New Peak-to-Average Power-Ratio Reduction Algorithms for Multicarrier Communications

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***Abstract*—Two new peak-to-average power-ratio (PAPR) reduction algorithms for passband multicarrier systems are deduced by applying optimization to modulation constellations. The associated optimization problem can be solved by two different methods, namely, the second-order cone programming method and the least-pth unconstrained optimization method. Our simulations show that the proposed PAPR-reduction algorithms outperform several existing algorithms.**

I. Introduction

Multicarrier modulation finds applications in both wired and wireless communications [1]-[2]. Well known examples of multicarrier modulation-based systems include digital audio broadcasting (DAB) and digital video broadcasting (DVB) using orthogonal frequency-division multiplexing (OFDM).

A major problem of multicarrier modulation is its large peak-to-average power-ratio (PAPR) which makes system performance very sensitive to distortion introduced by nonlinear devices, e.g., power amplifiers (PAs). In order to mitigate nonlinear distortion, a number of PAPR-reduction techniques and algorithms have been proposed. In [3][4], the authors proposed an approach that generates a set of multicarrier signals and selects the transmit signal with the lowest peak power. This method is computationally efficient but it requires transmission of side information. In [5], PAPR reduction is achieved by modifying the modulation constellation over active subcarriers in a way that will not degrade the BER performance. However, this method does not guarantee an optimal PAPR-reduction solution.

In this paper, two new PAPR-reduction algorithms that lead to improved performance are proposed. In the proposed algorithms, the associated optimization problem can be solved using either a second-order cone programming (SOCP) method [6] or an accelerated least pth unconstrained optimization method [7]. Simulation results are presented to demonstrate that the proposed algorithms outperform the algorithm in [5] and that improved PAPR reduction can be obtained when the algorithm of [4] is combined with the algorithms proposed in this paper.

II. Signal Model

Consider a typical multicarrier transmitter as illustrated in Fig. 1, where S/P, P/S, and D/A represent serial-to-parall-el, parallel-to-serial, and digital-to-analog converter, respectively, and the block labeled as "Amp." represents a power amplifier. The available bandwidth B in the system is divided into N subchannels whose center frequencies are sep-erated by B/N. Each of the subchannels is independently modulated using phase-shift keying (PSK) or quadrature amplitude modulation (QAM). The modulated symbol X_k is referred to as the *subsymbol* in the kth subchannel, and vector $\mathbf{X} = [X_1, \ldots, X_N]^T$ is referred to as the multicarrier symbol.

Fig. 1. Block diagram of a typical multicarrier transmitter.

The multicarrier transmit signal $\mathbf{x}$ can be obtained using the inverse discrete Fourier transform (IDFT) as

$$\mathbf{x} = \mathbf{QX} \tag{1}$$

where $\mathbf{Q}$ is the IDFT matrix with elements $q_{n,k} = (1/N)\, e^{j2\pi(k-1)(n-1)/N}$ for $n,\ k = 1, 2, \ldots, N$. For the multicarrier system in Fig. 1, the PAPR of signal $\mathbf{x}$ is defined as

$$\text{PAPR}_0 = \frac{\|\mathbf{x}\|_\infty^2}{\mathcal{E}[\|\mathbf{x}\|_2^2]/N} \tag{2}$$

where $\mathcal{E}[\cdot]$ denotes expectation, and $\|\mathbf{x}\|_\infty$ and $\|\mathbf{x}\|_2$ represent the infinity- and 2-norm of vector $\mathbf{x}$, respectively.

To minimize the PAPR of the transmit signal, one seeks to find a vector $\mathbf{c}$ that modifies $\mathbf{x}$ to $\mathbf{x} + \mathbf{c}$ such that the PAPR for the modified signal, i.e.,

$$\text{PAPR}\,(\mathbf{c}) = \frac{\|\mathbf{x} + \mathbf{c}\|_\infty^2}{\mathcal{E}[\|\mathbf{x}\|_2^2]/N} \tag{3}$$

is minimized such that the BER performance is not degraded.

Denote the number and index set of active subchannels that are chosen for PAPR reduction as n_a and $\mathcal{I}_a$, respectively. Denote the subvector of $\mathbf{C}$ composed of the elements with indices in $\mathcal{I}_a$ as $\tilde{\mathbf{C}}$. Denote the submatrix of $\mathbf{Q}$ composed of columns with indices in $\mathcal{I}_a$ as $\tilde{\mathbf{Q}}$. On the basis of these assumptions, the above optimization problem can be formulated as

$$\underset{\tilde{\mathbf{C}}}{\text{minimize}} \quad \|\mathbf{x}+\tilde{\mathbf{Q}}\tilde{\mathbf{C}}\|_\infty \tag{4a}$$

$$\text{subject to: } X_k + C_k \text{ feasible for } k \in \mathcal{I}_a \tag{4b}$$

which can be solved via two different approaches.

III. PAPR Reduction for Passband Multicarrier Systems

A. PAPR-reduction algorithm based on SOCP method

The optimization problem in (4) can be formulated as a SOCP problem as follows. Let τ be the upper bound of the objective function in (4a). It can be readily shown that by including parameter τ as an additional design variable, the constrainted optimization problem in (4) can be formulated as

$$\text{minimize} \quad \tau \tag{5a}$$

$$\text{subject to: } \|x_k + \tilde{\mathbf{q}}_k^T\tilde{\mathbf{C}}\| \le \tau \quad \text{for } k=1,\ldots,N \tag{5b}$$

$$X_k + C_k \text{ feasible} \quad \text{for } k \in \mathcal{I}_a \tag{5c}$$

where $\tilde{\mathbf{q}}_k^T$ denotes the kth row of $\tilde{\mathbf{Q}}$ and x_k denotes the kth component of $\mathbf{x}$. Note that in general x_k and vectors $\tilde{\mathbf{q}}_k^T$ and $\tilde{\mathbf{C}}$ are complex valued. Let $x_k = x_{r,k} + jx_{i,k}$, $\tilde{\mathbf{q}}_k = \tilde{\mathbf{q}}_{r,k} + j\tilde{\mathbf{q}}_{i,k}$ and $\tilde{\mathbf{C}} = \tilde{\mathbf{C}}_r + j\tilde{\mathbf{C}}_i$, the constraints in (5b) can be expressed in matrix form as

$$\left\| \begin{bmatrix} x_{r,k} \\ x_{i,k} \end{bmatrix} + \begin{bmatrix} \tilde{\mathbf{q}}_{r,k}^T & -\tilde{\mathbf{q}}_{i,k}^T & 0 \\ \tilde{\mathbf{q}}_{i,k}^T & \tilde{\mathbf{q}}_{r,k}^T & 0 \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{C}}_r \\ \tilde{\mathbf{C}}_i \\ \tau \end{bmatrix} \right\| \le \tau \tag{6}$$

where $k = 1,\ldots,N$. Let $\mathbf{d} = [0\cdots 0\ 1]^T \in \mathcal{R}^{2n_a+1}$ and $\mathbf{y} = [\tilde{\mathbf{C}}_r^T\ \tilde{\mathbf{C}}_i^T\ \tau]^T$, the objective function in (5a) becomes $\mathbf{d}^T\mathbf{y}$ and the constraints in (6) become

$$\|\mathbf{A}_k\mathbf{y} + \mathbf{b}_k\| \le \mathbf{d}^T\mathbf{y} \quad \text{for } k=1,\ldots,N \tag{7a}$$

where

$$\mathbf{A}_k = \begin{bmatrix} \tilde{\mathbf{q}}_{r,k}^T & -\tilde{\mathbf{q}}_{i,k}^T & 0 \\ \tilde{\mathbf{q}}_{i,k}^T & \tilde{\mathbf{q}}_{r,k}^T & 0 \end{bmatrix},\ \mathbf{b}_k = \begin{bmatrix} x_{r,k} \\ x_{i,k} \end{bmatrix}. \tag{7b}$$

Let $S_{rk} = \text{sgn}[\text{real}(X_k)]$, $S_{ik} = \text{sgn}[\text{imag}(X_k)]$, $C_{rk} = \text{real}(C_k)$, and $C_{ik} = \text{imag}(C_k)$. The constraint in (5c) can be formulated as

$$S_{rk}C_{rk} \ \ge\ 0 \quad \text{for } k \in \mathcal{I}_a \tag{8a}$$

$$S_{ik}C_{ik} \ \ge\ 0 \quad \text{for } k \in \mathcal{I}_a. \tag{8b}$$

and can be put together in matrix form as

$$\begin{bmatrix} \mathbf{S}_r & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_i & \mathbf{0} \end{bmatrix} \mathbf{y} \ge \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix} \tag{9}$$

where $\mathbf{S}_r$ and $\mathbf{S}_i$ are $n_a \times n_a$ matrices that can be generated by filling the $(k, i_k)th$ component of the zero matrix of the same size with S_{rk} and S_{ik}, respectively, for $k = 1, 2, \ldots, N$. Under these circumstances, the optimization problem at hand can be formulated as

$$\text{minimize} \quad \mathbf{d}^T\mathbf{y} \tag{10a}$$

$$\text{subject to: } \|\mathbf{A}_k\mathbf{y} + \mathbf{b}_k\| \le \mathbf{d}^T\mathbf{y} \text{ for } k=1,\ldots,N \tag{10b}$$

$$\mathbf{S}\mathbf{y} \ge \mathbf{0} \tag{10c}$$

where

$$\mathbf{S} = \begin{bmatrix} \mathbf{S}_r & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_i & \mathbf{0} \end{bmatrix}. \tag{10d}$$

The problem in (10) is a SOCP problem with $2n_a+1$ variables and $N+2n_a$ constraints, and can be solved efficiently using the method in [6].

B. PAPR-reduction algorithm based on least-pth unconstrained optimization method

The optimization problem in (4) can also be solved in another way. If we denote the kth row of $\tilde{\mathbf{Q}}$ as $\tilde{\mathbf{q}}_k^T$ and kth component of $\mathbf{x}$ by x_k , then the objective function in (4a) is the infinity norm of an N-dimensional vector whose kth component is $x_k + \tilde{\mathbf{q}}_k^T\tilde{\mathbf{C}}$, and the problem in (4) can be formulated as the *minimax* problem

$$\underset{\tilde{\mathbf{C}}}{\text{minimize}}\ \underset{1\le k\le N}{\text{maximize}} \quad \|x_k + \tilde{\mathbf{q}}_k^T\tilde{\mathbf{C}}\| \tag{11a}$$

$$\text{subject to: } X_k + C_k \text{ feasible} \quad \text{for } k \in \mathcal{I}_a \tag{11b}$$

where the norm in (11a) assumes the form

$$\|\hat{\mathbf{x}}_k + \mathbf{P}_k\hat{\mathbf{C}}\| \tag{12a}$$

with

$$\hat{\mathbf{x}}_k = \begin{bmatrix} x_{r,k} \\ x_{i,k} \end{bmatrix},\ \mathbf{P}_k = \begin{bmatrix} \tilde{\mathbf{q}}_{r,k}^T & -\tilde{\mathbf{q}}_{i,k}^T \\ \tilde{\mathbf{q}}_{i,k}^T & \tilde{\mathbf{q}}_{r,k}^T \end{bmatrix},\ \hat{\mathbf{C}} = \begin{bmatrix} \tilde{\mathbf{C}}_r \\ \tilde{\mathbf{C}}_i \end{bmatrix}. \tag{12b}$$

The problem in (11) can be expressed as

$$\underset{\hat{\mathbf{C}}}{\text{minimize}}\ \underset{1\le k\le N}{\text{maximize}} \quad \|\hat{\mathbf{x}}_k + \mathbf{P}_k\hat{\mathbf{C}}\| \tag{13a}$$

$$\text{subject to: } X_k + C_k \text{ feasible} \quad \text{for } k \in \mathcal{I}_a. \tag{13b}$$

As in Sec. III. A, the constraints in (13b) can be formulated as the inequalities in (9). Essentially, these constraints require that the signs of C_{rk} and C_{ik} be identical to those of S_{rk} and S_{ik}, respectively. If we define a sign vector $\mathbf{s}$ as

$$\begin{aligned} \mathbf{s} &= [s_1 \cdots s_{n_a}\ s_{n_a+1} \cdots s_{2n_a}]^T \\ &= [S_{r1} \cdots S_{rn_a}\ S_{i1} \cdots S_{in_a}]^T \end{aligned} \tag{14}$$

then the constraints in (13b) can be eliminated by assuming vector $\hat{\mathbf{C}}$ to be of the form

$$\hat{\mathbf{C}} = [s_1y_1^2 \ \cdots\ s_{n_a}y_{n_a}^2\ s_{n_a+1}y_{n_a+1}^2 \ \cdots\ s_{2n_a}y_{2n_a}^2]^T \tag{15}$$

where y_i is the ith component of an unconstrained parameter vector $\mathbf{y} = [y_1 \cdots y_{2n_a}]^T$, and the problem in (13) is converted to

$$\underset{\mathbf{y}}{\text{minimize}}\ \underset{1\le k\le N}{\text{maximize}} \quad \|\hat{\mathbf{x}}_k + \hat{\mathbf{P}}_k\mathbf{y}^2\| \tag{16}$$

where $\hat{\mathbf{P}}_k$ is obtained by multiplying each column of $\mathbf{P}_k$ by the corresponding component of vector $\mathbf{s}$, and $\mathbf{y}^2$ denotes the vector obtained by componentwise square of vector $\mathbf{y}$. Efficient optimization algorithms are available in the literature that can be used to solve the minimax problem in (16), see, for example, [7], where an accelerated least-pth method is proposed for minimax optimization.

We conclude this section with two remarks:

Remark 1: The proposed PAPR-redcution algorithms do not require transmission of any side information.

Remark 2: For the algorithm in [4], the receiver needs critical side information about the generation process to recover the transmit data. If such side information is manageable, then the proposed algorithms can be combined with the algorithm in [4] to further improve the PAPR-reduction performance.

IV. Simulations

The proposed PAPR-reduction algorithms were applied to several multicarrier communication systems and the PAPR reduction performance was evaluated and compared with that of the algorithms proposed in [4][5]. A commonly used performance measure for PAPR-reduction algorithms is the clipping probability which is defined as the probability that the PAPR of the multicarrier signal exceeds a given PAPR threshold PAPR_0. Since high peaks of analog signal may occur after the D/A conversion [8], oversampling was applied to approximate the analog signal. In our simulations, all algorithms were applied using signals that were oversampled by a factor of 2, the sampling rate is then increased to 8 times the Nyquist sampling rate by a root-raised cosine filter with a rolloff factor of 0.12.

The example presented below consists of two parts. In the first part, the algorithms proposed in Sec. III are simulated and compared with that of [5]. In the second part, under the assumption that the side information required by the method of [4] is available, it is demonstrated that the application of the proposed algorithms with the multicarrier signal selected by the method of [4] as an initial point can further reduce the PAPR considerablely.

Example 1: We considered a multicarrier transmitter with $N = 64$ active subchannels. First, the proposed PAPR-reduction algorithms with various n_a were applied to obtain the optimal peak reduction vector $\mathbf{C}^*$. The clipping probability of the algorithms using SOCP and least-pth methods is plotted in Fig. 2 as dash-dot and solid curves, respectively. Then, the performance of the algorithm in [5], with the target PAPR set to 6 dB, was evaluated and the clipping probability for various numbers of iterations, i.e., $itr = 20$ and 100 is plotted in the same figure as dashed curves. It is observed that performance improvement can be achieved by Jones' method with 20 iterations, but the algorithm's capability reaches its limit after 100 iterations. For example, for a clipping probability of 10^{-3}, 1-dB PAPR-reduction improvement can be achieved by Jones' method using 20 iterations, but only 1.2-dB improvement can be obtained by Jones' method using 100 iterations. On the other hand, the proposed algorithms using the SOCP and least-pth methods achieve a similar performance when the same n_a is used, and they offer a PAPR-reduction performance gain over the method in [5] if n_a is properly assigned. For example, for a clipping probability of 10^{-3} and n_a set to 28, the proposed algorithms using SCOP and least-pth methods offered a 1.1- and 1-dB gain over the algorithm in [5] with $itr = 100$, respectively. As for computational complexity, as can be seen from Fig. 2, both of the proposed PAPR-reduction algorithms with $n_a = 10$ achieve a performance similar to that of Jones' method with 100 iterations. It turned out, however, that the amount of computation required by the proposed algorithm using SOCP and least-pth methods is 17% and 20% less than that required by Jones' method, respectively. With n_a set to 28, the amount of computation required by the proposed algorithm using least-pth method is approximately 75% of that required by the proposed algorithm using SOCP method.

Example 2: We now consider a multicarrier transmitter with 64 active subcarrier where the side information required by method of [4] is available. The performance of the algorithm in [4] with the number of candidate sequences, $U = 4$ and 16, was evaluated and plotted in Fig. 3 and 4 as the dashed curves, respectively. In each case, the proposed algorithm using the least-pth method with the best multicarrier signal selected by the method of [4] as an initial point was then applied. The PAPR-reduction performance achiev-ed after 1 and 2 iterations is shown in the same figures as solid lines. For the case of $U = 4$, the performance gain was 0.9 dB after 1 iteration and 1.5 dB after 2 iterations. For the case of $U = 16$, the performance gain was 0.2 dB after 1 iteration and 0.7 dB after 2 iterations.

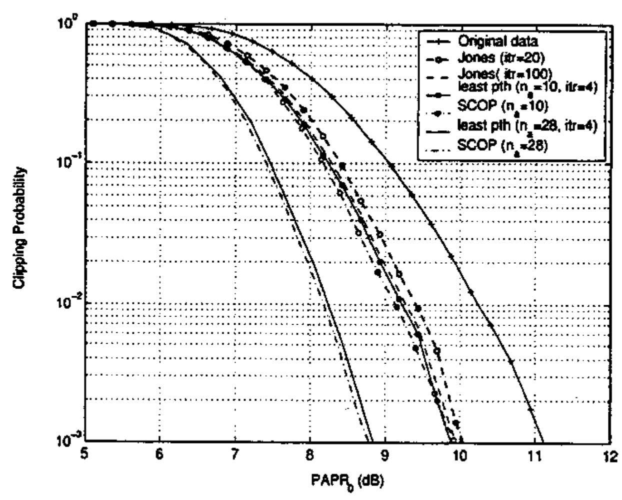

Fig. 2. Performance comparison of different PAPR-reduction algorithms.

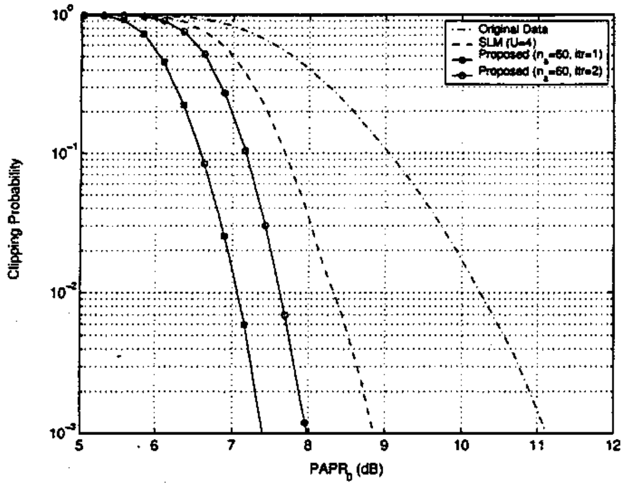

Fig. 3. Performance comparison of different PAPR-reduction algorithms.

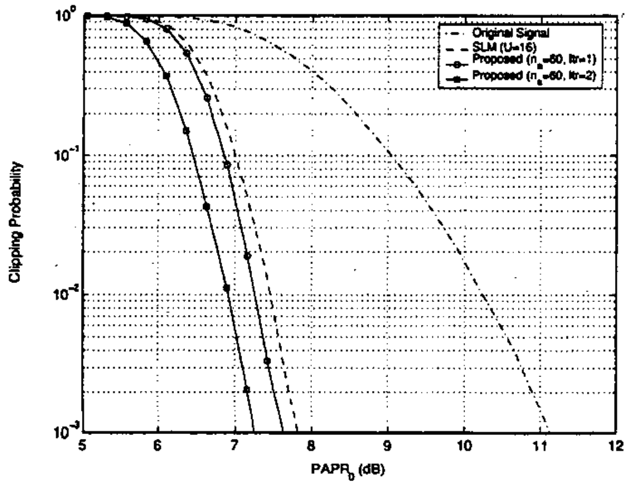

Fig. 4. Performance comparison of different PAPR-reduction algorithms.

V. Conclusions

Two new PAPR-reduction algorithms for passband multicarrier systems have been proposed by applying optimal constellation modification in active subcarriers. Our simulations have demonstrated that, in many practical situations, considerable performance improvement can be achieved by the proposed PAPR-reduction algorithms over several existing algorithms.

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