Optimization of Power Allocation in a Multicell DS/CDMA System with Heterogeneous Traffic

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Abstract - This paper outlines an efficient optimization method to determine the optimal power allocation plan that guarantees the quality of service contracts for multimedia traffic while ensuring minimal power consumption in a systematic manner. Both perfect and imperfect power-controlled cases in a multicell operating environment have been considered. This method may also be used to determine the capacity region of the integrated network for a given set of system parameters.

I INTRODUCTION

It is well known that the "Achilles Heel" of DS/CDMA is multiple access interference. As a consequence, its system capacity is tightly coupled with power control error. An accurate power control mechanism is a vital necessity in cellular DS/CDMA networks, even with homogeneous traffic, in order to alleviate the near-far problem as well as to minimize the co-channel interference. The effective use of power control in DS/CDMA cellular systems enables the frequency to be reused in every cell, which in turn exploits features such as soft-handoff and base-station diversity. However, the traditional power control law for single rate traffic (which maintains the received bit energy of all the users at a prescribed level) may no longer be applicable for multimedia networks where both information rates and grade of service requirements vary considerably for different classes of users. In such systems, a power control scheme not only must mitigate the near-far problem, but it should also balance the natural dissimilarities between the various classes of users.

Both power control and power allocation are essential to the operation of multi-service DS/CDMA networks. Power control is a mechanism that aims to keep the power level of all the received signals at a preferred level, while power allocation determines what the "preferred level" is for each service. Proper power assignment can control the mutual interference among different services while maintaining the quality of service (QoS) requirements of all services. The issue of power allocation will be the focus of this study.

Our present work builds on the analysis of a DS/CDMA integrated wireless access network recently presented by Zou et. al. [1][2]. A theoretical framework for modelling the multi-cell and multiuser co-channel interference has been developed in [1] using the concept of area averaged probability density function of the intercell interference and by exploiting the user membership statistics. Subsequently in [2], the authors presented a method for optimizing the power allocation in a mixed traffic scenario for capacity calculation. However, their approach was restricted to only two types of service, and assumed an identical outage requirement for all service classes. In this paper we present a more general framework and a systematic method for finding the optimal power allocation criterion in an integrated network with an arbitrary number of traffic types. The "optimality" here refers to the maximization of system capacity or throughput, and applies to the QoS guarantee for the multimedia traffic with minimal power consumption. Obviously, a power allocation strategy that achieves these goals will make an effective use of the resources available. For instance, minimizing the power consumption in a partially loaded system not only helps control the mutual interference but it also translates into an extended battery-life, which is an important design consideration particularly in the reverse-link communication.

Whether or not a traffic composition (denoting the number of calls for each service class) is admissible (i.e., capacity region) can be determined by numerically solving the nonlinear equations representing the service requirements. In our framework, we accomplish this task efficiently with the aid of a logarithmic barrier-function optimizer. Different from [2], our technique handles dissimilar outage requirements for different services as well as the imperfect power control case. Besides quantifying the capacity degradation due to power control error (PCE), it is important to understand the impact of this form of error on the power allocation plan. The power allocation plan developed in this paper also provides a guideline for setting the maximum transmit power levels for each of the distinct service classes so that the network throughput will be maximized for a specified fixed total transmit power. Alternatively, one could dynamically adjust the transmission rate (i.e., by varying the spreading gain) of a particular service to another service class whose power control saturation level has not been reached (i.e., this improvement is attained at the expense of increased task on resource manager) [4]. The organization of this paper is as follows. Section II details the problem formulation and the application of an interior point

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method to find the optimal solution. Selected numerical results are presented in Section III. Finally in Section IV, the main points are summarized and conclusions restated.

II CONSTRAINED OPTIMIZATION

A PROBLEM FORMULATION

Our system model is similar to that presented in [1], the effects of multipath fading are considered separately for inter-cell and intra-cell interferences. For inter-cell interference, we assume that multipath fading is Rayleigh distributed on top of the lognormal shadowing. Since propagation path loss is severe, the possibility that a subscriber belongs to a distant base station is very small. It is also assumed that the three closest base stations are involved in the soft handoff operation of a mobile. In order to achieve the goal of minimizing power consumption for a given throughput or traffic composition, we have developed an optimization approach to find the optimal power allocation plan for more than two service classes with transmit power constraints. First, a logarithmic barrier function is used to convert our constrained optimization to a parameterized unconstrained optimization problem. For a fixed value of the barrier parameter, the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm is employed to find the solution of the unconstrained problem. This solution then serves as an initial point for the unconstrained optimization with a reduced barrier parameter, and the BFGS algorithm is applied again to obtained an improved solution. This iterative process continues until the value of the barrier parameter is less than a prescribed tolerance.The optimization problem may be formulated separately for the following two cases: (a) perfect power control; (b) imperfect power control.

A.1 Perfect Power Control

The optimization problem in this case can be formulated as

$$\text{minimize} \quad \sum_{j=1}^{K} P_j \tag{2a}$$

subject to

$$\frac{t_j - M(I_j)}{\sqrt{2Var(I_j)}} \geq \phi_j \qquad j = 1, \dots, K \tag{2b}$$

$$0 \leq P_j \leq P_j^{(MAX)} \qquad j = 1, \dots, K \tag{2c}$$

$$Var(I_j) \geq 0 \qquad j = 1, \dots, K \tag{2d}$$

$$T - \eta \leq \sum_{j=1}^{K} N_j R_j \leq T + \eta \tag{2e}$$

$$\tag{2f}$$

where K is the number of traffic types supported by the system, $\phi_j = \text{erfc}^{-1}(2P_{out,j})$ and the subscript j denotes the traffic type. P_j, $P_j^{(MAX)}$, N_j, R_j, and T denote the power allocated to the j th service class, transmit power constraint (i.e., maximum power allocated to the j th traffic type), number of active users for each type of traffic, data rate, and throughput, respectively; and $\eta > 0$ is a user- defined constant for limiting the variation of the throughput. The outage threshold t_j is the same as defined in [1], and for the sake of notation simplicity, $M(I_j)$ and $Var(I_j)$ denote the mean and variance of the total multiuser interference including all types of traffic to a reference user of the j th type.

A.2 Imperfect Power Control

The optimization problem in this case can be formulated as

$$\text{minimize} \quad \sum_{j=1}^{K} P_j \tag{2g}$$

subject to

$$\frac{1}{2}\int_0^{\infty} f_j(w)\, \text{erfc}\left(\frac{\dfrac{wPG_jP_j}{(E_{bj}/N_o)_{QR}} - \sigma_T^2 - M(I_j)}{\sqrt{2Var(I_j)}} \right) dw \leq P_{out,j}$$

$$j = 1, \dots, K \tag{2h}$$

$$0 \leq P_j \leq P_j^{(MAX)} \qquad j = 1, \dots, K \tag{2i}$$

$$Var(I_j) \geq 0 \qquad j = 1, \dots, K \tag{2j}$$

$$T - \eta \leq \sum_{j=1}^{K} N_j R_j \leq T + \eta \tag{2k}$$

where $f_j(w)$ is the probability density function of the desired signal power (i.e., lognormal distribution) given in [1].

B INTERRIOR-POINT METHOD

Barrier-function based interior point algorithms are among the most efficient for constrained nonlinear optimization [5]. A barrier method attempts to solve the constrained nonlinear programming problem by solving a sequence of unconstrained subproblems, where the objective function is modified by adding a parameterized barrier term to the original objective function. In this paper, a logarithmic barrier method [5] is used to convert our nonlinear constrained problem to a parameterized unconstrained optimization problem, which is in turn solved using the BFGS algorithm, a well-known quasi-Newton optimization method [7].

B.1 Logarithmic Barrier Function

Consider a constrained optimization problem of minimizing $f(x) = C^T x$, subject to $x \geq 0$. The inequality constraints can be incorporated into the objective function by adding a logarithmic barrier function. Therefore, the subproblem obtained by the logarithmic barrier algorithm is the following unconstrained problem

$$\text{minimize } f_\mu(x) = C^T x - \mu \sum_{i=1}^{n} \ln x_i \tag{3}$$

where μ is a strictly positive scalar known as the barrier parameter. Given an initial strictly feasible point, the effect of the barrier function mainly depends on the magnitude of μ. The optimization begins with solving problem (3) with a fixed $\mu = \mu_1 > 0$, say $\mu_1 = 0.1$. Next, the solution obtained is used as an initial point for problem (3) with a reduced $\mu = \mu_2$, say $\mu_2 = 0.01$. And this iteration continues with a monotonically decreasing μ_k. When the barrier term is heavily weighted, a minimizer of the composite function will lie far away from the boundary. By reducing the coefficient of the barrier parameter μ in each iteration, an unique solution of subproblem can be obtained. As the barrier parameter μ_k decreases to zero, the sequence minimizers will converges to a constrained solution x^*. By applying the above basic idea of the logarithmic barrier algorithm in both perfect power control and imperfect power control scenario, the optimal power assignment strategy for a DS/CDMA system with a number of distinct services yields.

B.2 Identifying a Strictly Feasible Initial Point

Again we consider the constrained problem in Eq. (1). A point x is said to be strictly feasible if it satisfies

$$C_l(x) > 0 \text{ for } l = 1, \ldots, 4K \tag{4}$$

This problem can be tackled by considering the following auxiliary problem

$$\text{minimize } \tilde{c}^T\tilde{x} = \sum_{l=1}^{6K} r_l \tag{5a}$$

$$\text{subject to } r_l + C_l(x) \geq 0 \quad l = 1, \ldots, 4K \tag{5b}$$

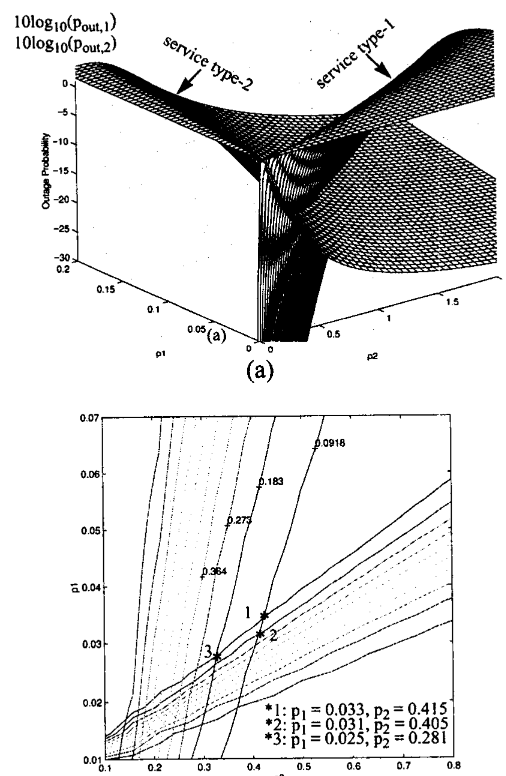
$$r_l \geq -\varsigma \qquad l = 1, \ldots, 4K \tag{5c}$$

where $\tilde{x} = [P_1 \cdots P_K\, r_1 \cdots r_{6K}]^T$, $\tilde{c} = [0 \cdots 0\ 1 \cdots 1]^T$, and ς is a small positive constant. A strictly feasible initial point $\tilde{x}_0$ for problem (4) can be found by first choosing an arbitrary point x_0 and then choosing sufficiently positive r_l such that $r_l + C_l(x_0) \geq 0$ for $1 \leq l \leq 4K$. Obviously, $\tilde{x}_0 = [\, x_0^T\, r_1 \cdots r_{6K}]^T$ is a strictly feasible point for problem (30) with this initial point. As pointed out earlier, this procedure can also be used to find the capacity region of a DS/CDMA system supporting multiple service classes. The capacity region is determined by numerically solving the nonlinear equations representing the service requirements and transmit power constraints, which are in the form of Eqs. (1) and (2). If a feasible point cannot be found for a specified number of calls for each service types and QoS requirements, it means that the system cannot support that traffic composition because it is beyond the system capacity. In this paper, we always assume that problems (1) and (2) are solvable, therefore the approach we just described should provide us with a strictly feasible initial point.

III NUMERICAL RESULTS

In this section, we provide selected numerical examples to demonstrate the effectiveness of the optimization technique to find the optimal power allocation strategy in multimedia DS/CDMA systems. For comparison purpose, we also present a conventional "brute-force" approach (using a graphical method) to determine this optimal value in a network supporting two services ($(E_{b1}/N_o)_{QR,1} = 7dB$, and $PG_1 = 1024$ $(E_{b2}/N_o)_{QR,2} = 9dB$. $PG_2 = 128$

In Fig. 1, the outage probabilities for voice (service type 1) and data (service type 2) are plotted by sweeping the power allocated to both service classes when $N_1 = 25$ and $N_2 = 6$. This figure illustrates the existence of an optimal power allocation law which minimizes the power consumption for a given traffic composition while ensuring all the quality of service (QoS) contracts are met. For instance, when $P_{out,1} = 0.183$ and $P_{out,2} = 0.092$, we can extract the optimal power allocation for the two services from the contour plot (see Fig. 1(b)) as $P_1 = 0.031$ and $P_2 = 0.405$. It is important to stress at this point that the graphical method cannot be used to determine the optimal power assignment for more than two traffic types. Nevertheless, the results obtained from this approach can be used to verify the accuracy of our optimizer (i.e., to see if the minimizer converges to the correct solution).



(b)

Fig. 1. The surface and contour map of the outage probabilities for an integrated network supporting two service classes.

From Fig. 2, we can see that during the first 4 iterations the powers and ratios for all three services (e.g. service 1, service 2 and service 3) are decreasing rapidly. After the 4th iteration, the changes tend to be rather slow till it converges to its optimal value. It is interesting to observe the trail of the barrier function and perform comparison with the curve corresponding to the total power. During the first 2 iterations, the change of the barrier function doesn't match exactly with the change in the total power (constrained objective function). But after a few iterations, the barrier function and objective function almost follows the same trail. This is because the effect of the barrier function mainly depends on barrier parameter μ, and when μ is large, the minimizer of the composite function is far away from the boundary. By reducing μ in each iteration, an unique minimizer of the original objective function $f(x)$ can be obtained by minimizing logarithmic barrier function $F_\mu(x)$. It is noted that the number iterations required to achieve the suboptimal solution heavily rely on the tolerance. If we fix $\varepsilon = 10^{-5}$, the number of iterations required is 7, and as the tolerance becomes stricter, the iterations times needed to converge to optimal point increases exponentially, as anticipated. However, as we can see from Fig. 2, it is not necessary to assign a very small value for the tolerance since the suboptimal power was obtained only after a few iterations.

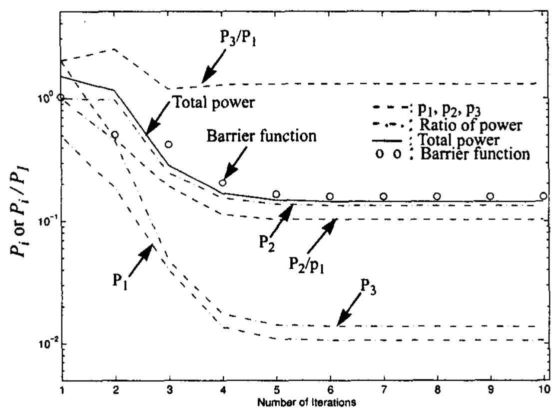


Fig. 2. Logarithmic barrier method for optimal power assignment.

Table 1. The optimal power allocation plan for an integrated network supporting four distinct services with identical outage probability for each traffic types: $QR = [7dB, 9dB, 5dB, 10dB]$, $PG = [1024, 128, 512, 256]$, $\upsilon = [0.375, 1, 0.5, 0.2]$, $P^{(MAX)} = [1, 5, 3, 2]$.

Outage Probability	Number of users N_1	N_2	N_3	N_4	P_1	P_2	P_3	P_4	P_2/P_1	P_3/P_1	P_4/P_1
$P_{out,1} = 0.01$	20	5	5	8	0.162	2.004	0.204	1.228	7.959	1.261	12.382
$P_{out,2} = 0.01$	10	6	5	3	0.236	2.917	0.297	1.875	7.959	1.261	12.383
$P_{out,3} = 0.01$	30	3	8	20	0.136	1.681	0.171	1.081	7.959	1.261	12.381
$P_{out,4} = 0.01$	60	2	10	20	0.091	1.126	0.115	0.724	7.958	1.261	12.376

In Table 1, the optimal power allocation strategy for a multirate multiservice scenario is investigated for the case where all the services have identical outage requirement. It can be seen that the power consumption is largely dependent on the users with high bit rates (i.e., corresponding to lower PGs) and high QoS requirements. For instance, power increases significantly to 2.917 from 2.004 as the number of service type 2 user increases by 1. As well, the ratio P_i/P_1 can be coarsely estimated via relationship,

$$\frac{P_i}{P_1} \approx \frac{PG_1}{PG_i}\frac{(E_{bi}/N_0)}{(E_{b1}/N_0)} \text{ for } i = 2, \ldots, K. \tag{6}$$

Hence, the power assignment strategy is tightly coupled with the ratio of spreading gains as well as the ratio between the quality requirements.

In [2], the authors only determined the power control law in terms for maximizing the capacity (i.e., fully loaded system). Therefore the method developed therein does not address the situations usually encountered in practice, namely, how should the power be allocation in a partially loaded system. In this case, the goal of minimizing power consumption is of paramount importance. Specifically, using our optimization approach we have shown that the ratio $P_i/P_1, i = 2, \ldots, K$ should be kept almost at a constant value regardless whether the system is fully loaded or only partially loaded. This implies that the power control law is not very sensitive to the user traffic composition.

Table 2. Sensitivity of the optimal power allocation criteria to call activity factor: $K = 3$, $P^{(MAX)} = [1, 4, 2]$ $QR = [7dB, 9dB, 5dB]$, $PG = [1024, 128, 512]$, and $P_{out} = [0.01, 0.01, 0.01]$.

Activity factor	Number of users N_1	N_2	N_3	P_1	P_2	P_3	P_2/P_1	P_3/P_1	P_3/P_2
X	20	6	4	0.1271	1.5740	0.1631	12.381	1.2614	0.1019
	20	2	6	0.0113	0.1398	0.0143	12.347	1.2614	0.1022
Y	20	6	4	0.0658	0.8155	0.0829	12.403	1.2605	0.1016
	20	2	6	0.0113	0.1401	0.0143	12.366	1.2604	0.1019
Z	20	6	4	0.0083	0.1050	0.01049	12.631	1.2622	0.0999
	20	2	6	0.0075	0.0940	0.0094	12.612	1.2622	0.1001

Note: X: $\upsilon_1 = 0.375, \upsilon_2 = 1, \upsilon_3 = 0.5$, Y: $\upsilon_1 = 0.375, \upsilon_2 = 0.90, \upsilon_3 = 0.85$, Z: $\upsilon_1 = 0.375, \upsilon_2 = 0.1, \upsilon_3 = 0.2$.

Table 2 shows that the ratio of P_2/P_1, P_3/P_1 and P_3/P_2 remain almost unchanged or vary only slightly by changing ACF for each service. When 1/ACF is increased, the transmission duty cycle is reduced, and thus the probability of simultaneous transmissions of multiple users is reduced. This leads to a reduction in multiuser interference. However, the decrease in the total power consumption is not very substantial by increasing 1/ACF. Our numerical results also reveal that ACF has only minimal influence on the power control law since the ratio does not change very much.

In Table 3, the effect of different outages for different services on the power allocation strategy and total power consumption are examined. It is obvious that power consumption increases as the outage condition of a more vulnerable service becomes stricter. For instance, the total power consumption increases to 0.4643 when $P_{out,2}$ is reduced from 0.05 to 0.01, but it declines to 0.4144 when $P_{out,2} = 0.05$ as compared to the case when $P_{out,2} = 0.005$. It is also apparent that the ratio P_i/P_1 varies with the outage values and the power control law can no longer be estimated via a simple relationship as illustrated in Eq. (6). The high data rate users (i.e., corresponding to a lower PG) would have to reduce the transmit power level in order not to affect the quality of the low data rate users too severely. On the other hand, service type 2 users need to transmit higher power if its outage requirement is made more stringent.

Table 3. The optimal power assignment strategy for a DS/CDMA system supporting three distinct services with dissimilar outage probabilities: $QR = [7dB, 9dB, 5dB]$, $PG = [1024, 128, 512]$, $P^{(MAX)} = [1, 4, 2]$ and $\upsilon = [0.375, 1, 0.5]$.

Outage Probability	Number of users			P_1	P_2	P_3	P_2/P_1	P_3/P_1	P_3/P_2
	N_1	N_2	N_3						
$P_{out,1} = 0.01$	20	6	4	0.1271	1.5740	0.1631	12.381	1.2614	0.1019
$P_{out,2} = 0.01$	70	2	45	0.0318	0.3924	0.0401	12.355	1.2614	0.1021
$P_{out,3} = 0.01$	20	2	6	0.0113	0.1398	0.0143	12.347	1.2614	0.1022
$P_{out,1} = 0.01$	20	6	4	0.0517	0.5881	0.0672	11.366	1.2997	0.1143
$P_{out,2} = 0.05$	70	2	45	0.0297	0.3465	0.0382	11.678	1.2874	0.1102
$P_{out,3} = 0.005$	20	2	6	0.0110	0.1290	0.0141	11.745	1.2847	0.1094
$P_{out,1} = 0.01$	20	6	4	0.2922	3.7394	0.3342	12.798	1.1438	0.0894
$P_{out,2} = 0.005$	70	2	45	0.0306	0.3864	0.0364	12.614	1.1866	0.0941
$P_{out,3} = 0.05$	20	2	6	0.0114	0.1438	0.0136	12.581	1.1933	0.0949

Table 4. The optimal power assignment strategy with imperfect power control for a DS/CDMA system supporting two distinct services with identical outage probability: $QR = [7dB, 9dB]$, $PG = [512, 256]$, $P^{(MAX)} = [1, 4]$ and $\upsilon = [0.375, 1]$.

Number of users	PCE		P_1	P_2	P_2/P_1
	σ_1	σ_2			
$N_1 = 20$, $N_2 = 3$	1.5	1.5	0.39465	1.3860	3.51120
	1.5	1	0.3666	1.1443	3.12177
	2.0	0.5	0.3347	0.9834	2.93796
	2.0	1.5	0.47217	1.46275	3.09828

Table 4 shows the effect of power control errors on the power assignment strategy. The ratio P_2/P_1 increases with a decrease in power control error σ_1. On the other hand P_2/P_1 decreases with a decline in σ_2. Since the requirement for service type 1 is more liberal as compared to that of service type 2, the fluctuation of the power decreases more rapidly with decreasing σ_2 than with decreasing σ_1. It is evident that higher power control error for type 1 service necessitates that service type 2 transmits at less power, thereby causing the ratio P_2/P_1 to decline.

IV. CONCLUSIONS

In this paper, we have developed an efficient optimization technique to determine the appropriate power assignment to distinct services in multimedia networks in a systematic manner. This task of power allocation is crucial in cellular DS/CDMA systems to utilize the available system resources efficiently. Besides reducing the multiple access interference, the battery life can also be extended. Appropriate power assignment controls the mutual interference seen by other users in the system, thereby maximizing the system throughput or minimizes the total power usage for a specified number of active users. Hence, the power allocation rule also provides a guideline for setting the maximum transmit power levels for each of the service classes. We have presented a graphical method as well as an interior point optimization method to optimize the power allocation in the reverse-link for an integrated network with multiple traffic types. In particular, the optimization technique is useful in many practical situations where a large number of variables need to be optimized simultaneously because the graphical method cannot handle the case where a system supports more than two distinct service types. Both perfect and imperfect power-controlled situations have been analyzed. As well, our optimization framework can be directly applied to other constrained optimization related problems (i.e., for different applications) encountered in practice without requiring much programming effort.

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