RECENT RESULTS ON MODEL REDUCTION METHODS FOR 2-D DISCRETE SYSTEMS

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ABSTRACT

Model reduction methods for two-dimensional (2-D) discrete systems are reviewed. Specifically, the methods of pseudo-balanced, quasi-balanced, structurally balanced, weighted quasi-balanced, and weighted structurally balanced model reduction for 2-D systems with input and output weights will be discussed. Particular attention will be given to the issues of approximation accuracy and error bounds, computational complexity of the methods, and the stability of the reduced-order systems.

1. INTRODUCTION

It is often desirable to represent a high-order system by a reasonablly accurate lower-order system for reduced system delay, improved computation efficiency, and lower implementation cost. Two important issues in the evaluation of a model reduction method are the approximation accuracy and the stability of the reduced-order system. For one-dimensional (1-D) systems, model reduction has been a subject of intensive research during the last two decades, and methods that yield excellent approximation accuracy with guaranteed stability have been established [1][2]. The most popular 1-D model reduction techniques are based on the concept of balanced realization which was originally proposed by Moore [1]. Given a discrete system, its balanced realization describes the system in a state-space representation in which the importance of the ith state variable can be measured by the ith Hankel singular value of the system. This suggests that one way of obtaining a low-order approximation of a state-space model is to form a balanced realization and then to retain those states corresponding to the k largest Hankel singular values where k is the order of the reduced-order system. For 2-D systems, the model reduction problem has been investigated by many researchers during the past several years [3]–[13], and the research results obtained have been found useful in, for example, digital filter design [12]–[14]. One of the problems in the study of 2-D model reduction is to extend the balanced realization concept to the 2-D case. Since a balanced realization is essentially determined by the controllability and observability Gramians of the system, and since there are several types of Gramians that can be properly defined for a given 2-D system, there are different types of balanced realizations for a given 2-D discrete system, leading to different balanced approximations.

This paper presents a survey of the recent progress in developing various 2-D model reduction methods with an emphasis on balanced-realization-related techniques. In the next section, we discuss three possible extensions of the controllability and observability Gramians for a discrete 2-D system and the three balanced realizations that are associated with these Gramians. Section 3 discusses the model reduction methods that are based on the balanced realizations described in Section 2. These methods are evaluated in terms of approximation accuracy, computation complexity, and stability. Section 4 discusses further extensions of the balanced realizations by including input and output weights, and their application to 2-D model reduction.

2. THREE TYPES OF GRAMIANS AND BALANCED 2-D DISCRETE SYSTEMS

2.1. Review of the 1-D Case

Let us begin by a brief review of the concept of Gramians for a 1-D discrete system represented by

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{b}u_k \tag{1a}$$
$$y_k = \mathbf{c}\mathbf{x}_k + du_k \tag{1b}$$

It is assumed that the system is controllable, observable, and stable [15]. Under these assumptions the realization (1) is said to be balanced if

$$\mathbf{P} = \mathbf{Q} = \mathrm{diag}\{\sigma_1,\ \sigma_2,\ \cdots,\ \sigma_n\} \tag{2}$$

where

$$\mathbf{P} = \sum_{k=0}^{\infty} \mathbf{A}^k \mathbf{b}\mathbf{b}^T(\mathbf{A}^T)^k, \quad \mathbf{Q} = \sum_{k=1}^{\infty} (\mathbf{A}^T)^k \mathbf{c}^T \mathbf{c}\mathbf{A}^k \tag{3}$$

are called the controllability and observability Gramians of the system, respectively. There are two advantages associated with these Gramians. First, for a balanced realization, the Gramians given by (2) provide a set of monotonically decreasing Hankel values $\sigma_1 \geq \sigma_1 \geq \cdots \geq \sigma_n > 0$ that indicate the significance of the associated state variables, leading to the well-known method of balanced model reduction [1]. Second, if the state-space representation given by (1) is not yet balanced, then the Gramians $\mathbf{P}$ and $\mathbf{Q}$ defined by (3) can be used to find a transformation $\mathbf{T}$ such that the realization characterized by $(\mathbf{T}^{-1}\mathbf{A}\mathbf{T},\ \mathbf{T}^{-1}\mathbf{b},\ \mathbf{c}\mathbf{T},\ d)$ is balanced. Having obtained a balanced realization, a reduced-order system of order r can be obtained by truncation as $(\mathbf{A}_r,\ \mathbf{b}_r,\ \mathbf{c}_r,\ d)$ where $\mathbf{A}_r = \mathbf{A}(1:r,\ 1:r)$, $\mathbf{b}_r = \mathbf{b}(1:r)$, and $\mathbf{c}_r = \mathbf{c}(1:r)$. Two important features of balanced model reduction methods are that the reduced-order system remains stable if the original system is stable, and that the approximation error has a upper bound, which is determined by the last $n-r$ Hankel singular values, and is given

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by

$$||H(z)-H_r(z)||_\infty \equiv \max_{|z|=1}|H(z)-H_r(z)| \le 2\sum_{k=r+1}^{n}\sigma_k \quad (4)$$

where $H(z) = \mathbf{c}(z\mathbf{I}-\mathbf{A})^{-1}\mathbf{b}+d$ and $H_r(z) = \mathbf{c}_4(z\mathbf{I}-\mathbf{A}_r)^{-1}\mathbf{b}_r+d$ are the transfer functions of the original system and reduced-order system, respectively. In addition to (3), there are two alternatives to evaluate $\mathbf{P}$ and $\mathbf{Q}$: one involves the complex integrals

$$\mathbf{P} = \frac{1}{2\pi j}\oint_{|z|=1}\mathbf{F}(z)\mathbf{F}^*(z)\frac{dz}{z},\quad \mathbf{Q} = \frac{1}{2\pi j}\oint_{|z|=1}\mathbf{G}^*(z)\mathbf{G}(z)\frac{dz}{z} \quad (5)$$

where

$$\mathbf{F}(z) = (z\mathbf{I}-\mathbf{A})^{-1}\mathbf{b} \quad (6a)$$
$$\mathbf{G}(z) = \mathbf{c}(z\mathbf{I}-\mathbf{A})^{-1} \quad (6b)$$

and the other uses the Lyapunov equations

$$\mathbf{APA}^T-\mathbf{P} = -\mathbf{bb}^T \quad (7a)$$
$$\mathbf{A}^T\mathbf{QA}-\mathbf{Q} = -\mathbf{c}^T\mathbf{c} \quad (7b)$$

Among the three evaluation methods (3), (5) and (7), the Lyapunov equation method requires the minimum amount of computations.

2.2. Three Types of Gramians for 2-D Discrete Systems

Consider the Roesser state-space model

$$\begin{bmatrix}\mathbf{x}^h(i+1,j)\\ \mathbf{x}^v(i,j+1)\end{bmatrix} = \begin{bmatrix}\mathbf{A}_1 & \mathbf{A}_2\\ \mathbf{A}_3 & \mathbf{A}_4\end{bmatrix}\begin{bmatrix}\mathbf{x}^h(i,j)\\ \mathbf{x}^v(i,j)\end{bmatrix}+\begin{bmatrix}\mathbf{b}_1\\ \mathbf{b}_2\end{bmatrix}u(i,j)$$
$$\equiv \mathbf{Ax}(i,j)+\mathbf{b}i(i,j) \quad (8a)$$
$$y(i,j) = [\mathbf{c}_1\ \ \mathbf{c}_2]\mathbf{x}(i,j)+du(i,j)$$
$$\equiv \mathbf{cx}(i,j)+du(i,j) \quad (8b)$$

and define

$$\mathbf{F}(z_1,z_2) = [\mathbf{I}(z_1,z_2)-\mathbf{A}]^{-1}\mathbf{b} \quad (9)$$
$$\mathbf{G}(z_1,z_2) = \mathbf{c}[\mathbf{I}(z_1,z_2)-\mathbf{A}]^{-1} \quad (10)$$

where $\mathbf{I}(z_1,z_2) = z_1\mathbf{I}_{n1}\oplus z_2\mathbf{I}_{n2}$. The first type of 2-D Gramians, known as the pseudo-controllability and observability Gramians [7], are obtained by generalizing (5) and are defined as

$$\mathbf{P}^p = \frac{1}{(2\pi j)^2}\oint_{|z_1|=1}\oint_{|z_2|=1}\mathbf{F}(z_1,z_2)\mathbf{F}^*(z_1,z_2)\frac{dz_2}{z_2}\frac{dz_1}{z_1}$$
$$\mathbf{Q}^p = \frac{1}{(2\pi j)^2}\oint_{|z_1|=1}\oint_{|z_2|=1}\mathbf{G}^*(z_1,z_2)\mathbf{G}(z_1,z_2)\frac{dz_2}{z_2}\frac{dz_1}{z_1}$$

The second type of Gramians, known as the structured controllability and observability Gramians [11][13], are defined by the positive definite solutions $\mathbf{P}^s$ and $\mathbf{Q}^s$ of the Lyapunov inequalities

$$\mathbf{AP}^s\mathbf{A}^T-\mathbf{P}^s+\mathbf{bb}^T < \mathbf{0} \quad (11a)$$
$$\mathbf{A}^T\mathbf{Q}^s\mathbf{A}-\mathbf{Q}^s+\mathbf{c}^T\mathbf{c} < \mathbf{0} \quad (11b)$$

Note that the positive definite matrices $\mathbf{P}^s$ and $\mathbf{Q}^s$, if they exist, are not unique. This leads to the non-uniqueness of structurally balanced realizations that are based on $\mathbf{P}^s$ and $\mathbf{Q}^s$, and therefore raises the question of how to find a structurally balanced realization that yields the best reduced-order system according to certain performance criteria [12].

The third type of Gramians, known as the quasi-controllability and observability Gramians [12], are defined by the positive definite block-diagonal matrices $\mathbf{P}^q = \text{diag}\{\mathbf{P}_1,\ \mathbf{P}_2\}$ and $\mathbf{Q}^q = \text{diag}\{\mathbf{Q}_1,\ \mathbf{Q}_2\}$ where $\mathbf{P}_i$ and $\mathbf{Q}_i$ $(i=1,2)$ satisfy the Lyapunov equations

$$\mathbf{A}_1\mathbf{P}_1\mathbf{A}_1^T-\mathbf{P}_1+\mathbf{b}_1\mathbf{b}_1^T+\mathbf{A}_2\mathbf{P}_2\mathbf{A}_2^T = \mathbf{0} \quad (12a)$$
$$\mathbf{A}_4\mathbf{P}_2\mathbf{A}_4^T-\mathbf{P}_2+\mathbf{b}_2\mathbf{b}_2^T+\mathbf{A}_3\mathbf{P}_1\mathbf{A}_3^T = \mathbf{0} \quad (12b)$$
$$\mathbf{A}_1^T\mathbf{Q}_1\mathbf{A}_1-\mathbf{Q}_1+\mathbf{c}_1^T\mathbf{c}_1+\mathbf{A}_3^T\mathbf{Q}_2\mathbf{A}_3 = \mathbf{0} \quad (12c)$$
$$\mathbf{A}_4^T\mathbf{Q}_1\mathbf{A}_4-\mathbf{Q}_2+\mathbf{c}_2^T\mathbf{c}_2+\mathbf{A}_2^T\mathbf{Q}_1\mathbf{A}_2 = \mathbf{0} \quad (12d)$$

In terms of computation complexity, the structured Gramians are most expensive to evaluate while the quasi-Gramians are most economical. An optimization approach to solving the Lyapunov inequalities (11a) and (11b) was proposed in [16], and a recursive method for the computation of the quasi-Gramians is described in [13].

2.3. Balanced Approximations

As in the 1-D case, balanced realizations of a 2-D discrete system are obtained by applying appropriate transformations to the state variables, and these transformations are determined by using the controllability and observability Gramians. Since there are at least three types of Gramians, as described in Sec. 2.2, one can accordingly derive three different types of balanced realizations. In effect, once a certain type of Gramians is chosen, the upper left and lower right diagonal blocks of the Gramians are used to compute the transformation matrix $\mathbf{T} = \mathbf{T}_1\oplus\mathbf{T}_2$ by using, for example, Laub's algorithm [17] such that the realization characterized by $\{\mathbf{T}^{-1}\mathbf{AT},\ \mathbf{T}^{-1}\mathbf{b},\ \mathbf{cT},\ d\}$ is balanced.

Once a balanced realization is obtained, a reduced-order system of order (r_1, r_2), denoted by $\{\mathbf{A}_r,\ \mathbf{b}_r,\ \mathbf{c}_r,\ d\}$, can be obtained by truncating matrices $\mathbf{A}$, $\mathbf{b}$, and $\mathbf{c}$ as

$$\mathbf{A}_r = \begin{bmatrix}\mathbf{A}_{1r} & \mathbf{A}_{2r}\\ \mathbf{A}_{3r} & \mathbf{A}_{4r}\end{bmatrix},\ \mathbf{b}_r = \begin{bmatrix}\mathbf{b}_{1r}\\ \mathbf{b}_{2r}\end{bmatrix},\ \mathbf{c}_r = [\mathbf{c}_{1r}\ \ \mathbf{c}_{2r}]$$

with

$$\mathbf{A}_{1r} = \mathbf{A}_1(1:r_1,\ 1:r_1),\ \mathbf{A}_{2r} = \mathbf{A}_2(1:r_1,\ 1:r_2)$$
$$\mathbf{A}_{3r} = \mathbf{A}_3(1:r_2,\ 1:r_1),\ \mathbf{A}_{4r} = \mathbf{A}_4(1:r_2,\ 1:r_2)$$
$$\mathbf{b}_{1r} = \mathbf{b}_1(1:r_1),\ \mathbf{b}_{2r} = \mathbf{b}_2(1:r_2)$$
$$\mathbf{c}_{1r} = \mathbf{c}_1(1:r_1),\ \mathbf{c}_{2r} = \mathbf{c}_2(1:r_2)$$

3. ERROR BOUNDS AND STABILITY OF REDUCED-ORDER SYSTEMS

3.1. Pseudo-Balanced Approximations

A reduced-order system obtained by using the pseudo-balanced approximation maintains stability if the original system has separable denominators [10]. For systems with nonseparable denominators, although the stability of the reduced-order system may be preserved in many circumstances, it was shown in [10] by an example that the reduced-order system may be unstable.

With the exception of some coarse upper bounds for the approximation error for special types of separable 2-D systems, no error bounds are available for the class of general

2-D discrete systems when the pseudo-balanced approximation method is used.

3.2. Structurally Balanced Approximations

Although the structurally balanced approximation requires the most intensive computations, the reduced-order system obtained always preserves stability. Moreover, it can be shown that

$$\|H(z_1,z_2) - H_r(z_1,z_2)\|_\infty \le 2\left(\sum_{i=r_1+1}^{n_1} \sigma_{1i} + \sum_{i=r_2+1}^{n_2} \sigma_{2i}\right) \tag{13}$$

where $H(z_1,z_2)$ and $H_r(z_1,z_2)$ are the transfer functions of the original and reduced-order systems, respectively, and $\{\sigma_{1i},\ i=r_1+1,\ \ldots,\ n_1\}$ and $\{\sigma_{2i},\ i=r_2+1,\ \ldots,\ n_2\}$ are the last $n_i - r_i$ $(i=1,\ 2)$ Hankel singular values associated with the ith dimension [11].

As was pointed out in Sec. 2.2, the Hankel singular values appearing on the right-hand side of (13) are not unique. Since the major concerns in model reduction are stability and reduction accuracy, it was proposed in [13] that solutions of the Lyapunov equalities (11a) and (11b) be chosen by solving an unconstrained optimization problem in which the objective function takes both the system stability and the magnitude of the sum of the Hankel singular values into account.

3.3. Quasi-Balanced Approximations

Although the quasi-balanced approximation method does not preserve the stability of the reduced-order system in general, the reduced-order system remains stable if the original system has a separable denominator [12]. For systems with separable denominators, [12] also provides several error bounds for the approximation error. Like those derived in [10], these error bounds tend to be quite conservative in many cases.

Some frequency-compensation techniques can be used in conjunction with the quasi-balanced approximation method to improve the performance of the reduced-order system in certain frequency region. It has been shown in [12] that the pseudo-balanced method or the quasi-balanced approximation method combined with a singular perturbation technique can lead to reduced-order systems with significantly improved performance in the low frequency region.

4. WEIGHTED BALANCED APPROXIMATIONS

If the requirements imposed on the approximation error are different for different frequency ranges, improved reduced-order systems can be obtained by applying a weighted balanced approximation method. The continuous-time 1-D case is considered in [18] and the discrete-time 1-D case is considered in [19][20].

In [13], a model reduction algorithm that yields a weighted structually balanced approximation was proposed. The method starts with defining two auxiliary transfer function matrices called the weighted-input-to-state and state-to-weighted-output transfer function matrices, and denoted as $H_i(z_1,z_2)$ and $H_0(z_1,z_2)$, respectively. If the transfer functions of the system, input weight, and output weight are given by

$$\begin{aligned}
\mathbf{H}(z_1,z_2) &= \mathbf{c}[\mathbf{I}(z_1,z_2) - \mathbf{A}]^{-1}\mathbf{B} + \mathbf{D} \\
\mathbf{W}_i(z_1,z_2) &= \mathbf{c}_i[\mathbf{I}(z_1,z_2) - \mathbf{A}_i]^{-1}\mathbf{B}_i + \mathbf{D}_i \\
\mathbf{W}_o(z_1,z_2) &= \mathbf{c}_o[\mathbf{I}(z_1,z_2) - \mathbf{A}_o]^{-1}\mathbf{B}_o + \mathbf{D}_o
\end{aligned}$$

respectively, then

$$\begin{aligned}
\mathbf{H}_i(z_1,z_2) &= [\mathbf{I}(z_1,z_2) - \mathbf{A}]^{-1}\mathbf{B}\mathbf{W}_i(z_1,z_2) \\
\mathbf{H}_o(z_1,z_2) &= \mathbf{W}_o(z_1,z_2)\mathbf{C}[\mathbf{I}(z_1,z_2) - \mathbf{A}]^{-1}
\end{aligned}$$

It can be shown that if $\mathbf{H}(z_1,z_2)$, $\mathbf{W}_i(z_1,z_2)$ and $\mathbf{W}_o(z_1,z_2)$ are stable transfer function matrices, then $\mathbf{H}_i(z_1,z_2)$ and $\mathbf{H}_o(z_1,z_2)$ are stable as well [13]. Consequently, state-space realizations of $\mathbf{H}_i(z_1,z_2)$ and $\mathbf{H}_o(z_1,z_2)$ can be used to define structured controllability and observability Gramians of the weighted systems. Once the Gramians are defined and evaluated, they can be used to obtain a block-diagonal transformation $\mathbf{T} = \mathbf{T}_1 \oplus \mathbf{T}_2$ such that the realization characterized by $\{\mathbf{T}^{-1}\mathbf{A}\mathbf{T},\ \mathbf{T}^{-1}\mathbf{B},\ \mathbf{C}\mathbf{T},\ \mathbf{D}\}$ is weighted structually balanced, and a reduced-order system can be generated by truncating the balanced realization obtained.

As in the unweighted case, it can be shown that for single-input single-output systems, the reduced-order system is stable for a stable original system and stable input and output weights [13]. For the multi-input multi-output case, if $\mathbf{H}(z_1,z_2)$ is stable with either unit input weight, $\mathbf{W}_i(z_1,z_2) = \mathbf{I}$, or unit output weight, $\mathbf{W}_o(z_1,z_2) = \mathbf{I}$, then the reduced-order system obtained from the weighted structually balanced realization is also stable.

By using the auxiliary transfer function matrices defined above, it was shown in [21] that it is possible to generalize the method of quasi-balanced approximation to include a stable input weight and a stable output weight.

5. CONCLUSION

From the work surveyed in this paper, it can be concluded that a considerable amount of effort has been devoted to the study of model reduction of 2-D discrete systems, and a number of reliable algorithms for various kinds of balanced and weighted balanced approximations have been proposed. The structurally balanced approximation method based on Lyapunov inequalities has shown many desirable properties although it is computationally much more involved than other types of balanced approximations. Research topics related to this method that appear to be worthwhile for further study include developing new methods for solving the Lyapunov inequalities and methods for selecting a solution of these Lyapunov inequalities that yields minimum approximation error.

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