DESIGN OF STABLE IIR DIGITAL FILTERS WITH EQUIRIPPLE PASSBANDS AND PEAK-CONSTRAINED LEAST SQUARES STOPBANDS

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ABSTRACT

This paper describes an algorithmic development for the design of stable IIR digital filters with equiripple passbands and peak-constrained least squares stopbands (ERPP-CLSS). Central to the development is a re-formulation of the design problem as an iterative quadratic programming problem where the stability and ERPPC requirements are met by imposing a system of linear inequalities constraints while the LSS property is satisfied by employing a weighted least squares type objective function.

1. INTRODUCTION

Recursive digital filters offer improved selectivity, computation efficiency and reduced system delay compared to what can be achieved by nonrecursive digital filters of comparable approximation accuracy [1]. However, popular design methods such as minimax design techniques [2][3] that are based on the Remez exchange algorithm, various window methods [4]-[6], and weighted or unweighted least squares methods proposed in [7]-[10] are difficult to extend to the IIR case particularly when filter stability is an integral part of the design specifications.

As was argued in [11], practically useful bandpass filters should possess equiripple passbands and peak-constrained least squares stopbands (ERPPCLSS). In a series of publications [11]-[14], the design of linear-phase ERPPCLSS FIR digital filters is investigated using a weighted least squares approach and efficient numerical optimization methods for solving the associated quadratic programming problem are proposed. This paper describes an algorithmic development for the design of stable ERPPCLSS IIR filters that approximate given frequency responses. We look at the design problem as approximating a given compelx-valued data acquence by a proper rational function with real-valued or complex-valued coefficients over the *entire* fequency baseband subject to various constraints. Design algorithms developed in this general framework are directly applicable to the design of symmetric or asymmetric frequency responses. Central to the development is a re-formulation of the design problem as an iterative quadratic programming (QP) problem where the stability and ERPPC requirements are met by imposing a system of linear inequality constraints while the LSS property is satisfied by employing a weighted least squares objective function with properly chosen weights. An early effort of designing stable IIR filters using a weighted least squares approach was made in [15], however the method was only able to achieve least squares designs rather than filters with ERPPCLSS.

2. PROBLEM STATEMENT

Let $h_d(\omega)$ be the desired frequency response of bandpass nature, which is given on $[-\pi, \pi]$. For notation simplicity, here (and throughout the paper) we use $h_d(\omega)$ to mean $h_d(e^{j\omega})$, and assume that $h_d(\omega)$ has only one passband and one stopband. We seek to find a causal IIR transfer function of order n, i.e.,

$$h(z) = \frac{b(z)}{d(z)} = \frac{\sum_{i=0}^{n} b_i z^{-i}}{\sum_{i=0}^{n} d_i z^{-i}} \qquad \text{with } d_0 = 1 \tag{1}$$

such that

$$J(x) = \frac{1}{2}\int_{-\pi}^{\pi} W(\omega)|h_d(\omega) - h(\omega)|^2 d\omega \tag{2}$$

is minimized subject to the following constraints: (i) $h(z)$ is stable; (ii) $h(z)$ is peak-constrained in the stopband, i.e., $|h(\omega)| \le \delta_a$ for $\omega \in \mathcal{S}_a$ where $\mathcal{S}_a$ denotes a set of grid points in the stopband; (iii) $h(z)$ is equiripple in the passband. It follows from [11] that with appropriately chosen weight $W(\omega) \ge 0$, $h(\omega)$ becomes equiripple in the passband if the constraints

$$||h(\omega)| - 1| \le \delta_p \qquad \omega \in \mathcal{S}_p \tag{3}$$

are imposed where $\mathcal{S}_p$ is a set of grip points in the passband. The vector x in (2) collects the coefficient vectors of $d(z)$ and $b(z)$, denoted by d and b respectively, as $x^T = [d^T\ b^T]^T$. Note that the above problem formulation is general enough to include designing real-valued as well as complex-valued coefficient digital filters, and approximating symmetric or asymmetric frequency responses.

3. ANALYSIS AND PROBLEM REFORMULATION

3.1. Formulating the design task as an iterative QP problem

We write (2) as

$$J(x) = \frac{1}{2}\int_{-\pi}^{\pi} \hat{W}(\omega)|h_d(\omega)d(\omega) - b(\omega)|^2 d\omega \tag{4}$$

where $\hat{W}(\omega) = W(\omega)/|d(\omega)|^2$. It follows that $\hat{W}(\omega) \ge 0$ for $\omega \in [-\pi, \pi]$ as long as $d(z)$ is stable. In such a case we may view (4) as a "weighted" least squares problem if $\hat{W}(\omega)$ were a *known* weight. This suggests an iterative weighted least squares scheme in which one starts by selecting a stable polynomial $d^{(0)}(z)$ (e.g., $d^0(z) \equiv 1$) and finds at the kth iteration polynomials $b^{(k)}(z)$ and $d^{(k)}(z)$ that minimize

$$J^{(k)} = \frac{1}{2}\int_{-\pi}^{\pi} \hat{W}_{k-1}(\omega)|h_d(\omega)d^{(k)}(\omega) - b^{(k)}(\omega)|^2 d\omega \quad (5)$$

subject to the stability of $d^{(k)}(z)$. Note that the weight $\hat{W}_{k-1}(\omega)$ given by $\hat{W}_{k-1}(\omega) = W(\omega)/|d^{(k-1)}(\omega)|^2$ is a known, positive function on $[-\pi, \pi]$, hence $J^{(k)}$ is quadratic with respect to $x_k = [d^{(k)T} \ b^{(k)T}]^T$. Now by writing $d^{(k)}(\omega) = 1 + \Omega_1^T d^{(k)}$, $b^{(k)}(\omega) = \Omega_0^T b^{(k)}$ with $\Omega_1 = [e^{-j\omega} \ e^{-j2\omega} \ \cdots \ e^{-jn\omega}]^T$, $\Omega_0 = [1 \ e^{-j\omega} \ \cdots \ e^{-jn\omega}]^T$, the error function $J^{(k)}$ can be expressed as

$$J^{(k)}(x_k) = \frac{1}{2}x_k^T H_k x_k + x_k^T p_k + \text{const} \quad (6)$$

$$H_k = \begin{bmatrix} H_{11k} & H_{12k} \\ H_{12k}^T & H_{22k} \end{bmatrix} \quad (7a)$$

$$H_{11k} = \int_{-\pi}^{\pi} \hat{W}(\omega)|h_d(\omega)|^2(\Omega_1\Omega_1^H)\,d\omega \quad (7b)$$

$$H_{12k} = -\int_{-\pi}^{\pi} \hat{W}(\omega)\mathrm{Re}[h_d(\omega)\Omega_1\Omega_0^H]\,d\omega \quad (7c)$$

$$H_{22k} = \int_{-\pi}^{\pi} \hat{W}(\omega)(\Omega_0\Omega_0^H)\,d\omega \quad (7d)$$

$$p_k = \begin{bmatrix} p_{1k}^T & p_{2k}^T \end{bmatrix} \quad (8a)$$

$$p_{1k} = \int_{-\pi}^{\pi} \hat{W}(\omega)|h_d(\omega)|^2 \mathrm{Re}(\Omega_1)\,d\omega \quad (8b)$$

$$p_{2k} = -\int_{-\pi}^{\pi} \hat{W}(\omega)\mathrm{Re}[h_d(\omega)\bar{\Omega}_0]\,d\omega \quad (8c)$$

Stability of $d^{(k)}(z)$ is guaranteed if $\mathrm{Re}[d^{(k)}(\omega)] \geq \delta > 0$ for $\omega \in [-\pi, \pi]$ where $\delta > 0$ is a positive lower bound to ensure a reasonable stability margin [16]. In practice the above stability constraint is implemented on a set of grid points, $\mathcal{S}_s = \{\omega_i^{(s)}, \ i = 1, \ \ldots, \ n_s\}$, which leads to

$$A_s x_k \leq (1-\delta)e_{n_s} \quad (9a)$$

$$A_s = -\begin{bmatrix} \cos\omega_1^{(s)} & \cdots & \cos n\omega_1^{(s)} & \\ \vdots & & \vdots & 0 \\ \cos\omega_{n_s}^{(s)} & \cdots & \cos n\omega_{n_s}^{(s)} & \end{bmatrix}_{n_s\times(2n+1)} \quad (9b)$$

$$e_{n_s} = [1 \ 1 \ \ldots \ 1]_{n_s\times 1}^T \quad (9c)$$

So now we have a typical QP problem

$$\min_{x_k} J^{(k)}(x_k) \quad (10)$$
$$\text{subject to (9a)}$$

It can readily be shown that the Hessian mateix of $J^{(k)}$, namely H_k in (7), is positive definite. Therefore unique solution to the QP problem (10) exists [17][18]. This solution provides a stable IIR transfer function $h^{(k)}(z) = b^{(k)}(z)/d^{(k)}(z)$. If $h^{(k)}(z)$ converges to $h(z) = b(z)/d(z)$ as $k \to \infty$, then (9) leads to $\mathrm{Re}[d(\omega)] \geq \delta > 0$ for $\omega \in [-\pi, \pi]$ which guarantees the stability of $h(z)$, $\hat{W}_{k-1}(\omega) \to W(\omega)/d(z)$, and $J^{(k)} \to J(x)$. Thus we see $h(z) = b(\omega)/d(\omega)$ as a stable and truly least squares solution.

3.2. Constraints for ERPPCLSS

For notation simplicity here we only consider designing lowpass filters with nearly constant group delay in the passband. Throughout the paper, the normalized passband and stopband edges are denoted by ω_p and ω_a, respectively.

A. Weighting Function $W(\omega)$

We shall use a piecewise constant weighting function defined by $W(\omega) = 1$ for $|\omega| \leq \omega_p$, and w_a for $\omega_a \leq |\omega| \leq \pi$. With a sufficiently large w_a and a set of amplitude constraints on $[-\omega_p, \omega_p]$, the filter obtained becomes virtually equiripple in the passband [11].

B. Amplitude Constraints on $[0, \omega_p]$

To explain how appropriate amplitude constraints in the passband for the transfer function in the kth iteration can be deduced, let us suppose the algorithm converges. Then for an $\omega \in [0, \omega_p]$ and sufficiently large k, we have

$$h(\omega) \approx \frac{b^{(k)}(\omega)}{d^{(k-1)}(\omega)} \approx e^{-jK\omega}R(\omega) \quad (11)$$

where K is the desired group delay of the filter in the passband and

$$R(\omega) = \left|\frac{b^{(k)}(\omega)}{d^{(k-1)}(\omega)}\right| \approx |h(\omega)| \quad (12)$$

Now we write $d^{(k-1)}(\omega)$ as $d^{(k-1)}(\omega) = e^{j\theta_{k-1}(\omega)}|d^{(k-1)}(\omega)|$ It follows that

$$R(\omega) \approx \frac{b^{(k)T}(e^{j[K\omega-\theta_{k-1}(\omega)]}\Omega_0)}{|d^{(k-1)}(\omega)|} \quad (13)$$

which implies $R(\omega) \approx c_k^T(\omega)b_k/|d^{(k-1)}(\omega)|$ with

$$c_k(\omega) = \begin{bmatrix} \cos[K\omega - \theta_{k-1}(\omega)] \\ \cos[(K-1)\omega - \theta_{k-1}(\omega)] \\ \vdots \\ \cos[(K-n)\omega - \theta_{k-1}(\omega)] \end{bmatrix} \quad (14)$$

By (3) and (12), we see that in order to impose the amplitude constraints for $\omega \in [0, \omega_p]$ we would require that $|R(\omega) - 1| \leq \delta_p$ which leads to

$$c_k^T(\omega)b^{(k)} \leq |d^{(k-1)}(\omega)|(1+\delta_p) \quad \omega \in [0, \omega_p] \quad (15a)$$
$$-c_k^T(\omega)b^{(k)} \leq |d^{(k-1)}(\omega)|(\delta_p - 1) \quad \omega \in [0, \omega_p] \quad (15b)$$

Note that at the kth iteration $c_k(\omega)$ defined in (14) and $|d^{(k-1)}(\omega)|$ are *known*, hence contraints (15) are linear with respect to $b^{(k)}$. In practice the constraints (15) can only be imposed at a set of grid points on $[0, \omega_p]$, $\mathcal{S}_p = \{\omega_i^{(p)}, \ i = 1, \ \ldots, \ n_p\}$. Constraints (15) on $\mathcal{S}_p$ can be put together in a matrix form as

$$A_p^{(k)} x_k \leq q_p^{(k)} \quad (16a)$$

where

$$A_p^{(k)} = \begin{bmatrix} & c_k^T(\omega_1^{(p)}) \\ & \vdots \\ & c_k^T(\omega_{n_p}^{(p)}) \\ 0 & -c_k^T(\omega_1^{(p)}) \\ & \vdots \\ & -c_k^T(\omega_{n_p}^{(p)}) \end{bmatrix}, \quad q_p^{(k)} = \begin{bmatrix} (1+\delta_p)|d^{(k-1)}(\omega_1^{(p)})| \\ \vdots \\ (1+\delta_p)|d^{(k-1)}(\omega_{n_p}^{(p)})| \\ (\delta_p - 1)|d^{(k-1)}(\omega_1^{(p)})| \\ \vdots \\ (\delta_p - 1)|d^{(k-1)}(\omega_{n_p}^{(p)})| \end{bmatrix} \quad (16b)$$

Note that if $h^{(k)}(z) \rightarrow h(z) = b(z)/d(z)$ as $k \rightarrow \infty$, then $c_k(\omega) \rightarrow \text{real}(\Omega_0)$, and (30) implies that $|b(\omega)/d(\omega) - 1| \leq 1 - \delta_p$ for $\omega \in \mathcal{S}_p$. With sufficiently large number of sampling points in $\mathcal{S}_p$, this leads to an IIR filter with equiripple passband.

C. Peak-Constrained Stopband

In stopband $[\omega_a,\ \pi]$, for sufficiently large k the peak-constraints can be expressed approximately as

$$\left| \frac{b^{(k)}(\omega)}{d^{(k-1)}(\omega))} \right| \leq \delta_a \tag{17}$$

where δ_a is the maximum stopband ripple. Let $c(\omega) = [1\ \cos\omega\ \cdots\ \cos n\omega]^T$, $s(\omega) = [0\ \sin\omega\ \cdots\ \sin n\omega]^T$, it is easy to verify that constraints

$$\left|c^T(\omega)b^{(k)}\right| \leq \frac{1}{2}\delta_a \left|d^{(k-1)}(\omega)\right| \tag{18a}$$

$$\left|s^T(\omega)b^{(k)}\right| \leq \frac{1}{2}\delta_a \left|d^{(k-1)}(\omega)\right| \tag{18b}$$

imply (17). Compared to (17), constraints (18) are easier to work with as all entities associated with $b^{(k)}$ there are of real value. Imposing (18) on the set of grid points $\mathcal{S}_a = \{\omega_i^{(a)},\ i = 1,\ \ldots,\ n_a\}$ in the stopband, we can write the modified constraints as

$$A_a^{(k)} x_k \leq q_a^{(k)} \tag{19a}$$

$$A_a^{(k)} = \begin{bmatrix} & C(\omega) \\ & -C(\omega) \\ 0 & S(\omega) \\ & -S(\omega) \end{bmatrix},\ q_a^{(k)} = \frac{\delta_a}{2}\begin{bmatrix} D(\omega) \\ D(\omega) \\ D(\omega) \\ D(\omega) \end{bmatrix} \tag{19b}$$

$$C(\omega) = \begin{bmatrix} c^T(\omega_1^{(a)}) \\ \vdots \\ c^T(\omega_{n_a}^{(a)}) \end{bmatrix},\ S(\omega) = \begin{bmatrix} s^T(\omega_1^{(a)}) \\ \vdots \\ s^T(\omega_{n_a}^{(a)}) \end{bmatrix},$$

$$D(\omega) = \begin{bmatrix} |d^{(k-1)}(\omega_1^{(a)})| \\ \vdots \\ |d^{(k-1)}(\omega_{n_a}^{(a)})| \end{bmatrix}$$

It follows that if $h^{(k)}(z) \rightarrow h(z) = b(z)d(z)$ as $k \rightarrow \infty$, then (18) leads to $|c^T(\omega)b| \leq \delta_a|d(\omega)|/2$, $|s^T(\omega)b| \leq \delta_a|d(\omega)|/2$ for $\omega \in \mathcal{S}_a$. Hence $|b(\omega)| \leq |c^T(\omega)b| + |s^T(\omega)b| \leq \delta_a|d(\omega)|$ i.e., $|h(\omega)| \leq \delta_a$, for $\omega \in \mathcal{S}_a$ suggesting that the peak of $|h(\omega)|$ is constrained in the stopband.

4. THE DESIGN ALGORITHM

In summary, a core step in our algorithm has now been formulated as an iterative QP problem where the objective function in the kth iteration is given by

$$J^{(k)}(x_k) = \frac{1}{2}x_k^T H_k x_k + x_k^T p_k + \text{const} \tag{20}$$

The stability and ERPPCLSS properties of the filter are assured by choosing weight w_a sufficiently large (say, $w_a = 10^3$) and by imposing linear inequality constraints

$$A_k x_k \leq q_k \tag{21}$$

where $A_k = [A_s^T\ A_p^{(k)T}\ A_a^{(k)T}]^T$ and q_k is a column vector that stacks $(1-\delta)e_n$, in (9), $q_p^{(k)}$ in (16), and $q_a^{(k)}$ in (19) as $q_k = [(1-\delta)e_n^T,\ q_p^{(k)^T}\ q_a^{(k)^T}]^T$. There are several numerical algorithms that can be used to solve QP problems efficiently with the active set method as the most noticeable [17]–[18]. Among other numerical computation software, the optimization toolbox associated with MATLAB includes a command qp that can be used as an efficient QP solver.

Once a solution to the QP problem

$$\min_{x_k} J^{(k)}(x_k) \tag{22a}$$

$$\text{subject to } A_k x_k \leq q_k \tag{22b}$$

say x_k, is obtained, x_k is modified by combining it linearly with x_{k-1} obtained from the preceding iteration to yield a *refined* $x_k := \tau x_k + (1-\tau)x_{k-1}$ where $0 < \tau < 1$ is a relaxation parameter. This refined x_k is then used in the next iteration. The iteration continues until the difference $||x_k - x_{k-1}||_2$ is less than a prescribed tolerance ε.

Although solutions to which the algorithm converges from any initial parameter vector x_0 are stable, the high degree of nonlinearity of the associated minimization problem implies that many of these solutions do not correspond to satisfactory designs. A feasible way to obtain a good initial point for the QP problem (22) is to find a solution x that minimuzes $J(x)$ subject only to the stability constraint $\text{Re}[d(\omega)] \geq \delta > 0$. This can also be accomplished in an iterative manner. That is, we solve the QP problem (10) to find x_k, then obtain a refined x_k, and repeat this procedure until $||x_k - x_{k-1}|| < \varepsilon$. An initial point for this simplified problem can be assigned as $x_0 = [1\ 0\ \ldots\ 0]^T \in R^{2n \times 1}$ which corresponds to $d^{(0)}(z) = 1$ and $b^{(0)}(z) = 0$. Obviously, the solution to the above problem is a weighted least squares design which turns out to be good enough to serve as an initial point for the QP problem (22).

5. AN ILLUSTRATIVE EXAMPLE

Let us consider designing a stable, 15th-order, lowpass IIR filter with ERPPCLSS. The design specifications are as follows: the normalized passband and stopband edges are $\omega_p = 0.2$ and $\omega_a = 0.28$, maximum passband ripple ≤ 0.2 dB, minimum stopband attenuation ≥ 30 dB, and maximum deviation of group delay in passband $\leq 1.5\%$. To meet these design specifications δ_p and δ_a were set to be $\delta_p = 0.012$ and $\delta_a = 0.032$. Since the desired frequency response is symmetric with respect to $\omega = 0$, the integrals involved in computing H_k and p_k in (6) can be carried out on $[0,\ \pi]$ rather than $[-\pi,\ \pi]$. One hundred frequency samples evenly placed on (normalized) frequency range $[0,\ 0.5]$ were used to numerically evaluate H_k and p_k. The weights are assigned as $W(\omega_i) = 1$ for $1 \leq i \leq 42$, 0 for $43 \leq i \leq 58$, and 100 for $59 \leq i \leq 100$. To ensure the stability of the filter designed, 20 frequency samples evenly placed on $[0,\ 0.5]$ were used to form set $\mathcal{S}_s$. The set $\mathcal{S}_p$ contains 20 frequency samples that are evenly plased on passband $[0,\ 0.2]$ to ensure an equiripple passband. Since large stopband ripples usually occur in a vicinity of stopband edge ω_a, set $\mathcal{S}_a$ contains only two frequencies $\omega_1^{(a)} = 0.28$ and $\omega_2^{(a)} = 0.33$. As is described above, a simplified QP problem, namely problem (10) was first solved to obtain an adequate initial point. The corresponding coefficients of $d^{(0)}(z)$ and $b^0(z)$ are listed in Table 1. With a torelance $\varepsilon = 5 \times 10^{-4}$, the algorithm took 26 iterations to converge, the optimized transfer function coefficients are given in Table 1. It was found that maximum modulas of the filters poles $= 0.8263$, maximum passband ripple = 0.1884 dB, minimum stopband attenuation = 31.8428 dB, and maximum deviation of group delay in passband = 1.25%. Graphic display of the filter's frequency responses is shown in Figures 1 and 2. As can be observed from Fig. 2(a), the passband is practically equiripple. From Fig. 2(b), we see that the phase response in passband is nearly linear.

Table I. Design Results

Initial		Optimized	
Denominator Polynomial	Numerator Polynomial	Denominator Polynomial	Numerator Polynomial
1.00000e+0	9.78850e-3	1.00000e+0	-5.22654e-3
-1.60788e+0	4.11375e-3	-1.56463e+0	1.94457e-2
1.75411e+0	-1.30125e-2	1.73980e+0	-1.15035e-2
-1.55198e+0	-1.18216e-2	-1.57008e+0	-7.20118e-3
1.35927e+0	1.89069e-2	1.39721e+0	5.93816e-5
-1.20941e+0	1.99740e-2	-1.27637e+0	2.79188e-2
1.00570e+0	-2.27399e-2	1.10667e+0	-1.48195e-2
-7.85493e-1	-3.16836e-2	-9.11686e-1	-2.49149e-2
6.06522e-1	2.34823e-2	7.59812e-1	2.88456e-3
-4.42517e-1	5.03890e-2	-6.20832e-1	5.06928e-2
2.84839e-1	-2.38328e-2	4.59430e-1	-1.14557e-2
-1.68250e-1	-7.17259e-2	-3.16294e-1	-6.05705e-2
9.16838e-2	-3.25680e-4	2.19661e-1	-2.08870e-2
-3.17702e-2	1.33954e-1	-1.39322e-1	1.26918e-1
-3.21645e-3	1.47605e-1	6.19802e-2	1.60440e-1
6.27767e-3	7.80542e-2	-1.31287e-2	1.03666e-1

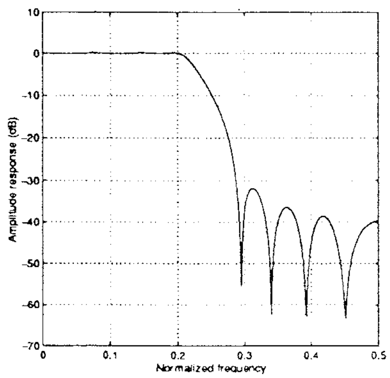

Figure 1. Amplitude response of the filter.

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References

[1] A. Antoniou, *Digital filters: Analysis, Design, and Applications*, 2nd ed., McGraw-Hill, 1993.

[2] T.W. Parks and J.H. McClellan, "Chebyshev approximation for nonrecursive digital filters with linear phase," *IEEE Trans. Circuits Theory*, vol. 19, pp. 189–194, March 1972.

[3] T.W. Parks and J.H. McClellan, "A program for the design of linear phase finite impulse response digital filters," *IEEE Trans. Audio Electroacoust.*, vol. 20, pp. 195–199, Aug. 1972.

[4] J.F. Kaiser, "Nonrecursive digital filter design using the I_0-sinh window function," in *Proc. IEEE Int. Symp. Circuit Theory*, pp. 20–23, 1974.

[5] L.R. Rabiner and B. Gold, *Theory and Application of Digital Signal Processing*, Prentice-Hall, 1975.

[6] A.V. Oppenheim and R.W. Schafer, *Discrete-Time Signal Processing*, Prentice-Hall, 1989.

[7] D.C. Farden and L.L. Scharf, "Statistical design of nonrecursive digital filters," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 22, pp. 188–196, June 1974.

[8] V.R. Algazi, M. Suk, and C.S. Rim, "Design of almost minimax FIR filters in one and two dimensional by WLS technique," *IEEE Trans. Circuits Syst.*, vol. 33, pp. 590–596, June 1986.

[9] Y.C. Lim, J.H. Lee, C.K. Chen, and R.H. Yang, "A weighted least squares algorithm for quasi-equiripple FIR and IIR digital filter design," *IEEE Trans. Acoust, Speech, Signal Processing*, vol. 40, pp. 551–558, 1992.

[10] S.C. Pei and J.J. Shyu, "Design of arbitrary FIR log filters by weighted least squares tehnique," *IEEE Trans. Signal Processing*, vol. 42, pp. 2495–2499, Sept. 1994.

[11] J.W. Adams, "FIR digital filters with least-squares stopbands subject to peak-gain constraints," *IEEE Trans. Circuits Syst.*, vol. 38, pp. 376–388, 1991.

[12] J.W. Adams, J.L. Sullivan, R. Hashemi, C. Ghadimi, J. Franklin, and B. Tucker, "New approaches to constrained optimization of digital filters," in *Proc. IEEE Int. Symp. Circuits Syst.*, pp. 80–83, May 1993.

[13] J.W. Adams, P. Kruethong, R. Hashimi, J.L. sullivan, and D. Gleeson, "New quadratic programming algorithms for designing FIR digital filters," in *Proc. 27th Asilomar Conf.*, pp. 1206–1210, Nov. 1993.

[14] J.L. Sullivan and J.W. Adams, "A new nonlinear optimization algorithm for asymmetric FIR digital filters," in *Proc. IEEE Int. Symp. circuits Syst.*, pp. 541–544, May 1994.

[15] W.-S. Lu, S.-C. Pei, and C.-C Tseng, "A weighted least squares method for the design of stable 1-D and 2-D IIR digital filters," submitted to *IEEE Trans. Signal Processing*, May 1995.

[16] E.A. Robinson, *Statistical Communication and Detection*, New York: Hafner, 1967.

[17] D.G. Luenberger, *Linear and Nonlinear Programming*, 2nd ed., Addison-Wesley, 1984.

[18] R. Fletcher, *Practical Methods of Optimization*, 2nd ed., John Wiley, 1987.

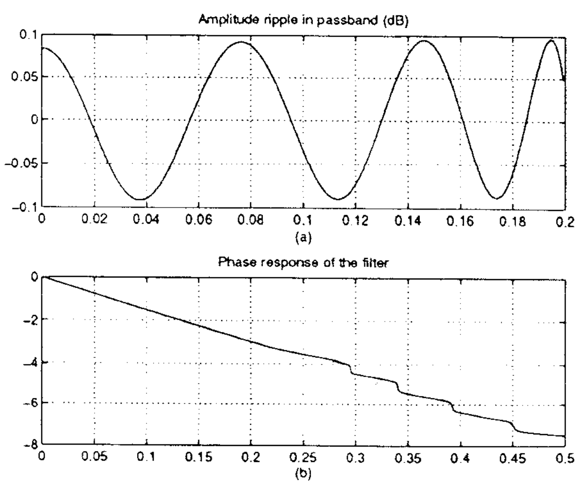

Figure 2. (a) Amplitude ripple in passband, (b) phase response of the filter.