Sequential Design of FIR Digital Filters for Low-Power DSP Applications

W.-S. Lu, A. Antoniou, and S. Saab
Dept. of Electrical and Computer Engineering
University of Victoria
Victoria, B.C., Canada, V8W 3P6
wslu, andreas, ssaab@ece.uvic.ca

Abstract

A method for the design of FIR digital filters with low power consumption is proposed. In this method, the digital filter is implemented as a cascade arrangement of low-order sections. The first section is designed through optimization so as to satisfy as far as possible, the overall required specifications. The first section is then fixed and a second section is added, which is designed so that the first two sections in cascade satisfy again as far as possible the overall required specifications. This process is repeated until a multisection filter is obtained that would satisfy the required specifications under the most critical circumstances imposed by the application at hand. In multisection filters of this type, the minimum number of sections required to process the current input signal can be switched in through the use of a simple adaptation mechanism and, in this way, the power consumption can be minimized. This design strategy is achieved by formulating the design of the k-th section as a weighted least-squares minization problem, assuming that an optimum (k −1)-section design is available.

1. Introduction

Digital filters are commonly designed to assure a desirable performance for all possible signals that may be encountered. However, for many DSP applications, the worst-case conditions that set the filter specifications are rarely present. This leads to wasted power and computational resources. On the other hand, many electrical systems such as portable wireless systems are critically dependent on power availability. Many other systems such as multitasking systems depend, in addition, on the availability of computational resources. Recent studies have shown that these dependencies can be alleviated by dynamically adjusting the complexity of the algorithms used according to predetermined criteria and the changing signal characteristics [1]–[3].

In this paper, a method for the design of FIR digital filters for low-power applications is proposed. The main idea is to design the digital filter as a collection of low-order cascaded sections that can be turned off when they are not needed so as to conserve power and/or reduce the computational complexity. The sections are designed individually from left to right such that the sections to the left satisfy specifications in an optimal manner. It is shown that this design problem can be solved in a sequential weighted least-squares (WLS) minimization setting where each WLS minimization yields the impulse response of one more section to be added to the sections that have been previously designed.

2. Problem formulation

The FIR filter to be design in order to approximate a desired frequency response, $H_d(\omega)$, assumes the casecade structure shown in Fig. 1. For the sake of convenience, we consider a typical problem of designing a linear-phase, lowpass digital filter. Let the first section be a linear-phase lowpass filter of length $L+1$ and assume that its transfer function is of the form

$$H_0(z) = \left(\frac{1+z^{-1}}{2}\right)^L \tag{1}$$

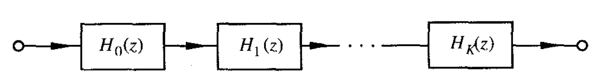


Figure 1: Cascade FIR filter.

If we use $H_0(\omega)$ to denote its frequency response, then

$$H_0(0) = 1$$
$$\frac{d^l}{d\omega^l}[H_0(\omega)]_{\omega=\pi} = 0, \quad 0 \le l \le L-1$$

1058-6393/98 $10.00 © 1998 IEEE

The L zeros at $\omega = \pi$ imply that $H_0(\omega)$ is *flat* at $\omega = \pi$. Since the overall transfer function is given by

$$G_K(z) = \prod_{k=0}^{K} H_k(z)$$

it is easy to see that $G_K(z)$ has at least L zeros at $\omega = \pi$. The number of zeros of a lowpass filter at $\omega = \pi$ can be related to the number of *vanishing moments* of an associated wavelet function [4] which has been found desirable for multirate DSP systems when the smoothness of the associated wavelet is of importance for the application at hand.

The sequential design strategy is as follows. Given $H_0(z)$, the transfer function of section 1, i.e., $H_1(z)$, is obtained. With $H_0(z)$ and $H_1(z)$ known, section 2, represented by $H_2(z)$, is designed, and so on. When section $k-1$ has been designed, $H_0(z)$, $H_1(z)$, ..., $H_{k-1}(z)$ are known, and the overall transfer function of sections 0 to $k-1$ in cascade is given by

$$G_{k-1}(z) = \prod_{l=0}^{k-1} H_l(z)$$

At this point, we need to obtain the transfer function of section k, namely, $H_k(z)$, such that

$$J_k = \int_0^\pi W(\omega)|H_d(\omega) - G_{k-1}(\omega)H_k(\omega)|^2 d\omega \qquad (2)$$

is minimized, where $W(\omega) \geq 0$ is a weighting function. If the desired phase response as well as the phase responses of sections 0 to k are all linear, then the phase factors in $H_d(\omega)$, $G_{k-1}(\omega)$, and $H_k(\omega)$ can be eliminated. In such a case, $H_d(\omega)$, $G_{k-1}(\omega)$, and $H_k(\omega)$ can be assumed to be real-valued functions, and (2) can be written as

$$J_k = \int_0^\pi W(\omega)[H_d(\omega) - G_{k-1}(\omega)H_k(\omega)]^2 d\omega \qquad (3)$$

with

$$H_d(\omega) = \begin{cases} 1 & \omega \in [0,\ \omega_p] \\ 0 & \omega \in [\omega_a,\ \pi] \end{cases} \qquad (4)$$

where ω_p and ω_a denote the normalized passband and stopband edges, respectively.

3. Weighted least-squares design method

Let $H_k(z)$ represent a linear-phase FIR filter of odd length N, i.e.,

$$H_k(z) = \sum_{n=0}^{N-1} h_n z^{-n}$$

with $h_n = h_{N-1-n}$. We can write

$$H_k(\omega) = \mathbf{x}^T \mathbf{c}(\omega) \qquad (5)$$

where

$$\mathbf{x} = \begin{bmatrix} 2h_{(N-1)/2} \\ \vdots \\ 2h_1 \\ h_0 \end{bmatrix}, \quad \mathbf{c}(\omega) = \begin{bmatrix} 1 \\ \cos\omega \\ \vdots \\ \cos\frac{(N-1)\omega}{2} \end{bmatrix}$$

From (3)–(5), it follows that

$$J_k = \mathbf{x}^T \mathbf{Q} \mathbf{x} - 2\mathbf{x}^T \mathbf{b} + c \qquad (6a)$$

with

$$\mathbf{Q} = \int_0^{\omega_p} W(\omega) G_{k-1}^2(\omega) \mathbf{C}(\omega) d\omega + \int_{\omega_a}^{\pi} W(\omega) G_{k-1}^2(\omega) \mathbf{C}(\omega) d\omega \qquad (6b)$$

$$\mathbf{b} = \int_0^{\omega_p} W(\omega) G_{k-1}(\omega) \mathbf{c}(\omega) d\omega \qquad (6c)$$

where $\mathbf{C}(\omega) = \mathbf{c}(\omega)\mathbf{c}^T(\omega)$, c is a constant independent of $\mathbf{x}$, and weighting function $W(\omega)$ has been assumed to be zero in the transition band $(\omega_p,\ \omega_a)$. Since $\mathbf{Q}$ in (6b) is positive definite, the unique minimizer of J_k in (6a) is given by

$$\mathbf{x}^* = \mathbf{Q}^{-1}\mathbf{b} \qquad (7)$$

Notice that $\mathbf{x}^*$ is dependent on weighting function $W(\omega)$. Once an $\mathbf{x}^*$ is obtained, $H_k(\omega)$ can be evaluated using (5). The error function

$$e(\omega) = H_d(\omega) - G_{k-1}(\omega)H_k(\omega) \qquad (8)$$

can then be used to update the weighting function. Several methods for updating $W(\omega)$ are available in the literature, see, for example, [5] and [6]. A modified version of the method proposed in [6] is used in our design, and is as follows. Let $\Omega_p = \{\omega_{pi},\ 1 \leq i \leq N_p\}$ and $\Omega_a = \{\omega_{ai},\ 1 \leq i \leq N_a\}$ be the two sets of frequencies in the passband and stopband over which the two integrals in (6b) are evaluated. The values of $W(\omega)$ at $\omega_{pi} \in \Omega_p$ and at $\omega_{ai} \in \Omega_a$ are updated as

$$\hat{W}(\omega_{pi}) = \frac{w[W(\omega_{pi}) + \sigma]|e(\omega_{pi})|^\eta}{S}, \quad 1 \leq i \leq N_p \qquad (9a)$$

$$\hat{W}(\omega_{ai}) = \frac{[W(\omega_{ai}) + \sigma]|e(\omega_{ai})|^\eta}{S}, \quad 1 \leq i \leq N_a \qquad (9b)$$

$$S = w \sum_{\omega \in \Omega_p} [W(\omega) + \sigma]|e(\omega)|^\eta + \sum_{\omega \in \Omega_a} [W(\omega) + \sigma]|e(\omega)|^\eta \qquad (9c)$$

where $\hat{W}(\omega)$ denotes the updated value of the weighting function at frequency ω, $w > 0$ is a scalar weight used to emphasize ($w > 1$) or deemphasize ($w < 1$) the filter's performance in the passband as compared to that in the stopband, $\eta \in [1, 1.5]$ is an acceleration factor for the weight-updating process, and σ is a small positive constant used to prevent the propagation of a zero weight value. When the weighting function is updated using (9), $\mathbf{Q}$ and $\mathbf{b}$ are re-calculated with (6), and are then used to find an updated minimizer, $\hat{\mathbf{x}}^*$, with (7). The new error function $e(\omega)$ in (8) is then used to update the weighting function again until $||\mathbf{x}^* - \hat{\mathbf{x}}^*||$ is less than a prescribed tolerance.

Once $H_k(z)$ is obtained, section k is connected in cascade with the sections designed before to obtain the overall transfer function

$$G_k(z) = \prod_{l=0}^{k} H_l(z)$$

and $H_{k+1}(z)$ is deduced by minimizing objective function J_{k+1}, which is defined by (3) with index k replaced by $k+1$. The design continues until the last section, say section K, is designed.

This sequential design is illustrated in the flow chart shown of Fig. 2, and the features of the design method are as follows:

1. Given a sequence of low-order sections $0, 1, \ldots, k-1$, the next section, section k, is optimally designed so as to minimize the error measure in (3).
2. As each section is of low-order, each WLS minimization can be carried out with high computational efficiency.
3. The overall length of the cascade filter is $K(N-1)+L+1$. For linear-phase filters, the K subproblems for the design of sections $1, \ldots, K$, involve a total of $K(N+1)/2$ parameters. Note that in a conventional design of a linear-phase FIR filter of order $K(N-1)$, the number of parameters involved is $[K(N-1)+2]/2$. Hence there are $K-1$ more parameters in a sequential design than in a conventional design.

4. A design example

We have used the method to design a linear-phase lowpass cascade filter of the type shown in Fig. 1 of overall length 29 and normalized passband and stopband edges $\omega_p = 0.37\pi$ and $\omega_a = 0.58\pi$, respectively. The filter was designed to have four zeros at $\omega = \pi$, which implies that $L = 4$. Four sections were used, i.e., $K = 4$, each of order 6, which were designed with the WLS method described in Sec. 3 using $w = 2$. For an accurate evaluation of $\mathbf{Q}$ and $\mathbf{b}$, 200 equally spaced grid points were used for each of Ω_p and Ω_a.

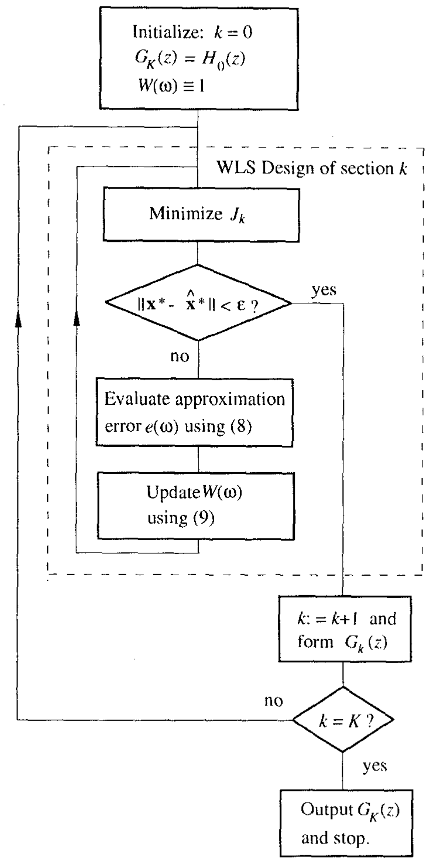
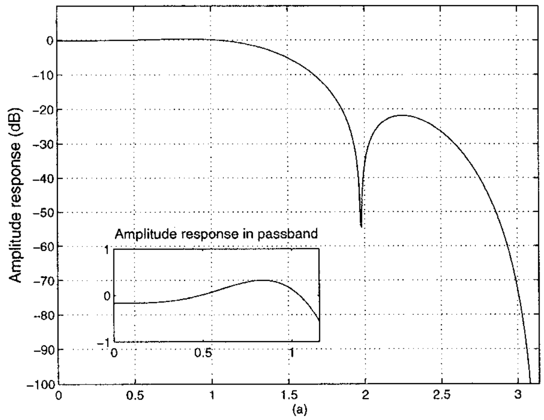


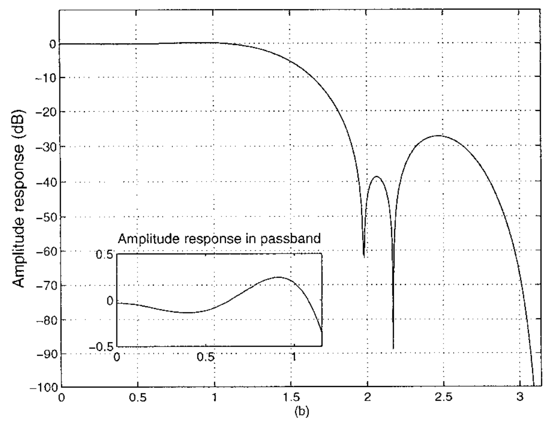
Figure 2: An illustration of the proposed design algorithm.

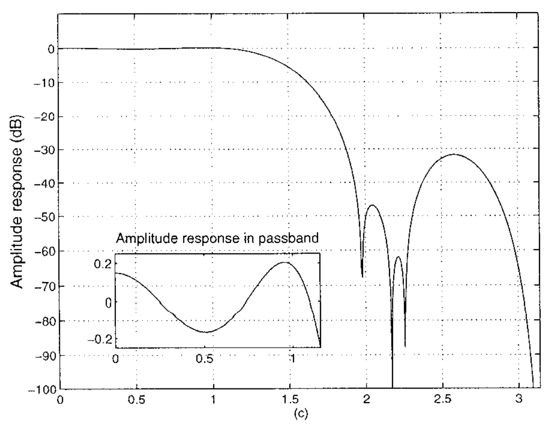
Fig. 3 depicts the amplitude responses of the multisection filter with $k = 1, 2, 3$, and 4. The maximum approximation errors in the passband and stopband are listed in Table 1. It is observed from Fig. 3 and Table 1, that the amplitude response is gradually improved as more sections are added.

Table 1 Approximation error in $G_k(\omega)$

k	$\max_{\omega \in \Omega_p} \lvert e(\omega) \rvert$	$\max_{\omega \in \Omega_a} \lvert e(\omega) \rvert$
1	0.0691	0.1291
2	0.0454	0.0893
3	0.0346	0.0652
4	0.0336	0.0527







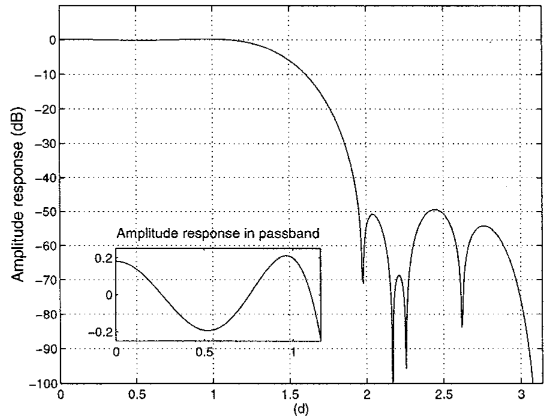


Figure 3: Amplitude response $20 \log |G_k(\omega)|$ of cascade FIR filter: (a) $k = 1$; (b) $k = 2$; (c) $k = 3$; and (d) $k = 4$.

Acknowledgement

The authors are grateful to the Natural Science and Engineering Research Council of Canada and Micronet, Networks of Centres of Excellence Program, Canada, for supporting this work.

References

[1] J. T. Ludwig, S. H. Nawab, and A. P. Chandrakasan, "Low-power digital filtering using approximate processing," *IEEE J. Solid-State Circuits*, vol. 31, pp. 395-400, March 1996.

[2] V. R. Lesser, J. Pavlin, and E. Durfee, "Approximate processing in real-time problem solving," *AI Magazine*, pp. 49-61, Spring 1988.

[3] S. H. Nawab, A. V. Oppenheim, A. P. Chandrakasan, J. M. Winograd, and J. T. Ludwig, "Approximate signal processing," *J. VLSI Signal Processing*, vol. 15, pp. 177-200, 1997.

[4] G. Strang and T. Nguyen, *Wavelets and Filter Banks*, Wellesley, MA: Wellesley-Cambridge Press, 1996.

[5] Y. C. Lim, J. H. Lee, C. K. Chen, and R. H. Yang, "A weighted least squares algorithm for quasi-equiripple FIR and IIR digital filter design," *IEEE Trans. Signal Processing*, vol. 40, pp. 551-558, March 1992.

[6] Y.-D. Jou, C.-H. Hsieh, and C.-M. Kuo, "Efficient weighted least-squares algorithm for the design of FIR filters," *IEE Proc. Vis. Image Signal Process.*, vol. 144, pp. 244-248, Aug. 1997.