Wavelet Approaches to Still Image Denoising

W.-S. Lu
Dept. of Electrical and Computer Engineering
University of Victoria
Victoria, B.C., Canada, V8W 3P6
wslu@ece.uvic.ca

Abstract

This paper describes three wavelet-based methods for noise reduction of still images: (i) Hyperbolic shrinkage with a level-dependent thresholding policy; (ii) Hyperbolic shrinkage with a two-dimensional cross-validation-based thresholding; and (iii) block SVD-wavelet denoising. All three methods make use of the hyperbolic shrinkage rather than the conventional soft shrinkage. As the thresholding of wavelet coefficients is concerned, at each level of wavelet decomposition, the first method employs a level-dependent universal threshold determined by the coefficient variance and the number of the coefficients at that level; while the second method extends Nason's cross-validation approach to the 2-D case. In the third method, an image is divided into several subimages (blocks) and singular value decomposition (SVD) is applied to each block. The singular values obtained are then truncated and each pair of singular vectors are treated as 1-D noisy signals and are denoised using a wavelet-based method. The subimages are then reconstructed using the truncated singular values and denoised singular vectors.

1. Introduction

Since the work of Donoho and Johnstone, wavelet-based noise-reduction methods have been a subject of intensive research [1]–[8]. Three issues critical in implementing a wavelet denoising algorithm are the determination of the noise variance, the determination of a way to "shrink" the wavelet coefficients, and the determination of a threshold level to be used in the shrinkage step. In addition to these, wavelets utilized to perform signal decomposition must be chosen with care. The purpose of this paper is to describe three wavelet-based denoising algorithms for noise reduction of still images that, through extensive tests, have been found effective. These methods are: (i) Hyperbolic shrinkage with a level-dependent thresholding policy; (ii) Hyperbolic shrinkage with a two-dimensional cross-validation-based thresholding; and (iii) block SVD-wavelet denoising.

2. Hyperbolic shrinkage with level-dependent thresholding

The basic components of a wavelet-based denoising method are illustrated in Fig. 1. Notice that the generic structure in Fig. 1 applies in principle to denoising tasks regardless of signal dimension, thus can be used as a unified system framework in denoising 1-D discrete signal, discretized still images, as well as sequence of digital visual signals.

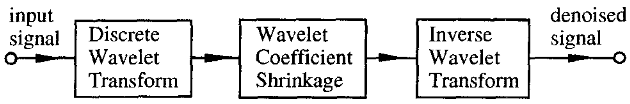

Figure 1: Wavelet-based signal denoising.

For a noise-corrupted still image of size $N \times N$ with $N = 2^n$, separable 2-D wavelets can be used to perform discrete wavelet transform (DWT) with up to n scales. For denoising purposes, the "MIT 9/7" wavelet filters suggested by T. Arias [9], which are biorthogonal linear-phase FIR filters with impulse responses

$$h_0 = [1 \;\; 0 \;\; -8 \;\; 16 \;\; 46 \;\; 16 \;\; -8 \;\; 0 \;\; 1]/64$$
$$f_0 = [-1 \;\; 0 \;\; 9 \;\; 16 \;\; 9 \;\; 0 \;\; -1]/32$$

have been found satisfactory.

Our first image denoising method employs the hyperbolic shrinkage initiated by Vidakovic [3]:

$$y(t) = \text{sgn}[x(t)] \cdot \sqrt{(x(t)^2 - \delta^2)_+} \qquad (1)$$

where $x(t)$ is the noise-contaminated input, $\text{sgn}(\cdot)$ is the signum function, $(\cdot)_+$ denotes the maximum of the variable and zero, and δ is a threshold to be determined shortly. When comparing the hyperbolic shrinkage with the hard and soft shrinkage [5], the hyperbolic shrinkage is "almost hard", but it is this subtle difference from the hard shrinkage that often leads to superior denoising results. When the noise is assumed to be white Gaussian, a conventional

1058-6393/98 $10.00 © 1998 IEEE

wavelet-based denoising method uses a single threshold δ across all levels of the decomposition, which in the case of universal thresholding is determined by

$$\delta = \sqrt{2\log(N)}\sigma \tag{2}$$

where N is the length of the input signal and σ^2 is an estimate of the noise variance [5]. The value of σ in (2) is usually estimated using the "detail" subimage obtained at the highest resolution level (the subsampled output from the first round of highpass-highpass filtering).

As was reported in [2], the wavelet denoising method of [1][5] works also for the case of color noise if σ in (2) is estimated at each decomposition level. This modification was also found advantageous even to the white noise case. In [8], this level-dependent thresholding was further modified such that the N in (2) is taken to be the length of the subsignal at each level, rather than the length of the entire input signal. Here this level-dependent universal thresholding policy is extended to the 2-D case to handle noisy images.

3. Hyperbolic shrinkage with cross-validation based thresholding

The second method also employs the hyperbolic shrinkage, while the threshold δ is determined using a 2-D extension of Nason's cross-validation [4]. For 1-D signals, the cross-validation based method works as follows: Given a 1-D noise-corrupted sequence $\{y_i,\ i = 1,\ \ldots,\ 2^p\}$, we define two sequences from it:

$$y_i^{odd} = \frac{y_{2i-1} + y_{2i+1}}{2}, \qquad i = 1,\ \ldots,\ \frac{N}{2}$$

with $y_{N+1} = y_{N-1}$, and

$$y_i^{even} = \frac{y_{2i} + y_{2i+2}}{2}, \qquad i = 1,\ \ldots,\ \frac{N}{2}$$

with $y_{N+2} = y_N$. For each fixed λ, define function $M(\lambda)$ as

$$M(\lambda) = \sum_{j,k}[T(d_{j,k}^{even};\lambda) - d_{j,k}^{odd}]^2 + \sum_{j,k}[T(d_{j,k}^{odd};\lambda) - d_{j,k}^{even}]^2$$

where $T(y;\lambda)$ is a sequence obtained by either soft-shrinkage or hyperbolic shringkage of y using λ as the threshold. It was shown [4] that function $M(\lambda)$ is convex with respect to λ. Let λ^* be the minimizer of $M(\lambda)$, then the cross-validation based threshold is given by

$$\delta = \lambda^*\left(1 - \frac{\log 2}{\log N}\right)^{-1/2}$$

The idea of the cross-validation can be extended to the two-dimensional case in a natural fashion as described below.

Given an image $\{y_{ij},\ i, j = 1,\ \cdots,\ N = 2^p\}$, we define two images from it as

$$y_{ij}^{odd} = \frac{y_{2i-1,2j-1} + y_{2i+1,2j+1}}{2} \qquad i, j = 1, \cdots, \frac{N}{2}$$

with $y_{i,N+1} = y_{i,N-1}$; $y_{N+1,j} = y_{N-1,j}$; $y_{N+1,N+1} = y_{N-1,N-1}$, and

$$y_{ij}^{even} = \frac{y_{2i,2j} + y_{2i+2,2j+2}}{2} \qquad i, j = 1, \cdots, \frac{N}{2}$$

with $y_{i,N+2} = y_{i,N}$; $y_{N+2,j} = y_{N,j}$; $y_{N+2,N+2} = y_{N,N}$. For each fixed λ, define function

$$M(\lambda) = \sum_{j,k,l}[T(d_{j;k,l}^{even},\ \lambda) - d_{j;k,l}^{odd}]^2 + \sum_{j,k,l}[T(d_{j;k,l}^{odd},\ \lambda) - d_{j;k,l}^{even}]^2$$

where $\{d_{j;k,l}^{even}\}$ and $\{d_{j;k,l}^{odd}\}$ denote the detail subimages obtained by applying 2-D wavelet transform to $y_{k,l}^{even}$ and $y_{k,l}^{odd}$ at the jth resolution, respectively, and $T(y,\ \lambda)$ is the result of hyperbolic shrinkage of image y using λ as the threshold. The threshold is then determined by

$$\delta = \lambda^*\left(1 - \frac{\log 2}{\log N}\right)^{-1/2}$$

where λ^* is the minimizer of $M(\lambda)$.

4. Block-SVD-wavelet denoising

The third method starts with subdividing a noisy image of size $N \times N$ into subimages of size $M \times M$. The singular value decomposition (SVD) is applied to each subimage . Recall that the SVD of a matrix $A \in F^{m\times n}$ factorizes A as

$$A = USV^T$$

where $U \in R^{m\times m}$ and $V \in R^{n\times n}$ are orthogonal and

$$S = \begin{bmatrix} \sigma_1 & & & \\ & \ddots & & 0 \\ & & \sigma_r & \\ & 0 & & 0 \end{bmatrix} \in R^{n\times m}$$

with singular values $\sigma_1 > \cdots \sigma_r > 0$ and $r = \text{rank}\ (A)$. Write $U = [u_1 \cdots u_r \cdots]$ and $V = [v_1 \cdots v_r \cdots]$, columns u_i's and v_i's are called the left and right singular vectors of A, respectively. As was observed in [10], noise in still images can be reduced when the singular values are truncated (i.e., use only the first K singular values $\sigma_1,\ \cdots,\ \sigma_K$) adequately. Our method does one more thing, that is to treat each pair of singular vectors that are associated with the significant singular values as 1-D noisy signals and are denoised using a 1-D wavelet-based algorithm. The truncated singular values and the denoised singular vectors are then used to reconstruct the subimages. The computational complexity of this method depends critically on the ratio $r = N/M$: in general a greater r means a less expensive denoising algorithm. Our simulation study indicates that for images of size 256×256 or 512×512, best denoising results usually occur when $r = 2$ or 4.

5. Simulations

The above methods were applied to a number of test images of sizes 256×256 and 512×512 which are corrupted with white Gaussian noise, with comparable signal-to-noise-ratio improvement. Fig. 2 shows the denoising result of the block SVD-wavelet method as applied to the 512×512 image "Boats" with SNR = 14.3717 dB. The denoised image has SNR = 21.0651 dB. Fig. 3 exhibits the denoising result of the first method as applied to the 512×512 Lena with SNR = 12.7778 dB. The denoised image has SNR = 20.2801 dB. Fig. 4 shows the denoising result of the second method as applied to the 512×512 image "Tiffany" with SNR = 18.4363 dB. The denoised image has SNR = 26.5064 dB. From the figure the convexity of function $M(\lambda)$ can also be observed.

6. Conclusion

We have described three wavelet-based methods for the noise removal of still digital images. Three key factors that influence the denoising result most are the choice of wavelets, the shrinkage method, and the way the threshold is determined. Since the MIT9/7 wavelets have consistently shown excellent denoising results, they often become the user's choice. From our simulations, it was found that the denoising performances of the three methods in terms of SNR improvement are often comparable each other without a decisive superiority; however, the method that uses a hyperbolic shrinkage policy with Donoho's universal thresholding requires substantially less computations compared to the other two methods.

Acknowledgement

The author is grateful to the Natural Science and Engineering Research Council of Canada (NSERC) and the MICRONET for supporting this work.

References

[1] D. L. Donoho and I. M. Johnstone, "Ideal spatial adaptation via wavelet shrinkage," *Biometrika*, vol. 81, pp. 425-455, 1994.

[2] I. M. Johnstone and B. W. Silverman, "Wavelet threshold estimators for data with correlated noise," *Standford Technical Report*, No. 496, Dec. 1994.

[3] B. Vidakovic, "Wavelet and baysian statistics," *Interface'94*, RTP, North Carolina, June 1994.

[4] G. P. Nason, "Wavelet regression by cross-validation," Dept. of Mathematics, Univ. of Bristol, Bristol, UK, March 1994.

[5] D. L. Donoho, "Denoising by soft-thresholding," *IEEE Trans. Information Theory*, vol. 41, pp. 613-627, May 1995.

[6] D. L. Donoho, I. M. Johnstone, G. Kerkyacharian, and D. Picard, "Wavelet shrinkage: Asymptopia?" *J. Roy. Stat. Soc.*, vol. 57, pp. 301-369, 1995.

[7] M. Lang, H. Guo, J. E. Odegard, and C. S. Burrus, "Noise reduction using undecimated discrete wavelet transform," *IEEE Signal Processing Lett.*, vol. 3, pp. , Jan. 1996

[8] R. S. C. Wong and V. K. Bhargava, "Denoising of low SNR signals using composite wavelet shrinkage," to be presented at *IEEE PACRIM'97*, Victoria, BC, August 1997.

[9] G. Strang, "Creating and comparing wavelets," in *Numerical Analysis: A. R. Mitchell Birthday Volume*, G. A. Watson and D. F. Griffiths eds., World Scientific 1996.

[10] K. Konstantinides, B. Natarajan, and G. S. Yovanof, "Noise estimation and filtering using block-based singular value decomposition," *IEEE Trans. Image Processing*, vol. 6, pp. 479-483, March 1997.

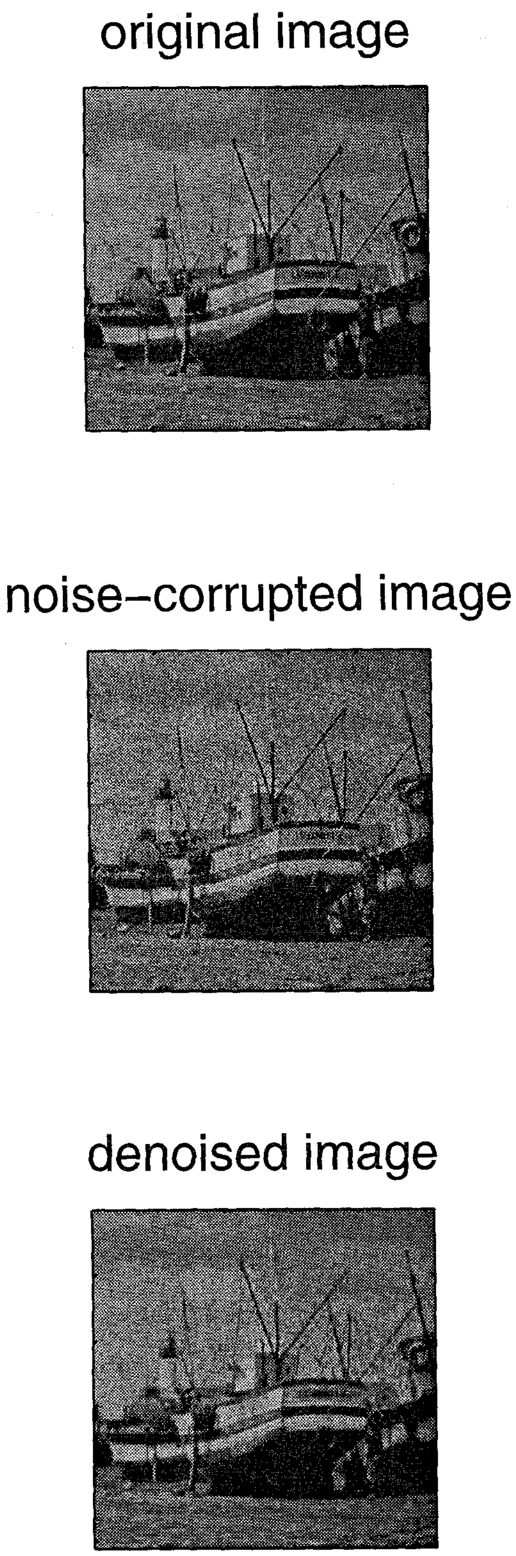

Figure 2: $M = 128$, $K = 21$, SNR1 = 14.3717 dB, SNR2 = 21.0651 dB.

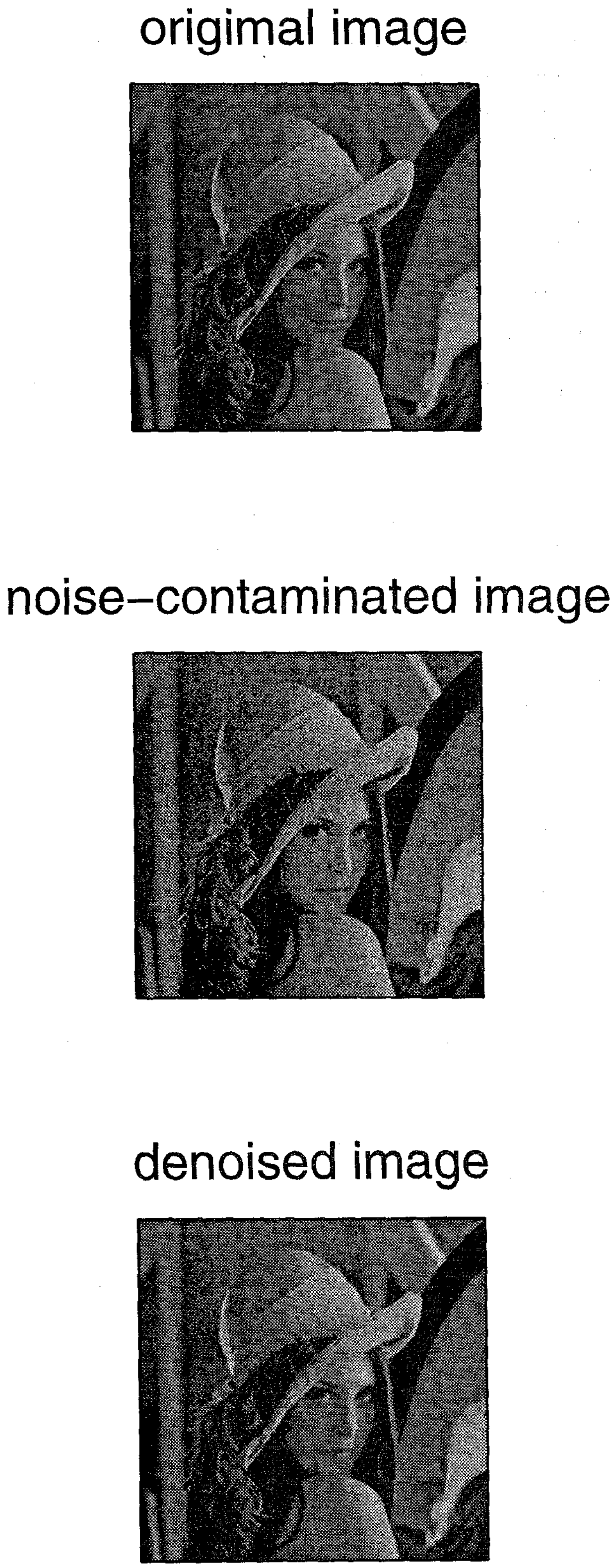

Figure 3: SNR1 = 12.7778 dB, SNR2 = 20.2801 dB.

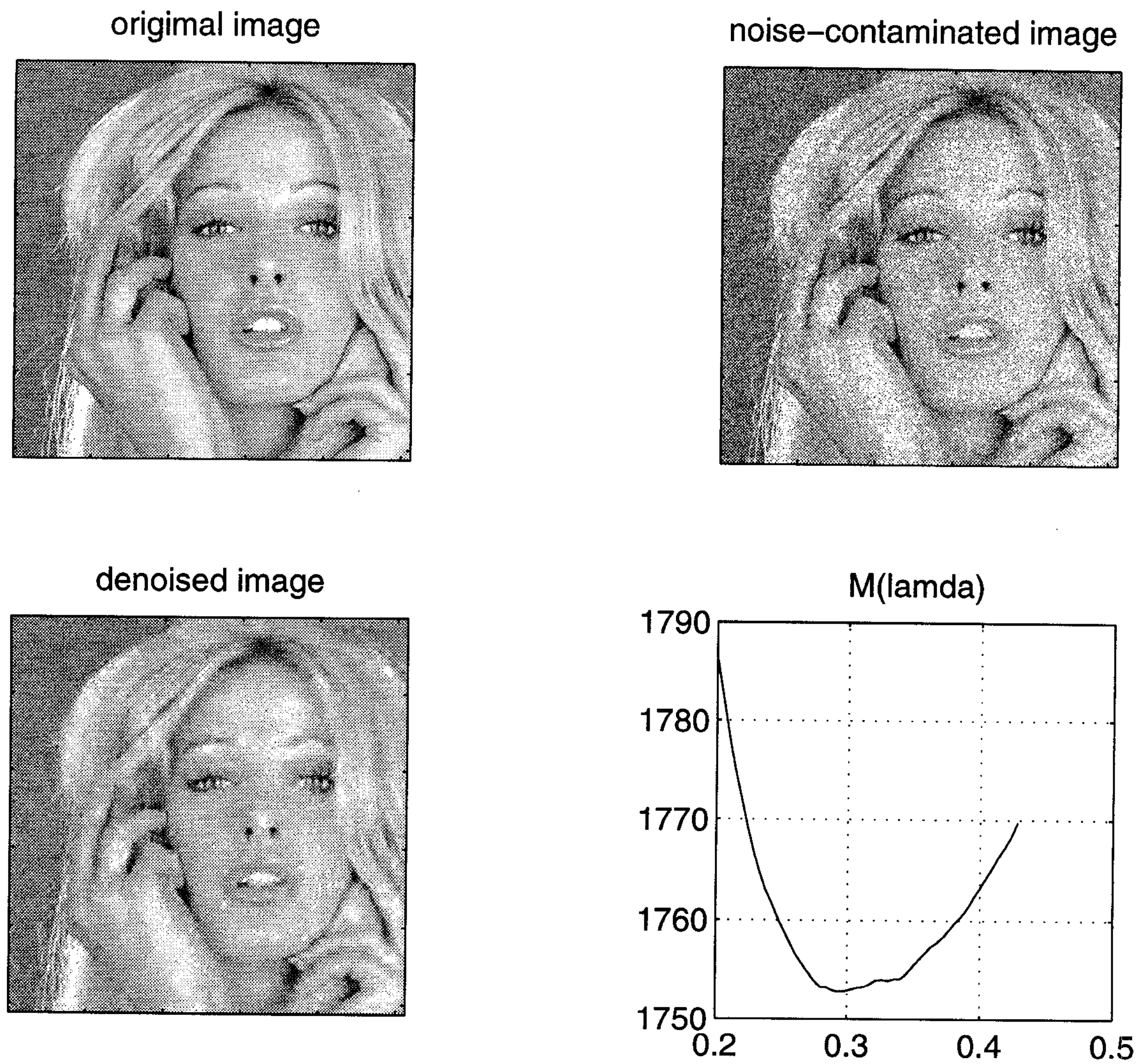

Figure 4: SNR1 = 18.4363 dB, SNR2 = 26.5064 dB.