Design of Quadrature Mirror-Image Filter Banks for Low-Power Applications

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Abstract—A method for the design of quadrature mirror-image filter QMF banks for low-power applications based on the algorithm of Chen and Lee is proposed. The method entails designing a sequence of cascaded filter sections such that any number of consecutive sections starting with the first one constitute an optimal design for a given set of specifications for the filter bank. The method includes modifications that allow for the increase in vanishing moments with the increase in the number of filter sections and it can incorporate techniques that facilitate control over the stopband attenuation or enable the design of low-reconstruction delay QMF banks. Using a simple adaptive mechanism, the input and output signals are used to determine the minimum number of sections that should be used in order to provide a desired performance. By turning off unrequired sections, power and computational complexity can be minimized.

I. Introduction

Many electrical systems such as portable wireless systems are dependent on power availability. Many other systems such as multitasking systems are, in addition, dependent on computational resources. These dependencies can be alleviated by dynamically adjusting the complexity of the algorithms used according to predetermined criteria and changing signal characteristics [1].

Digital filters are commonly designed to assure a desirable performance for all possible signals that may be encountered. However, for many applications, the worst-case conditions that set the filter specifications are rarely present. This leads to wasted power and computational resources.

In this paper, a method for the design of quadrature mirror-image filter (QMF) banks for low-power applications is proposed. The main idea is to design digital filters as a collection of low-order cascaded sections that can be turned off when they are not needed so as to conserve power and reduce the computational complexity. The sections are designed individually from left to right such that the sections to the left satisfy specifications in an optimal manner.

QMF banks have ideal properties which make low-power design simple, fast, and useful. As a matter of fact, the proposed design approach introduces more savings when applied to filter banks than to individual filters. As the signal changes, each filter in the filter bank adapts, thereby multiplying the savings by twice the number of channels.

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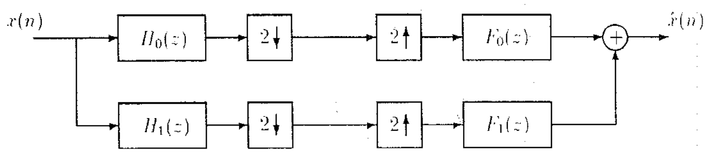

Fig. 1. A two-channel filter bank.

II. Problem Formulation

A. QMF Bank Design

Consider the two-channel filter bank in Fig. 1. Alias-cancellation and perfect-reconstruction can be achieved when the modulation matrices satisfy the system of equations

$$\begin{bmatrix} H_0(z) & H_1(z) \\ H_0(-z) & H_1(-z) \end{bmatrix} \begin{bmatrix} F_0(z) \\ F_1(z) \end{bmatrix} = \begin{bmatrix} F_0(z) & F_1(z) \\ F_0(-z) & F_1(-z) \end{bmatrix} \begin{bmatrix} H_0(z) \\ H_1(z) \end{bmatrix} = \begin{bmatrix} 2z^{-l} \\ 0 \end{bmatrix}$$

This can be accomplished by following the design method presented in [2]. The first step is to achieve alias cancellation with $F_0(z) = H_0(z)$, $H_1(z) = H_0(-z)$, and $F_1(z) = -H(-z)$. This reduces the number of filters to be designed to one, and the appropriate objective function for attaining perfect reconstruction is

$$\Psi_0(\mathbf{f}, \mathbf{h}) = \int_{-\pi}^{\pi} |P(\omega) - P(\omega + \pi) - 2e^{-jl\omega}|^2 d\omega \qquad (1)$$

where

$$P(\omega) = H_0(\omega)F_0(\omega) = H_0^2(\omega) = \sum_{i=0}^{2n} p_i e^{-ji\omega} = \mathbf{p}^{\mathrm{T}}\mathbf{e}(\omega)$$

$$H_0(\omega) = \sum_{i=0}^{n} h_i e^{-ji\omega} = \mathbf{h}^{\mathrm{T}}\mathbf{e}(\omega)$$

$$F_0(\omega) = \sum_{i=0}^{n} f_i e^{-ji\omega} = \mathbf{f}^{\mathrm{T}}\mathbf{e}(\omega)$$

Coefficient vectors $\mathbf{p}$ and $\mathbf{f}$ are related by

$$\mathbf{p} = \mathbf{H}\mathbf{f}$$

where $\mathbf{H}$ is the convolution matrix of $\mathbf{h}$ defined as

$$\mathbf{H} = \begin{bmatrix} h_0 & 0 & \cdots & 0 \\ h_1 & h_0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ h_n & h_{n-1} & \cdots & h_0 \\ 0 & h_n & \cdots & h_1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_n \end{bmatrix}$$

Using the optimization algorithm in [2] and [3], the objective function in (1) is minimized with respect to $\mathbf{f}$. Since $\mathbf{f}$ and $\mathbf{h}$ must be the same to cancel aliasing, a linear combination of $\mathbf{h}$ and the optimum $\mathbf{f}$ is reassigned to $\mathbf{h}$. This procedure is then repeated until $\mathbf{f}$ and $\mathbf{h}$ become equal to within a prescribed tolerance.

From equation (1), we can write

$$\Psi(\mathbf{f}) = \frac{\Psi_0(\mathbf{f},\mathbf{h})}{16\pi} = 0.5\mathbf{f}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\mathbf{S}\mathbf{H}\mathbf{f} - \mathbf{f}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\mathbf{s} + 0.5 \qquad (2)$$

where

$$\mathbf{S} = \begin{bmatrix} 0 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & 0 \end{bmatrix}$$

and $\mathbf{s}$ is a vector whose elements are zero except for a one in entry $l+1$ if l is odd. This means that for filter banks with an odd reconstruction delay, (2) can be minimized as a quadratic problem. For the sake of completeness, an even reconstruction delay filter bank can be designed by solving an eigenvalue problem, and a noninteger reconstruction delay filter bank can be achieved by using

$$\mathbf{s} = \frac{1}{2\pi} \begin{bmatrix} 0 \\ \frac{\sin[(l-1)\pi]-\sin[(l-2)\pi]}{l-1} \\ \frac{\sin[(l-2)\pi]-\sin[(l-4)\pi]}{l-2} \\ \vdots \\ \frac{\sin[(l-2n)\pi]-\sin[(l-4n)\pi]}{l-2n} \end{bmatrix}$$

If the filter to be designed is of high order, $\mathbf{H}^{\mathrm{T}}\mathbf{S}\mathbf{H}$ can become ill-conditioned and the extra term

$$2\alpha \int_{\omega_a}^{\pi} |F_0(\omega)|^2 d\omega$$

must be added to (1), where ω_a is the stopband edge, and α is used to control the attenuation in the stopband. This extra term changes $\mathbf{H}^{\mathrm{T}}\mathbf{S}\mathbf{H}$ to $\mathbf{H}^{\mathrm{T}}\mathbf{S}\mathbf{H} + \mathbf{R}$ where

$$\mathbf{R}(k, k \pm m) = -\frac{\alpha \sin(m\omega_a)}{4\pi m}$$

for $1 \le k \le n+1$, $0 \le m \le n$. If low-reconstruction-delay filter banks are to be designed, anomalies in the amplitude response can occur in the transition region. Such anomalies can be reduced by adding the extra term

$$2\beta \int_{\omega_{t_1}}^{\omega_{t_2}} |F_0(\omega) - \sqrt{2}e^{-j\omega l/2}|^2 d\omega$$

to (1), where β is used to control the flatness in the transition region $[\omega_{t_1}, \omega_{t_2}]$. This extra term changes $\mathbf{H}^{\mathrm{T}}\mathbf{S}\mathbf{H}$ to $\mathbf{H}^{\mathrm{T}}\mathbf{S}\mathbf{H} + \mathbf{T}$, and $\mathbf{H}^{\mathrm{T}}\mathbf{s}$ to $\mathbf{H}^{\mathrm{T}}\mathbf{s} + \mathbf{b}$ where

$$\begin{aligned} \mathbf{T}(k, k \pm m) &= \frac{\beta[\sin(m\omega_{t_2}) - \sin(m\omega_{t_1})]}{4\pi m} \\ \mathbf{b}(m+1) &= \frac{\beta\{\sin[\omega_{t_2}(l/2-m)] - \sin[\omega_{t_1}(l/2-m)]\}}{\sqrt{2}\pi(l-2m)} \end{aligned}$$

for $1 \le k \le n+1$, $0 \le m \le n$. If a complex reconstruction error whose magnitude is equiripple is desired, (1) must include a weighting function with respect to ω [2]-[4].

B. Low-Power Design

If a low-order filter is designed by obtaining the optimum $\mathbf{f}$ in (2), then another low-order filter section can be cascaded with the first design in order to improve the frequency responses of all the filter blocks in the filter bank. If the impulse responses of the first two filters are $\mathbf{f}_0$ and $\mathbf{f}_1$, then the impulse response of the cascaded filter is $\mathbf{F}_0\mathbf{f}_1$ where $\mathbf{F}_0$ is the convolution matrix of $\mathbf{f}_0$. When $k-1$ low-order sections have been designed, (2) can be formulated as

$$\Psi(\mathbf{f}_k) = 0.5\mathbf{f}_k^{\mathrm{T}}\mathbf{F}_{k-1}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\mathbf{S}\mathbf{H}\mathbf{F}_{k-1}\mathbf{f}_k - \mathbf{f}_k^{\mathrm{T}}\mathbf{F}_{k-1}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\mathbf{s} + 0.5 \quad (3)$$

where $\mathbf{F}_{k-1}$ is the convolution matrix of the impulse response of the first $k-1$ filter sections in cascade.

To assure an increase in the vanishing moments in the associated scaling and wavelet function, as the filter bank grows, a zero is forced at $\omega = \pi$ for each filter section. This can be done by moving the zero out of $\mathbf{f}_k$ and forming its convolution matrix. Equation (3) is then altered to

$$\begin{aligned} \Psi(\dot{\mathbf{f}}_k) &= 0.5\dot{\mathbf{f}}_k^{\mathrm{T}}\mathbf{Z}^{\mathrm{T}}\mathbf{F}_{k-1}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\mathbf{S}\mathbf{H}\mathbf{F}_{k-1}\mathbf{Z}\dot{\mathbf{f}}_k \\ &\quad -\dot{\mathbf{f}}_k^{\mathrm{T}}\mathbf{Z}^{\mathrm{T}}\mathbf{F}_{k-1}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\mathbf{s} + 0.5 \end{aligned} \qquad (4)$$

where $\mathbf{f}_k = \mathbf{Z}\dot{\mathbf{f}}_k$ and $\mathbf{Z}$ is the zero convolution matrix.

Using the cascaded structure of filters designed through this method, a low-power filter bank structure can be constructed based on the approach in [1]. By computing the energy difference between the input and output signals and multipling it by a proportionality constant and the attenuation of the active cascaded filter sections, an accurate estimation of the output energy in the stopband can be obtained. Based on this calculation, other low-order sections can be turned on, turned off, or left alone. The amount of extra computation is approximately equal to that required by an FIR filter of length six. Other approaches for deciding when to switch on and off sections can also be used depending on the desired application.

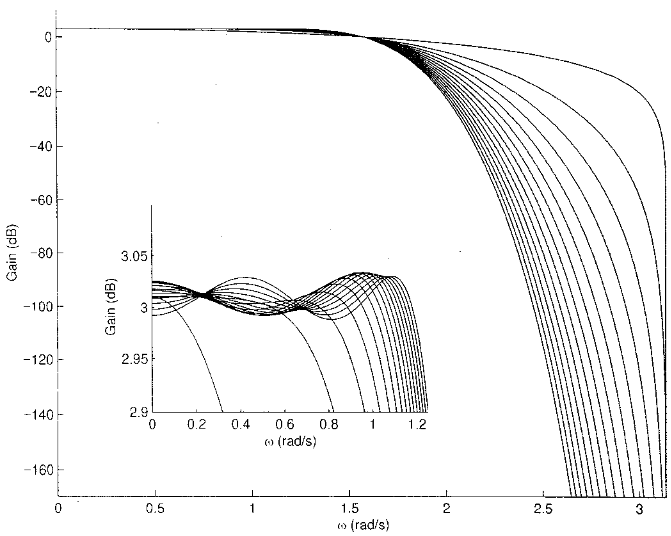

Fig. 2. Amplitude responses.

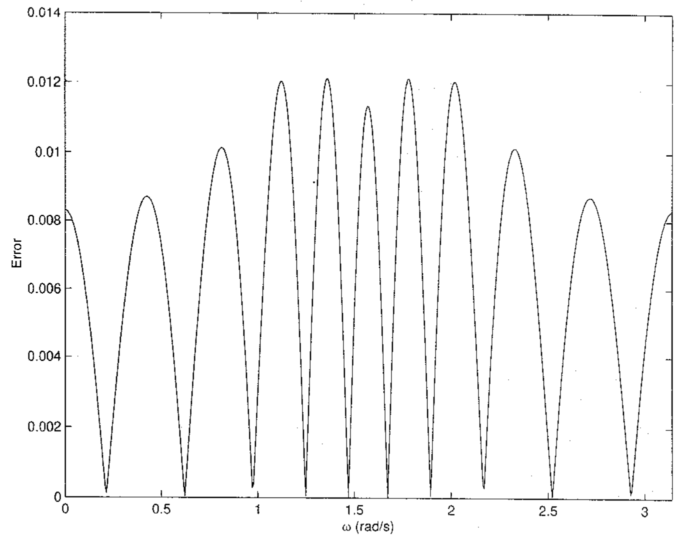

Fig. 3. Final absolute reconstruction error.

III. Algorithm Implementation

When optimizing $\mathbf{f}$ in (2) or any of its modified forms, a starting point $\mathbf{h}$ is required in order to construct $\mathbf{H}$. A good starting point is the rectangular window lowpass filter with a zero removed. After the first section of a low-power structure is completed, the starting points should be the rectangular window lowpass filter with the previous sections and current zero removed. This gives the advantage of comparing only the filter coefficients in $\mathbf{f}$ and $\mathbf{h}$ which contribute to a change from one iteration to the next, as well as reducing computational effort. After each iteration, $\mathbf{h}$ is updated using

$$\mathbf{h} = (1 - \tau)\mathbf{h} + \tau\mathbf{f}$$

where τ is a smoothing parameter between 0 and 1. It is found that $\tau = 0.5$ provides a satisfactory convergence rate [2]. Convergence can be tested by calculating the Euclidean norm of the difference between the two vectors. Iteration stops when the result is below a prescribed tolerance ϵ.

When calculating the optimum $\mathbf{f}$, computational effort can be further reduced by examining the form of (4). After differentiating, both the first and second terms start with $\mathbf{Z}^T\mathbf{F}^T\mathbf{H}^T$. This product can be removed from each term, and the resulting equation can be solved using least-squares techniques. This method is also more stable. If α or β is nonzero, only $\mathbf{Z}^T\mathbf{F}^T$ can be removed. Further reduction in computational effort can be achieved by taking advantage of the pattern of zeros in $\mathbf{S}$ and $\mathbf{s}$, and any symmetric patterns in other variables.

If desired, the design parameters ϵ, α, ω_a, β, ω_{t_1}, and ω_{t_2} can be assigned separately for each iteration in order to gain more control over the design process.

IV. Design Examples

Fig. 2 displays the amplitude responses of a first-order section followed by fourteen eighth-order sections, and Fig. 3 displays the final reconstruction error of the resulting 113th-order filter. Fig. 4 displays the minimized value of (1) after each section. The reconstruction delay for each filter bank is equal to the order of each filter within it. Other design parameters are $\omega_a = 0.5\pi$, $\epsilon = 10^{-12}$, and $\alpha = \beta = 0$.

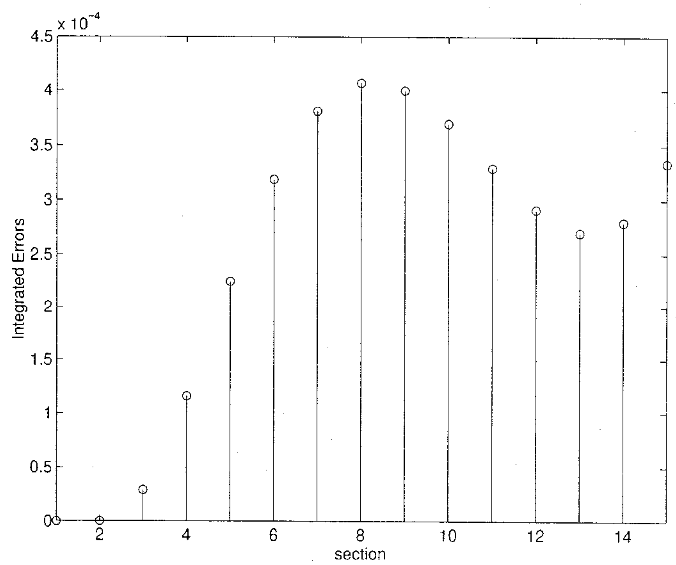

Fig. 4. Minimized values of (1).

Figs. 5-7 display the amplitude responses, final reconstruction error, and minimized values of (1), respectively, when α is changed to 0.05. As can be seen, the reconstruction error is on the average slightly higher compared to the design with $\alpha = 0$, but the attenuation in the stopband is significantly increased.

V. Conclusions

A method for the design of QMF banks for low-power applications based on the algorithm of Chen and Lee has been proposed. The method entails designing a sequence of cas-

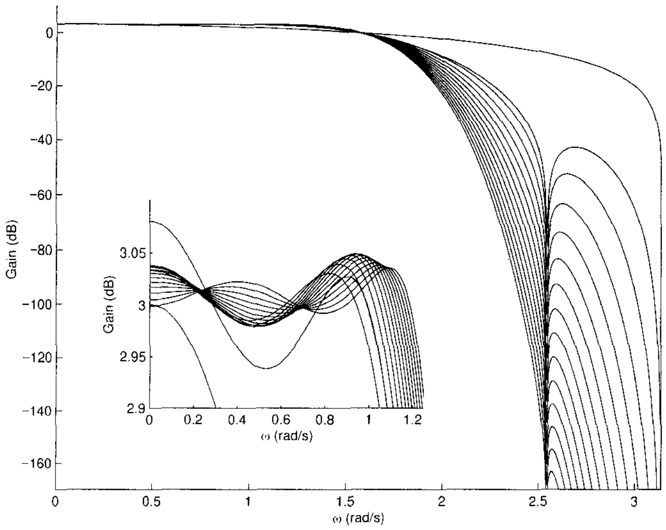

Fig. 5. Amplitude responses.

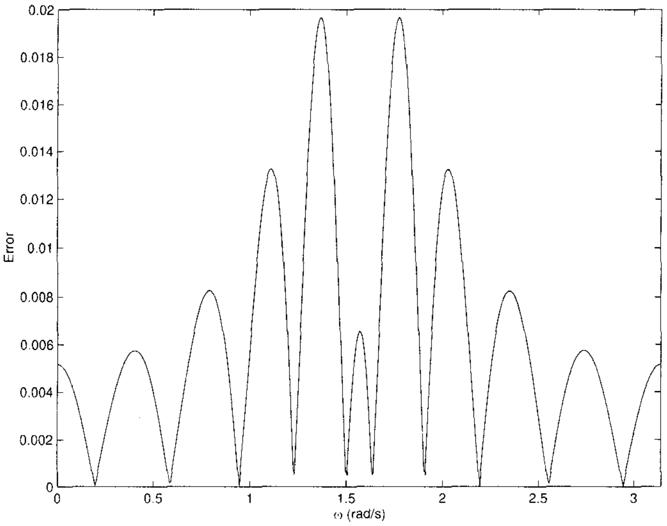

Fig. 6. Final absolute reconstruction error.

caded filter sections such that any number of consecutive sections starting with the first one constitute an optimal design for a given set of specifications for the filter bank. The method includes modifications that allow for the increase in vanishing moments with the increase in the number of filter sections and it can incorporate techniques that facilitate control over the stopband attenuation or enable the design of low-reconstruction delay QMF banks.

Using approximate processing techniques, the minimum number of sections to provide the required performance for the filter bank can be turned on. This allows for the dynamic reduction of power and computational complexity.

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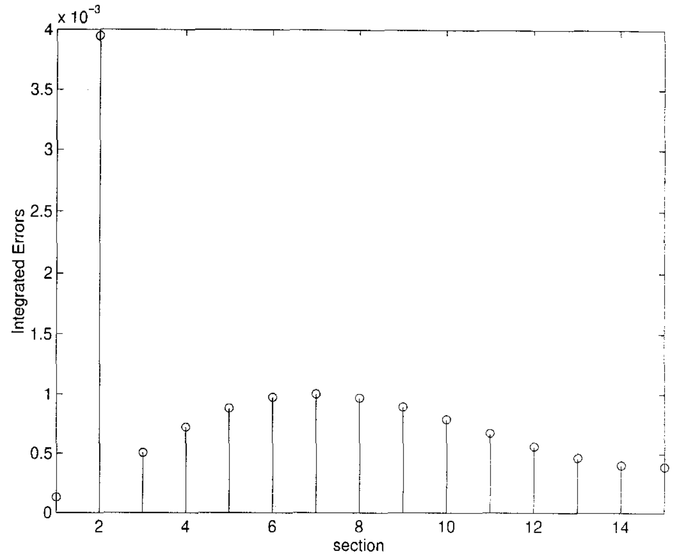

Fig. 7. Minimized values of (1).

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