Multiuser Detection for Multiple-Access Communications Using Wavelet-Packet Transforms

X. F. Wang, W.-S. Lu, and A. Antoniou
Department of Electrical and Computer Engineering
University of Victoria
Victoria, B.C., CANADA, V8W 3P6
Email: {xfwang, wslu, andreas}@ece.uvic.ca

Abstract— A wavelet-packet-based alternating projection joint detector (WPAPJD) for over-saturated multiple-access systems is proposed. By exploiting the structure of the multiple-access interference (MAI) associated with the wavelet packets used, the MAI seen by each user can only occur within a small-size subgroup of users. Further, MAI is reduced by using an APJD as applied to the subgroup. In comparison with a refined multistage joint detector, simulation studies show that our proposed method offers improved bit-error rate (BER) with reduced computational complexity in applications where a signal-to-noise ratio less than 16 dB is acceptable.

I. Introduction

Multiple-access interference (MAI) is a major limiting factor for conventional direct-sequence code-division multiple access (DS-CDMA) systems. As an efficient way to mitigate the effect of MAI, various multiuser detection methods have been proposed (see [1]-[7] and their references). An optimal multiuser detector was proposed in [1] and suboptimal detectors with much reduced computational complexity were proposed in [2][3] as linear joint detectors and in [4] as multistage joint detectors (MJD).

In [5], the wavelet packet transform (WPT) was applied to select signature waveforms and jointly design the receiver in order to increase the throughput of a multiple-access system. In a wavelet-packet-based multiple-access (WPMA) system, each user is assigned a distinct signature waveform generated by the inverse WPT. By exploiting the hierarchical structure of the WPT, a more efficient multiuser detector was developed. It was also shown that this WPT-based multiuser detection algorithm can be viewed as a refined version of the MJD proposed in [4]. A sufficient condition for the convergence of the WPT-based detection algorithm was presented in [8]. It turned out that for over-saturated systems, the algorithm in [5] does not, in general, satisfy the sufficient condition. In [6], the multiuser detection problem was examined through a geometric interpretation, and a detector known as alternating projection joint detector (APJD) was proposed to solve the problem. It is noted, however, that the APJD algorithm was not considered in a WPT framework.

In this paper, we first investigate the design of the signature set in over-saturated WPMA systems, and show that K user signature waveforms can be generated by the inverse WPT such that their cross-correlations exhibit a hieararchical structure. The users are then divided into several subgroups according to the cross-correlation structure. Based on this, a new multiuser joint detector, named WPAPJD, is developed in which APJD is applied to detect the users within each subgroup. This leads to a significant reduction in the computational complexity. The performance of the proposed WPAPJD is examined via simulations and comparisons are carried out with the method in [5].

II. Design of Signature Set in WPMA System

In a WPMA system, each user is assigned a distinct signature waveform which is derived by applying the inverse WPT to a vector at a certain location of a WPT tree. Since the vector and its location determine the time-frequency characteristic of the signature waveform derived, by defining locations in the WPT tree as levels and nodes, as shown in Fig. 1, the design of the signature set can be accomplished by carefully selecting the nodes and the vectors in the WPT binary tree.

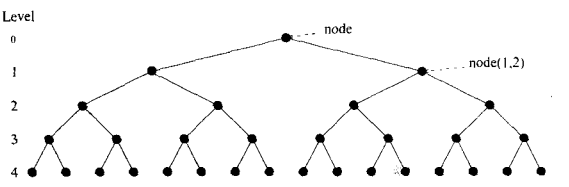

Fig. 1. A 4-level full binary tree of wavelet-packet decomposition.

Let $\mathbf{w}_{(l,c)}$ denote the waveform generated by the inverse WPT of a vector located at node (l,c) and, for convenience, call it the waveform at node (l,c). If the length (number of entries) of the vectors at certain levels is selected such that the waveforms generated have the same length, then

$$\mathbf{w}^t_{(l,c_1)}\mathbf{w}_{(l,c_2)} = \delta(c_1 - c_2) \tag{1a}$$

$$\mathbf{w}^t_{(l_1,c_1)}\mathbf{w}_{(l_1+k,c_2)} = 0 \tag{1b}$$

where $c2 \notin [(c_1-1)\times 2^k+1,\ \ c_1\times 2^k]$, and k is a positive integer.

0-7803-3905-3/97/$10.00©1997 IEEE

Eq. (1a) indicates that all the waveforms derived from the same level are orthogonal to each other, while Eq. (1b) shows that a waveform is correlated only to its ancestors and descendants (ancestors and descendants of a waveform are the waveforms at the nodes above or below the node where the waveform is located, respectively).

In an over-saturated system, the number of users, K, is larger than the dimension of a signature waveform. Consequently, the signature waveforms are linearly dependent, and the user detection problem becomes more challenging. As will be shown below, using the WPT approach allows one to consider the design of the signature set and receiver concurrently, which leads to improved system efficiency. To accommodate K users in an N-dimensional system with $K > N$, all the N nodes at level l ($l = \log_2 N$) are selected to generate N orthogonal signature waveforms. The remaining $K - N$ signature waveforms are generated to maximize the minimum Euclidean distance in the resulting signature set without changing the energy level of the signature waveforms.

As an example, let us consider a system that handles 36 users. The length of the signature waveforms used is 32. The first 32 signature waveforms can be derived from 32 nodes at level 5 of the WPT binary tree. The other 4 can be derived such that

(a) they are orthogonal to each other, and

(b) each new signature waveform is correlated to a maximized number of the 32 signature waveforms.

Requirement (a) is for the simplicity of the receiver design while requirement (b) leads to a maximized minimum Euclidean distance. Likewise, the 4 nodes at level 2 are selected to further maximize the minimum Euclidean distance. It can be readily proved that the vector at each node in level 2 should be selected in order for the derived signature waveform to be an evenly linear combination of its 8 descendants at level 5, i.e.,

$$\mathbf{w}_{(2,i)} = \sum_{j=8(i-1)}^{8i} \frac{1}{2\sqrt{2}} \mathbf{w}_{(5,j)} \qquad \text{for i=1,2,3,4.}$$

Notice that the cross-correlations of the resulting signature waveforms exhibit a hierarchical structure which will be used in the receiver design step to be described in the next section.

III. Multiuser Joint Detection

Through the rest of the paper, we assume that

- the user signature waveforms are known at the receiver,
- the channel is an additive white Gaussian noise channel,
- perfect synchronization and power control are assumed, thus all user signals arrive synchronously with the same power, and
- antipodal $\{+1, -1\}$ signaling is used.

If we let b_k denote the information bit of the kth user, and $\mathbf{w}_k$, a column vector of length N, represent the signature waveform of the kth user, then the received signal, $\mathbf{r}$, can be expressed as

$$\mathbf{r} = \sum_k b_k \mathbf{w}_k + \mathbf{n} = \mathbf{W}\mathbf{b} + \mathbf{n} \tag{2}$$

where $\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, ..., \mathbf{w}_K]$, $\mathbf{b} = [b_1, b_2, ..., b_K]^T$, and $\mathbf{n}$ is a Gaussian noise vector with variance σ^2.

A conventional matched-filter-based detector is optimal only when all other user signals are orthogonal to the signal of interest. However, when the above condition is not satisfied, as in conventional DS-CDMA and WPMA systems, a more efficient approach is to exploit the rich information content in the MAI and detect the user symbols jointly. The optimal multiuser joint detector proposed in [1] seeks to maximize the log-likelihood function and results in

$$\begin{aligned} \hat{\mathbf{b}} &= \arg\left[\max_{b_k \in \{1,-1\}} \|\mathbf{r} - \mathbf{W}\mathbf{b}\|_2^2\right] \\ &= \arg\left[\max_{b_k \in \{1,-1\}} 2\mathbf{r}^T\mathbf{W}\mathbf{b} - (\mathbf{W}\mathbf{b})^T(\mathbf{W}\mathbf{b})\right] \end{aligned} \tag{3}$$

However, there are 2^k possible symbol vectors for $\mathbf{b}$, and its computational complexity is exponential with respect to K. This makes the optimal detector impractical for systems with a large number of users. A number of suboptimal joint detectors have since been developed, seeking to offer a slightly degraded performance with much reduced computational complexity. In [6], the multiuser detection problem was considered using a geometric representation of multiuser detection, and an iterative detector known as APJD was proposed for over-saturated multiple-access systems. In the $(m+1)$th iteration, a more accurate estimate of the user information bits, $\hat{\mathbf{b}}^{\mathbf{m+1}}$, is obtained from the mth estimate as

$$\hat{\mathbf{b}}^{\mathbf{m+1}} = \mathcal{P}_\gamma \mathcal{P}_\omega \hat{\mathbf{b}}^{\mathbf{m}} \tag{4}$$

where γ denotes the space comprising 2^K possible user information vectors, ω denotes the space comprising all the solutions of equation $\mathbf{r} = \mathbf{W}\mathbf{x}$, and $\mathcal{P}$ denotes an projection operator. Using linear algebraic manipulations, the above equation can be wrriten as

$$\hat{\mathbf{b}}^{\mathbf{m+1}} = \text{sgn}[\mathbf{W}^\dagger\mathbf{r} + (\mathbf{I} - \mathbf{W}^\dagger\mathbf{W})\hat{\mathbf{b}}^{\mathbf{m}}] \tag{5}$$

where sgn denotes the *signum* function, $\mathbf{W}^\dagger$ denotes the pseudoinverse of $\mathbf{W}$, and $\mathbf{I}$ is the identity matrix. It is noted that one of the most attractive features of the APJD is that the algorithm always converges at a high rate (see [6] for details).

In (5) $\mathbf{W}^\dagger\mathbf{r}$ is a constant vector in each iteration; hence the numerically most expensive part of the algorithm is to compute $(\mathbf{I} - \mathbf{W}^\dagger\mathbf{W})\hat{\mathbf{b}}^{\mathbf{m}}$. In what follows, we show that this computation can be accomplished efficiently by exploiting the hierarchical structure of the cross-correlations of the user signature waveforms.

First, let us write the matrix $\mathbf{W}$ such that the signature waveforms with the common root in the WPT tree

are stacked to form submatrices in $\mathbf{W}$. This means that the users are grouped into several subgroups. In each submatrix, signature waveforms at lower levels are placed after the waveforms at higher levels. For example, in the system with 36 users discussed earlier, $\mathbf{W}$ is arranged as

$$\mathbf{W} = [\mathbf{W_a} \ \vdots \ \mathbf{W_b} \ \vdots \ \mathbf{W_c} \ \vdots \ \mathbf{W_d}] \tag{6}$$

where

$$\begin{aligned}
\mathbf{W_a} &= [\mathbf{w}_{(5,1)} \ \cdots \ \mathbf{w}_{(5,8)} \ \mathbf{w}_{(2,1)}] \\
\mathbf{W_b} &= [\mathbf{w}_{(5,9)} \ \cdots \ \mathbf{w}_{(5,16)} \ \mathbf{w}_{(2,2)}] \\
\mathbf{W_c} &= [\mathbf{w}_{(5,17)} \ \cdots \ \mathbf{w}_{(5,24)} \ \mathbf{w}_{(2,3)}] \\
\mathbf{W_d} &= [\mathbf{w}_{(5,25)} \ \cdots \ \mathbf{w}_{(5,32)} \ \mathbf{w}_{(2,4)}]
\end{aligned}$$

We now discard all the signature waveforms which are not from the highest level, and define the new matrix obtained as $\tilde{\mathbf{W}}$

$$\tilde{\mathbf{W}} = [\mathbf{W_1} \ \vdots \ \mathbf{W_2} \ \vdots \ \mathbf{W_3} \ \vdots \ \mathbf{W_4}] \tag{7}$$

where

$$\begin{aligned}
\mathbf{W_1} &= [\mathbf{w}_{(5,1)} \ \cdots \ \mathbf{w}_{(5,7)} \ \mathbf{w}_{(5,8)}] \\
\mathbf{W_2} &= [\mathbf{w}_{(5,9)} \ \cdots \ \mathbf{w}_{(5,15)} \ \mathbf{w}_{(5,16)}] \\
\mathbf{W_3} &= [\mathbf{w}_{(5,17)} \ \cdots \ \mathbf{w}_{(5,23)} \ \mathbf{w}_{(5,24)}] \\
\mathbf{W_4} &= [\mathbf{w}_{(5,25)} \ \cdots \ \mathbf{w}_{(5,31)} \ \mathbf{w}_{(5,32)}]
\end{aligned}$$

Notice that $\tilde{\mathbf{W}}$ is an orthogonal matrix, and multiplying its transpose by a vector does not change the norm of the vector. Thus, we can rewrite (3) as

$$\hat{\mathbf{b}} = \arg\left[\min_{b_k \in \{1,-1\}} \|\tilde{\mathbf{W}}^T\mathbf{r} - \tilde{\mathbf{W}}^T\mathbf{W}\mathbf{b}\|_2^2\right] \tag{8}$$

where

$$\tilde{\mathbf{W}}^T\mathbf{W} = \begin{bmatrix} \mathbf{W}_1^T\mathbf{W}_a & 0 & 0 & 0 \\ 0 & \mathbf{W}_2^T\mathbf{W}_b & 0 & 0 \\ 0 & 0 & \mathbf{W}_3^T\mathbf{W}_c & 0 \\ 0 & 0 & 0 & \mathbf{W}_4^T\mathbf{W}_d \end{bmatrix} \tag{9}$$

If we arrange the user information vector $\mathbf{b}$ in accordance with the structure of $\mathbf{W}$, we obtain

$$\mathbf{b} = [\mathbf{b_a^T} \ \vdots \ \mathbf{b_b^T} \ \vdots \ \mathbf{b_c^T} \ \vdots \ \mathbf{b_d^T}]^{\mathbf{T}},$$

and (8) becomes

$$\hat{\mathbf{b}} = \arg\left[\min_{b_k \in \{1,-1\}} \left\| \begin{pmatrix} \mathbf{W}_1^T\mathbf{r} - \mathbf{W}_1^T\mathbf{W}_a\mathbf{b}_a \\ \mathbf{W}_2^T\mathbf{r} - \mathbf{W}_2^T\mathbf{W}_b\mathbf{b}_b \\ \mathbf{W}_3^T\mathbf{r} - \mathbf{W}_3^T\mathbf{W}_c\mathbf{b}_c \\ \mathbf{W}_4^T\mathbf{r} - \mathbf{W}_4^T\mathbf{W}_d\mathbf{b}_d \end{pmatrix} \right\|_2^2\right] \tag{10}$$

Evidently, the objective function can be divided into several sub-objective functions, with each one dependent only on the information bits of a small-size group of users. For instance, the sub-objective function for the users from group 1 in the system with 36 users is

$$\hat{\mathbf{b}}_\mathbf{a} = \arg\left[\min_{b_k \in \{1,-1\}} \|\mathbf{W}_1^T\mathbf{r} - \mathbf{W}_1^T\mathbf{W}_a\mathbf{b}_a\|_2^2\right] \quad \text{for} \quad k \in [1,9] \tag{11}$$

By applying the APJD to each subgroup of users, we obtain the detector for user group 1 as

$$\hat{\mathbf{b}}_a^{m+1} = \mathrm{sgn}[(\mathbf{W}_1^T\mathbf{W}_a)^\dagger\mathbf{W}_1^T\mathbf{r} + (\mathbf{I} - (\mathbf{W_1^T}\mathbf{W_a})^\dagger\mathbf{W_1^T}\mathbf{W_a})\hat{\mathbf{b}}_\mathbf{a}^\mathbf{m}] \tag{12}$$

Since

$$(\mathbf{W}_1^T\mathbf{W}_a)^\dagger\mathbf{W}_1^T = \mathbf{W}_a^\dagger \tag{13}$$

(12) becomes

$$\hat{\mathbf{b}}_a^{m+1} = \mathrm{sgn}[\mathbf{W}_a^\dagger\mathbf{r} + (\mathbf{I} - \mathbf{W}_\mathbf{a}^\dagger\mathbf{W}_\mathbf{a})\hat{\mathbf{b}}_\mathbf{a}^\mathbf{m}] \tag{14}$$

In the proposed WPAPJD, the pseudoinverse is involved; hence the computation might be costly. In practice, however, all the pseudoinverse matrices can be computed off line. The only operation that has to be performed in each iteration is multiplying $\hat{\mathbf{b}}^\mathbf{m}$ by $\mathbf{I} - \mathbf{W}_\mathbf{a}^\dagger\mathbf{W}_\mathbf{a}$ and adding the result to $\mathbf{W}_\mathbf{a}^\dagger\mathbf{r}$. Due to the user grouping, the large-size matrix consisting of all the signature waveforms is divided into several submatrices. Consequently, the amount of computation is reduced significantly. In the example we used above, $\mathbf{I} - \mathbf{W}_\mathbf{a}^\dagger\mathbf{W}_\mathbf{a}$ is of dimension 9, while $\mathbf{I} - \mathbf{W}^\dagger\mathbf{W}$ is of dimension 36. The amount of computation required performing the user detection of all the 4 groups will be about 25% of the computation required by the original detector.

From our extensive computer simulations, we observed that the algorithm might converge to an undesirable point due to one of the projection operators in (4), $\mathcal{P}_\gamma$. This can be clarified if we assume the following:

(a) That no noise is present and hence (14) can be written as

$$\begin{aligned}
\hat{\mathbf{b}}_\mathbf{a}^{\mathbf{m+1}} &= \mathrm{sgn}[\hat{\mathbf{b}}_\mathbf{a}^\mathbf{m} + \mathbf{W}_\mathbf{a}^\dagger\mathbf{W}_\mathbf{a}(\mathbf{b}_\mathbf{a} - \hat{\mathbf{b}}_\mathbf{a}^\mathbf{m})] \\
&= \mathcal{P}_\gamma[\hat{\mathbf{b}}_\mathbf{a}^\mathbf{m} + \delta\hat{\mathbf{b}}_\mathbf{a}^\mathbf{m}]
\end{aligned} \tag{15}$$

where $\mathbf{b_a}$ denotes the transmitted information bits of users in group 1.

(b) After the mth iteration, the distance between $\hat{\mathbf{b}}_a^m$ and the true user information vector $\mathbf{b}_a$ is greater than 0, thus $\hat{\mathbf{b}}_a^\mathbf{m}$ is different from the true information vector.

(c) In the $(m+1)$th iteration, the absolute values of all entries in $\delta\hat{\mathbf{b}}_a^m$ are smaller than one.

After projection, we would have

$$\mathrm{sgn}[\hat{\mathbf{b}}_\mathbf{a}^\mathbf{m} + \delta\hat{\mathbf{b}}_\mathbf{a}^\mathbf{m}] = \hat{\mathbf{b}}_\mathbf{a}^\mathbf{m}$$

Thus, the algorithm would converge to an incorrect point. This suggests that one can improve the situation by changing the projector $\mathcal{P}_\gamma$ in the following manner:

- If $b_i \le \delta$ $(0 \le \delta < 1)$, keep b_i unchanged.
- If $b_i > \delta$, project it to '+1' or '−1'.

After these checks, the distance between $\hat{\mathbf{b}}_a^m$ and $\hat{\mathbf{b}}_a^{m+1}$ is examined. If the distance is less than a given tolerance,

the algorithm is deemed to converge, and the new estimate $\hat{\mathbf{b}}_a^{m+1}$ is projected into the information vector space. The value of δ affects the convergence rate as well as the performance of the detector. It has been observed that a δ in the interval $[0.3, 0.5]$ leads to the lowest bit-error rate (BER) and a slightly increased number of iterations.

IV. Simulation Results

In order to examine the merits of the proposed method, we have carried out simulations with WPAPJD and the method in [5], which is a refined version of the MJD. In each simulation, the systems were assumed to have 34 users with 32 samples for each signature waveform, and 5000 information bits per user were transmitted. Fig. 2 shows performance comparisons between the two methods. The performance is measured by the BER versus the signal-to-noise ratio (SNR). It is observed that the WPAPJD consistently achieves lower BER if the SNR is less than 16 dB. Fig. 3 shows that the WPAPJD requires much fewer iterations when the SNR is relative low, and almost the same number of iterations in the scenarios of very high SNR. Note that the amounts of computation required by the two methods in each iteration are nearly the same and, in effect, the WPAPJD has a lower computational complexity if the SNR is moderate.

V. Conclusion

A concurrent design of the signature set and receiver for over-saturated multiple-access systems has been presented. A multiuser joint detector using a wavelet-packet-based alternating projection has then been proposed. The simulation results show that the proposed detector is a promising one for applications in over-saturated multiple-access systems since improved BER and reduced computational complexity can be achieved for applications where a SNR less than 16 dB is acceptable. This work has also layed the groundwork for future developement of more general over-saturated multiple-access systems.

References

[1] S. Verdú, "Minimum probability of error for asynchronous Gaussian multiple-access channels," *IEEE Trans. Inform. Theory*, vol. 32, pp. 85-96, Jan. 1986.

[2] R. Lupas and S. Verdú, "Linear multiuser detectors for synchronous code-division multiple-access channels," *IEEE Trans. Inform. Theory*, vol. 35, pp. 123-136, Jan. 1989

[3] R. Lupas and S. Verdú, "Near-far resistance of multiuser detectors in asynchronous channels," *IEEE Trans. Commun.*, vol. 38, pp. 496-508, April 1989.

[4] M. Varanasi and B. Aazhang, "Multistage detection in asynchronous code-division multiple-access communications," *IEEE Trans. Commun.*, vol. 38, pp. 509-519, April 1990.

[5] R. E. Learned, H. Krim, B. Claus, A. S. Willsky and W. C. Karl, "Wavelet-packet-based multiple access communication," in *Proc. SPIE*, July 1994.

[6] R. E. Learned, S. Mallat, B. Claus and A. S. Willisky, "Geometric interpretation of multiaccess joint detection and the alternating projection algorithm," *IEEE Proc. ICASSP*, vol. 2, pp. 1193-1196, 1995.

[7] S. Moshavi, "Multi-user detection for DS-CDMA communications," *IEEE Commun. Mag.*, pp. 124-136, Oct. 1996.

[8] R. E. Learned, H. Krim, B. Claus, A. S. Willsky and W. C. Karl, "Convergence study of an iterative joint detector for wavelet packet multiple-access communication," in *Proc. IEEE Signal Processing Int. Conf. Time-Frequency and Time-Scale Analysis*, Oct. 1994.

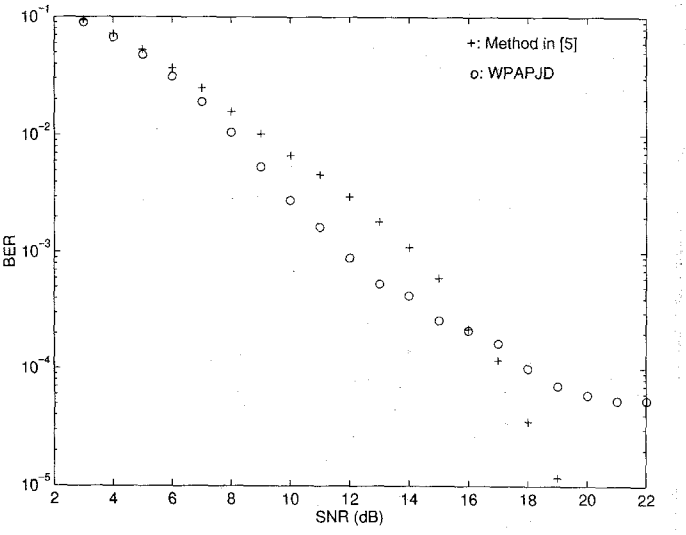

Fig. 2. Performance comparisons between WPAPJD and the method in [5].

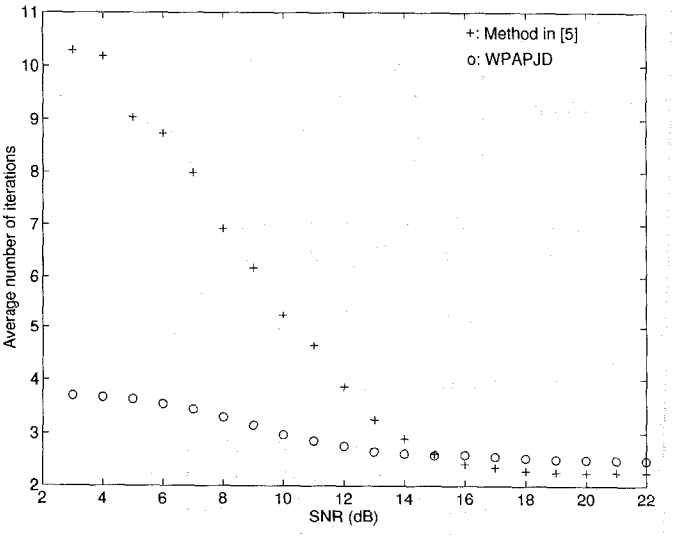

Fig. 3. Average number of iterations required by WPAPJD and the method in [5].