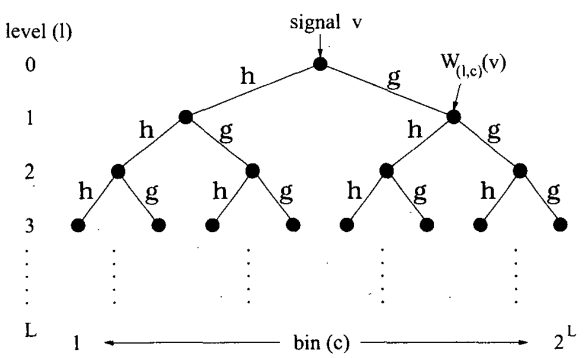

Figure 1: Wavelet Packet Binary Tree of L Levels.

In the above figure, we have that $L = \log_2(N)$ is the number of levels, $W_{(l,c)}(\mathbf{v})$ is the node denoting the vector or signal content at level l and bin c, and h and g are the branches of the tree corresponding to the lowpass and highpass wavelet filtering, respectively, followed by downsampling. Therefore, the WPT decomposes the original signal $\mathbf{v}$ into successive subsignals as we descend the tree structure. This yields the following relationships between two children bin vectors, $W_{(l+1,2c-1)}(\mathbf{v})$ and $W_{(l+1,2c)}(\mathbf{v})$, and their ancestor $W_{(l,c)}(\mathbf{v})$:

$$W_{(l+1,2c-1)}(\mathbf{v}) = h\, W_{(l,c)}(\mathbf{v}) \tag{6a}$$

$$W_{(l+1,2c)}(\mathbf{v}) = g\, W_{(l,c)}(\mathbf{v})\,. \tag{6b}$$

On the other hand, the inverse wavelet packet transform (IWPT) can be used to ascend the tree diagram. This inverse operation is equivalent to the substitution of the analysis filters h and g by the synthesis filters $\bar{h}$ and $\bar{g}$, and the downsampling process by the upsampling one. We then obtain an expression which relates a bin vector with its two descendants:

$$W_{(l,c)}(\mathbf{v}) = \bar{h}\, W_{(l+1,2c-1)}(\mathbf{v}) + \bar{g}\, W_{(l+1,2c)}(\mathbf{v})\,. \tag{7}$$

Moreover, the wavelet packet transform provides a *time-frequency picture* of the signal. The uppermost or level 0 corresponds to the time domain representation of the signal while the bottom or level L features the frequency domain. In other words, as we proceed down the tree, we observe a trade-off between time resolution and frequency resolution.

A wavelet-packet-based signature can be obtained by applying the IWPT to a vector in a node of the tree until the uppermost level, that is, the time domain representation of the signal is reached. Both the position of the node within the tree and the content of the vector, as well as the particular wavelet filter selected, will uniquely determine the signature waveform. Thus, a set of user signatures can be formed by strategically choosing a set of nodes and their respective vectors. We will graphically represent a set of user signatures by the user constellation tableau.

This technique of signature design has a remarkable property: users who do not share descendants are orthogonal, namely, their crosscorrelation is zero. However, this property assumes perfect user synchronization in the system. If the multiple access system is asynchronous, then the orthogonality among users will no longer hold and, in turn, multiuser interference will occur. Next, we will consider the minimization of multiuser interference in an asynchronous system.

IV. PROBLEM FORMULATION

Our main objective is to design signatures which minimize the interference, that is, signatures having the lowest crosscorrelation values. This can be achieved by employing the appropriate wavelet filter h in the construction of the signatures.

An important concept in wavelet theory [6] is the number of vanishing moments of a wavelet function. It can be shown [7] that if the number of vanishing moments need not be maximum, then we have some degree of freedom in the choice of h. This is exactly the case in designing wavelet-packet-based signatures. In fact, the number of degrees of freedom is given by

$$\lambda = \frac{R - 2P}{2} \tag{8}$$

where R denotes the length of the wavelet filter and P is the number of vanishing moments. If $\lambda = 0$ (P is maximum), then h reduces to the known Daubechies filter [6]. This filter will be used as the reference in our comparative study.

Next, we are interested in finding a wavelet filter h such that the aperiodic crosscorrelation (5) is minimized. Specifically, we want to minimize the sum of the peak aperiodic crosscorrelation ρ_i for all K users in the system. This leads to the formulation of an optimization problem as follows:

$$\text{minimize } f = \phi(h) = \sum_{i=1}^{K} \rho_i \tag{9}$$

$$\text{subject to} \begin{cases} \text{(a) } \left|\hat{h}(\omega)\right|^2 + \left|\hat{h}(\omega+\pi)\right|^2 = 2, \ \forall\omega \\ \text{(b) } \hat{h}(0) = \sqrt{2}, \\ \text{(c) } \hat{h} \text{ has } \lambda \text{ degrees of freedom.} \end{cases} \tag{10}$$

where $\hat{h}$ represents the Fourier Transform of h. The constraints shown in (10) can be categorized into two groups. Constraints (a)-(b) guarantee that the wavelet filter h is orthogonal. Constraint (c) sets the number of degrees of freedom λ which is directly related to the number of vanishing moments P via equation (8) once the length of the filter R is fixed. Since $P \in [1, \frac{R}{2}]$, λ can vary in the range $[0, \frac{R-2}{2}]$. As a result, the geometry of the feasible region defined by (10) will also change with λ.

In addition, it should be emphasized that the objective function (9) is nonlinear and computationally complex. Indeed, the input h and the output f are related by the following diagram:

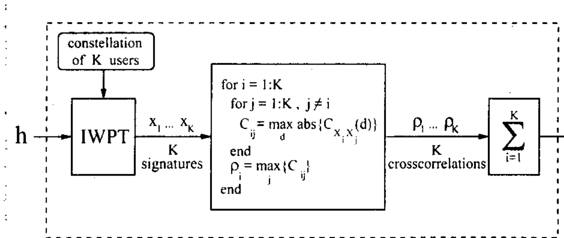

Figure 2: Objective Function $f = \phi(h)$

The solution of the above optimization problem involves: (a) finding a good initial point, and (b) applying an efficient search algorithm over the feasible region starting from the initial point. In practice, it was found that often the feasible region was neither a connected nor a convex set. In such cases we could not simply apply the Broyden-Fletcher-Goldfarb-Shanno (BFGS) method [8] which was otherwise the search algorithm utilized. Instead we had to resort to exhaustive search prior to the application of the BFGS technique in our quest for the minimizer.

V. NUMERICAL RESULTS

Given the complexity of the objective function, a system with a small spreading factor $N = 8$ was considered. Three different user scenarios displayed in Fig. 3 were analyzed: a regular system ($K = N$) with $K = 8$ users, and two oversaturated systems ($K > N$) with 12 and 15 users respectively. Wavelet filters of length $R = 14$ were employed in the generation of the signatures. Thus, the number of degrees of freedom λ was defined by the interval $[0, 6]$.

The crosscorrelation behaviour of each set of signatures for several filter cases ($\lambda = 0, 2,$ and 5) in different user systems is shown in Figs. 4 and 5. It is apparent from graphs that the performance improves as the number of degrees of freedom λ of the filter increases. In fact, a crosscorrelation reduction close to 20% relative to the Daubechies filter was reported by the optimal filter ($\lambda = 5$). Furthermore, the improvement is especially significant in the oversaturated scenario. For instance, the 15-user oversaturated system employing the optimal filter ($\lambda = 5$) clearly overpowers the regular system using the Daubechies filter. In brief, the optimal wavelet filter allows more users into the system without sacrificing crosscorrelation performance.

VI. CONCLUSIONS

In this study, we have examined a class of non-binary signatures, wavelet-packet-based in nature, to be utilized in an asynchronous CDMA system with multiuser detection. We have demonstrated that the crosscorrelation can be successfully reduced via optimization methods. Indeed, we have obtained the wavelets filters which minimize the crosscorrelation between user signatures and analyzed their behaviour for several user configuration systems. Additionally, we have discussed that these signatures are well suited to an oversaturated system since they provide a more efficient use of bandwidth resources. Finally, without embarking on a detailed analysis of how and why, we have mentioned that the studied signatures could improve user privacy and that their implementation entails minimal difficulty with today's current technology.

VII. REFERENCES

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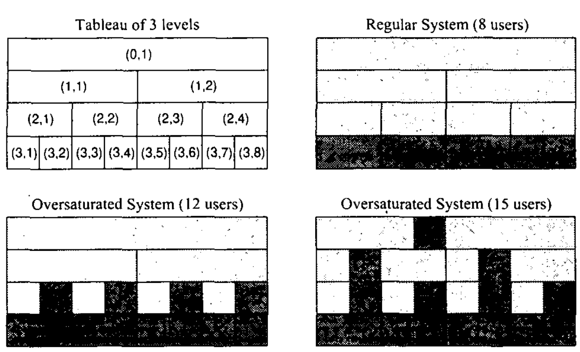

Figure 3: User Constellation Tableau of 3 Levels. Constellations of 8, 12, and 15 Users.

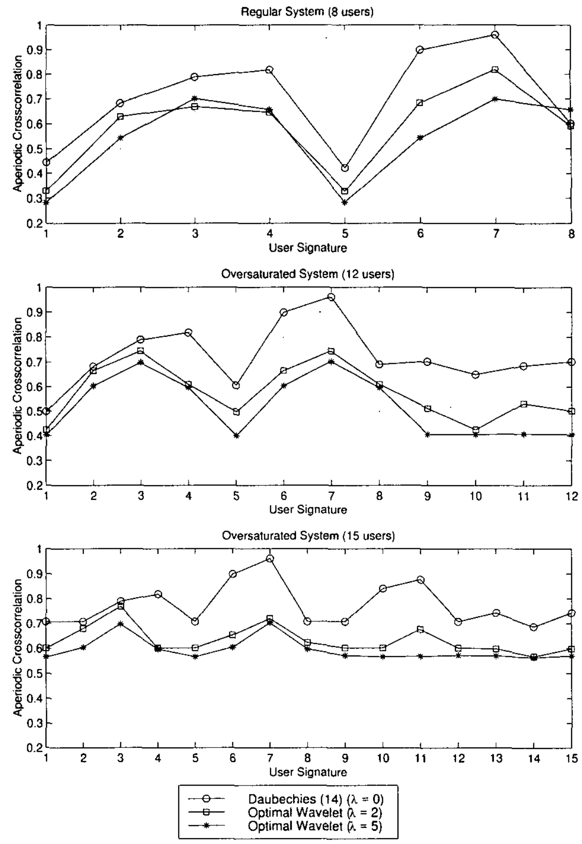

Figure 4: Peak Aperiodic Crosscorrelation for each User in the System.

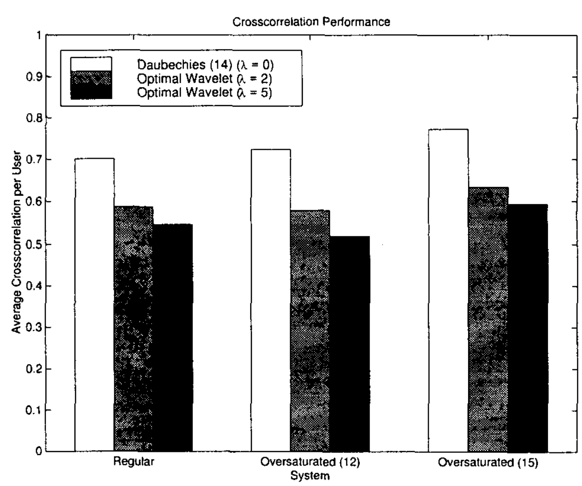

Figure 5: Average Peak Aperiodic Crosscorrelation per User.