SUBSPACE ESTIMATION-BASED CONSTRAINED OPTIMIZATION METHOD FOR MULTIPATH CDMA CHANNELS

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ABSTRACT

A subspace estimation-based constrained optimization method for the suppression of multiple-access interference (MAI) is proposed. The proposed method entails a new algorithm that approximates the noise subspace in a more robust manner compared with several existing methods. Simulations described in the paper demonstrate that the performance of the proposed method is almost the same as that of some known subspace methods but leads to a significant reduction in the amount of computation. Relative to some known constrained optimization methods, our method offers a significantly improved performance while requiring a comparable amount of computation.

1. INTRODUCTION

Blind adaptive multiuser detection in direct-sequence code-division multiple-access (DS-CDMA) systems has received a great deal of attention because, first, it offers improved detection performance relative to the traditional matched-filter detector and, second, detection can be achieved without extra information [1]. In a wireless channel, the signature of the desired signal is frequently distorted due to multipath propagation and is thus unknown to the receiver. As a result, the mismatched estimation of the desired user's signature leads to significant degradation in the performance of blind adaptive detectors [1]. A possible remedy is to estimate the impulse response of the unknown multipath channel and then use the reconstructed signature of the desired user to compute the detection vector. This approach was pursued by several authors using subspace methods [3][4] and constrained optimization methods [5-7]. Although subspace methods provide better estimates for the unknown channel impulse response than constrained optimization methods, especially in very noisy channels, they usually require more computation which could become a burden in some applications.

In this paper, we propose a subspace estimation-based constrained optimization method for the estimation of the channel impulse response in CDMA systems. By estimating the least eigenvalue of the data correlation matrix, the noise subspace can be well approximated. It is demonstrated that the proposed method can achieve nearly the same performance as the subspace methods in [3][4] with much reduced computational complexity. Compared with the existing constrained optimization methods [5-7], our method offers a much improved performance while requiring a comparable amount of computation.

2. SIGNAL MODEL

Consider a K-user CDMA system with a distinct signature sequence of length N, namely, $\mathbf{s}_k = [s_k(1)\ s_k(2)\ \cdots\ s_k(N)]^T$ with $||\mathbf{s}_k|| = 1$ assigned to user k for $k = 1, \ldots, K$. The transmitted signal at chip-rate of user k is given by

$$x_k(n) = \sum_{i=-\infty}^{\infty} b_k(i) s_k(n - iN) \tag{1}$$

where $\{b_k(i)\}$ is an information bearing sequence. The impulse response of a multipath channel of user k can be described as [2]

$$g_k(t) = \sum_{l=1}^{L_k} \alpha_{kl} p(t - \tau_{kl}) \quad \text{for } i = 1, 2, \ldots, K \tag{2}$$

where $p(t)$ is the unit impulse function, L_k is the number of resolvable paths for the signal of user k at the receiver, and τ_{kl} is the excess delay. The parameter $\alpha_{kl} = a_{kl} \exp(-j\theta_{kl})$ together with $p(t - \tau_{kl})$ describe the impulse response of the lth path for the signal of user k. Let the receiver collect M samples per chip duration. The received noise-free discrete-time signal related to user k is given by

$$y_k(n) = \sum_{j=-\infty}^{\infty} x_k(j) g_k(n - d_k M - j) \tag{3}$$

where $\{g_k(n)\}$ is the discrete-time model of the channel impulse response of user k, which satisfies the relation $g_k(n) = g_k(t)|_{t=nT_c/M}$, where T_c is the chip duration and d_k is the delay of user k in chip periods. From (1) and (3), it follows that

$$y_k(n) = \sum_{j=-\infty}^{\infty} b_k(j) h_k(n - d_k M - jMN) \tag{4}$$

$$h_k(n) = \sum_{m=-\infty}^{\infty} s_k(m) g_k(n - mM) \tag{5}$$

where $h_k(n)$ represents the distorted signature of user k. By taking the signals of all the users and the channel noise into account, the received signal $y(n)$ can be modeled as

$$y(n) = \sum_{k=1}^{K} y_k(n) + v(n) \tag{6}$$

where $v(n)$ denotes a zero-mean white Gaussian noise component with variance σ_v^2.

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In the rest of the paper, we assume that the impulse responses of the multipath channel for all users are of finite duration with lengths no greater than q and that the first user is the desired user to which the receiver is synchronized and $d_k > 0$ for $k = 2, 3, \ldots, K$. In the special case where $M = 1$ (one sample per chip duration), $h_k(n)$ in (5) becomes the convolution of $s_k(n)$ and $g_k(n)$. Defining $\mathbf{y}(n) = [y(nN+1)\ y(nN+2)\ \cdots\ y(nN+N+q-1)]^T$ and $\mathbf{S}_1$ as an $(N+q-1)\times q$ Toeplitz matrix generated by $[\mathbf{s}_1^T\ \mathbf{0}]^T$ as its first column and $[s_1(1)\ \mathbf{0}]$ as its first row, we obtain a signal model as

$$\mathbf{y}(n) = \mathbf{S}_1\mathbf{g}_1 b_1(n) + \mathbf{H}\tilde{\mathbf{b}}(n) + \mathbf{v}(n) \tag{7a}$$
$$= \mathbf{h}_1 b_1(n) + \mathbf{H}\tilde{\mathbf{b}}(n) + \mathbf{v}(n) \tag{7b}$$

The first term at the right-hand side of (7a) represents the signal of the desired user of the current bit with $\mathbf{g}_1 = [g_1(1)\ g_1(2)\ \cdots\ g_1(q)]^T$ and $\mathbf{h}_1 = [h_1(1)\ h_1(2)\ \cdots\ h_1(N+q-1)]^T$ being the channel impulse response and distorted signature of user 1, respectively. The second term in (7a) represents all interference which includes the signal of the desired user of the preceding and following bits and the signal of the interference users. $\mathbf{H}$ and $\tilde{\mathbf{b}}(n)$ can be written more explicitly as $[\mathbf{h}_{11}\ \mathbf{h}_{12}\ \mathbf{h}_{21}\ \mathbf{h}_2\ \mathbf{h}_{22}\ \cdots\ \mathbf{h}_{K1}\ \mathbf{h}_K\ \mathbf{h}_{K2}]$ and $[b_1(n-1)\ b_1(n+1)\ b_2(n-1)\ b_2(n)\ b_2(n+1)\ \cdots\ b_K(n-1)\ b_K(n)\ b_K(n+1)]^T$, respectively, where

$$\begin{aligned}
\mathbf{h}_{11} &= [h_1(N+1)\ \cdots\ h_1(N+q-1)\ 0\ \cdots\ 0]^T\\
\mathbf{h}_{12} &= [0\ \cdots\ 0\ h_1(1)\ \cdots\ h_1(q-1)]^T\\
\mathbf{h}_{k1} &= [h_k(N+1-d_k)\ \cdots\ h_k(N+q-1)\ 0\ \cdots\ 0]^T\\
\mathbf{h}_{k} &= [0\ \cdots\ 0\ h_k(1)\ \cdots\ h_k(N+q-1-d_k)]^T\\
\mathbf{h}_{k2} &= \begin{cases} [0\ 0\ \cdots\ 0]^T & \text{for}\ \ d_k > q-1\\ [0\ \cdots\ 0\ h_k(1)\ \cdots\ h_k(q-d_k-1)]^T & \text{otherwise}\end{cases}
\end{aligned}$$

for $k = 2, \ldots, K$. In (7), $\mathbf{v}(n) = [v(1)\ v(2)\ \cdots\ v(N)]^T$ is a vector of zero-mean Gaussian noise with a covariance matrix $\sigma_v^2\mathbf{I}$.

Our objective here is to estimate the channel impulse response $\mathbf{g}_1$ for the desired user from $\mathbf{y}(n)$ without the knowledge of the signatures $\{\mathbf{s}_k,\ k = 2, \ldots, K\}$ of the interfering users so as to extract the information transmitted by the desired user.

3. REVIEW OF EXISTING METHODS

3.1. Subspace Methods

Several subspace methods [3][4] have been proposed for the estimation of the channel impulse response $\mathbf{g}_1$ and the corresponding detection vector $\mathbf{w}_1$ using the eigen-decomposition (ED) of the correlation matrix $\mathbf{R}$ which is defined as

$$\mathbf{R} = \mathrm{E}[\mathbf{y}(n)\mathbf{y}^H(n)] \tag{8}$$

The ED of matrix $\mathbf{R}$ can be expressed as

$$\mathbf{R} = [\mathbf{U}_s\mathbf{U}_n]\begin{bmatrix}\mathbf{\Lambda}_s & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{U}_s^H\\ \mathbf{U}_n^H\end{bmatrix} + \sigma_v^2\mathbf{I} \tag{9}$$

where $[\mathbf{U}_s\mathbf{U}_n]$ is a unitary matrix, the columns of $\mathbf{U}_s$ and $\mathbf{U}_n$ generate the signal and noise subspaces, respectively, and $\mathbf{\Lambda}_s = \mathrm{diag}\{\lambda_1, \ldots, \lambda_\xi\}$ with ξ being the dimension of the signal space. Because the signature vector of the first user, $\mathbf{h}_1$, is in the signal subspace which is orthogonal to the noise subspace, we have

$$\mathbf{U}_n^H\mathbf{h}_1 = \mathbf{U}_n^H\mathbf{S}_1\mathbf{g}_1 = \mathbf{0} \tag{10}$$

By rewriting (9) as

$$\mathbf{R} = [\mathbf{U}_s\mathbf{U}_n]\begin{bmatrix}\mathbf{\Lambda}_s + \sigma_v^2\mathbf{I}_\xi & \mathbf{0}\\ \mathbf{0} & \sigma_v^2\mathbf{I}_{N-\xi}\end{bmatrix}\begin{bmatrix}\mathbf{U}_s^H\\ \mathbf{U}_n^H\end{bmatrix} \tag{11}$$

we observe that the eigenvectors contained in $\mathbf{U}_n$ are associated with the *least eigenvalues* which equal the noise variance σ_v^2.

In practice, matrix $\mathbf{R}$ is not available but can be approximated by its *moving average* estimation based on the J most recent observations as

$$\hat{\mathbf{R}} = \frac{1}{J}\sum_{n=i-J+1}^{i}\mathbf{y}(n)\mathbf{y}^H(n), \tag{12}$$

and equation (10) becomes

$$\hat{\mathbf{U}}_n^H\mathbf{S}_1\mathbf{g}_1 = \mathbf{0} \tag{13}$$

where $\hat{\mathbf{U}}_n$ is the counterpart of $\mathbf{U}_n$. Because of the approximation error introduced in (12), it is very likely that a nonzero vector $\mathbf{g}_1$ satisfying (13) does not exist [4]. A remedy for this problem is to compute an approximate solution of (11) by solving the constrained optimization problem

$$\text{minimize}\ \ \mathbf{g}_1^H\mathbf{S}_1^H\hat{\mathbf{U}}_n\hat{\mathbf{U}}_n^H\mathbf{S}_1\mathbf{g}_1 \tag{14a}$$
$$\text{subject to:}\ \ ||\mathbf{g}_1|| = 1 \tag{14b}$$

The solution $\mathbf{g}_1^*$ of the problem in (14) is the eigenvector corresponding to the smallest eigenvalue of $\mathbf{S}_1^H\hat{\mathbf{U}}_n\hat{\mathbf{U}}_n^H\mathbf{S}_1$. Having obtained $\mathbf{g}_1^*$, the detection vector, $\mathbf{w}_1^*$, can be deduced as

$$\mathbf{w}_1^* = \hat{\mathbf{R}}^{-1}\mathbf{S}_1(\mathbf{S}_1^H\hat{\mathbf{R}}^{-1}\mathbf{S}_1)^{-1}\mathbf{g}_1^* \tag{15}$$

3.2. Constrained Optimization Methods

In the constrained optimization method proposed in [5], the impulse response $\mathbf{g}_1$ of the multipath channel and the constrained minimum output energy (CMOE) detection vector $\mathbf{w}_1$ are the solutions of the constrained minimax problem

$$\underset{\mathbf{g}_1}{\text{maximize}}\ \underset{\mathbf{w}_1}{\text{minimize}}\ \ \mathbf{w}_1^H\mathbf{R}\mathbf{w}_1 \tag{16a}$$
$$\text{subject to:}\ \ \mathbf{S}_1^H\mathbf{w}_1 = \mathbf{g}_1 \tag{16b}$$
$$||\mathbf{g}_1||^2 = 1 \tag{16c}$$

where $\mathbf{R}$ is the data correlation matrix defined in (9). In (16), the output variance $\mathbf{w}_1^H\mathbf{R}\mathbf{w}_1$ is minimized in terms of $\mathbf{w}_1$ and maximized in terms of $\mathbf{g}_1$. The minimization problem can be solved by using the Lagrange multiplier method to obtain

$$\mathbf{w}_1^*(\mathbf{g}_1) = \mathbf{R}^{-1}\mathbf{S}_1(\mathbf{S}_1^H\mathbf{R}^{-1}\mathbf{S}_1)^{-1}\mathbf{g}_1 \tag{17}$$

where $\mathbf{w}_1^*$ depends on $\mathbf{g}_1$. Using (17), it can be shown that the problem in (16) is equivalent to

$$\underset{\mathbf{g}_1}{\text{maximize}}\ \mathbf{g}_1^H(\mathbf{S}_1^H\mathbf{R}^{-1}\mathbf{S}_1)^{-1}\mathbf{g}_1 \tag{18a}$$
$$\text{subject to:}\ \ ||\mathbf{g}_1||^2 = 1 \tag{18b}$$

The solution $\mathbf{g}_1$ of the problem in (18) is the eigenvector of $\mathbf{S}_1^H\mathbf{R}^{-1}\mathbf{S}_1$ corresponding to the least eigenvalue. Hence, the problem in (18) is equivalent to

$$\underset{\mathbf{g}_1}{\text{minimize}}\ \mathbf{g}_1^H\mathbf{S}_1^H\hat{\mathbf{R}}^{-1}\mathbf{S}_1\mathbf{g}_1 \tag{19a}$$
$$\text{subject to:}\ \ ||\mathbf{g}_1||^2 = 1 \tag{19b}$$

where matrix $\mathbf{R}^{-1}$ has been replaced by $\hat{\mathbf{R}}^{-1}$ defined in (12). After the solution $\mathbf{g}_1^*$ in (19) is obtained, the detection vector $\mathbf{w}_1$ can be computed using (17). It was shown in [7] that the minimax problem in (16) can be efficiently solved by using adaptation algorithms, which is advantageous because constrained optimization methods can be used to deal with CDMA systems where the multipath channel changes frequently.

Compared to the two methods reviewed above, subspace methods usually provide more accurate estimation of the channel impulse response and detection vector. The improvement becomes more significant when the channel noise is very large (i.e., low SNR). However, because of the ED involved in subspace methods, their computational complexity is rather high, which could become a serious burden in some applications.

3.3. Relation of the Two Methods

The two classes of methods reviewed above have an interesting relation. To see this, note that $\hat{\mathbf{R}}^{-1}$ can be expressed as [5]

$$\hat{\mathbf{R}}^{-1} = \hat{\mathbf{U}}_s\mathbf{D}\hat{\mathbf{U}}_s^H + \frac{1}{\sigma_v^2}\hat{\mathbf{U}}_n\hat{\mathbf{U}}_n^H \quad (20)$$

where $\mathbf{D} = \text{diag}\{\frac{1}{\lambda_1+\sigma_v^2}, \frac{1}{\lambda_2+\sigma_v^2}, \ldots, \frac{1}{\lambda_\xi+\sigma_v^2}\}$. This implies that

$$\sigma_v^2\hat{\mathbf{R}}^{-1} = \hat{\mathbf{U}}_s\mathbf{M}\hat{\mathbf{U}}_s^H + \hat{\mathbf{U}}_n\hat{\mathbf{U}}_n^H \quad (21)$$

where $\mathbf{M} = \sigma_v^2\mathbf{D} = \text{diag}\{\frac{\sigma_v^2}{\lambda_1+\sigma_v^2}, \frac{\sigma_v^2}{\lambda_2+\sigma_v^2}, \ldots, \frac{\sigma_v^2}{\lambda_\xi+\sigma_v^2}\}$. As a result,

$$\begin{aligned}\lim_{\sigma_v^2 \to 0}\sigma_v^2\hat{\mathbf{R}}^{-1} &= \lim_{\sigma_v^2 \to 0}\sigma_v^2\sum_{i=1}^{\xi}\frac{\hat{\mathbf{u}}_{s,i}\hat{\mathbf{u}}_{s,i}^H}{\lambda_i+\sigma_v^2} + \hat{\mathbf{U}}_n\hat{\mathbf{U}}_n^H \\ &= \hat{\mathbf{U}}_n\hat{\mathbf{U}}_n^H \end{aligned} \quad (22)$$

where $\hat{\mathbf{u}}_{s,i}$ is the eigenvector associated with λ_i. Consequently, (22) indicates that when the channel noise is insignificant (i.e., σ_v^2 is small), $\hat{\mathbf{R}}^{-1}$ is a good approximation of $\hat{\mathbf{U}}_n\hat{\mathbf{U}}_n^H$ to within a scalar multiplier. In other words, the solutions obtained by solving the problems in (14) and (19) are very close to each other when σ_v^2 is small. On the other hand, if σ_v^2 is large, the solution of the problem in (19) can only be a degraded version of that in (14). For this reason, constrained optimization methods can be viewed as modified subspace methods.

In an effort to improve the performance of constrained optimization methods, matrix $\hat{\mathbf{U}}_n\hat{\mathbf{U}}_n^H$ was replaced by $\hat{\mathbf{R}}^{-1} - \beta\mathbf{I}$ in [6] where β is the reciprocal of the largest eigenvalue of $\hat{\mathbf{R}}$. However, for systems with strong MAI, the value of β becomes very small which leads to a limited performance improvement.

4. SUBSPACE ESTIMATION-BASED CONSTRAINED OPTIMIZATION METHOD

The review in Sec. 3 reveals that the quality of estimation of the channel impulse response is largely determined by the accuracy of the approximating matrix $\hat{\mathbf{U}}_n\hat{\mathbf{U}}_n^H$ used in (14a). In this section, we propose a new way to obtain an approximation for this crucial matrix.

We start by subtracting a scaled identity matrix $\alpha\mathbf{I}$ from the data correlation $\mathbf{R}$, which yields

$$\mathbf{R} - \alpha\mathbf{I} = [\mathbf{U}_s\mathbf{U}_n]\begin{bmatrix}\mathbf{\Lambda}_s & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{U}_s^H\\ \mathbf{U}_n^H\end{bmatrix} + (\sigma_v^2 - \alpha)\mathbf{I} \quad (23)$$

If $\alpha < \sigma_v^2$, then matrix $\mathbf{R} - \alpha\mathbf{I}$ remains a positive definite matrix whose inverse is given by

$$(\mathbf{R} - \alpha\mathbf{I})^{-1} = \mathbf{U}_s\mathbf{E}\mathbf{U}_s^H + \frac{1}{\sigma_v^2 - \alpha}\mathbf{U}_n\mathbf{U}_n^H \quad (24)$$

where $\mathbf{E} = \text{diag}\{\frac{1}{\lambda_1+\sigma_v^2-\alpha}, \frac{1}{\lambda_2+\sigma_v^2-\alpha}, \ldots, \frac{1}{\lambda_\xi+\sigma_v^2-\alpha}\}$. By multiplying both sides of (24) by $(\sigma_v^2 - \alpha)$, (24) becomes

$$(\sigma_v^2 - \alpha)(\mathbf{R} - \alpha\mathbf{I})^{-1} = \mathbf{U}_s\mathbf{N}\mathbf{U}_s^H + \mathbf{U}_n\mathbf{U}_n^H \quad (25)$$

where $\mathbf{N} = (\sigma_v^2 - \alpha)\mathbf{E}$. If we use $\alpha \to \sigma_v^{2-}$ to denote a limiting process where α approaches σ_v^2 from the left side of the real axis, then (25) implies that

$$\begin{aligned}&\lim_{\alpha \to \sigma_v^{2-}}(\sigma_v^2 - \alpha)(\mathbf{R} - \alpha\mathbf{I})^{-1} \\ &= \lim_{\alpha \to \sigma_v^{2-}}(\sigma_v^2 - \alpha)\sum_{i=1}^{\xi}\frac{\mathbf{u}_{s,i}\mathbf{u}_{s,i}^H}{\lambda_i + \sigma_v^2 - \alpha} + \mathbf{U}_n\mathbf{U}_n^H \\ &= \mathbf{U}_n\mathbf{U}_n^H \end{aligned} \quad (26)$$

Equation (26) shows that if we can estimate the variance of the channel noise, σ_v^2, and choose a constant α which is close to but less than σ_v^2, then the matrix $\mathbf{U}_n\mathbf{U}_n^H$ can be well approximated by $(\mathbf{R} - \alpha\mathbf{I})^{-1}$ to within a scalar multiplier. It is important to stress that the introduction of parameter α in (26) allows one to use $(\mathbf{R} - \alpha\mathbf{I})^{-1}$ to approximate the noise subspace accurately for a wide range of noise variance values. It is because of this property that the performance of the proposed method remains comparable with that of the subspace methods in [3][4] even for very noisy channels. Concerning the computational complexity, since σ_v^2 is the least eigenvalue of $\mathbf{R}$, which can be computed efficiently by using, for example, the inverse-power method [8], the ED of $\mathbf{R}$ can be avoided.

As mentioned earlier, $\mathbf{R}$ is usually not available but can be approximated by $\hat{\mathbf{R}}$ as defined in (12). In this case, we simply compute the least eigenvalue of matrix $\hat{\mathbf{R}}$, denoted as γ_0. Then we choose α to be close to but less than γ_0 to within a certain tolerance so as to avoid numerical difficulties in the matrix inversion.

Next we make use of the matrix $(\hat{\mathbf{R}} - \alpha\mathbf{I})^{-1}$ in constrained optimization methods. The channel impulse response is then obtained as the solution of the optimization problem

$$\text{minimize} \quad \mathbf{g}_1^H\mathbf{S}_1^H(\hat{\mathbf{R}} - \alpha\mathbf{I})^{-1}\mathbf{S}_1\mathbf{g}_1 \quad (27a)$$

$$\text{subject to:} \quad ||\mathbf{g}_1||^2 = 1 \quad (27b)$$

Obviously, the solution $\hat{\mathbf{g}}_1$ in (27) is the eigenvector of $\mathbf{S}_1^H(\mathbf{R} - \alpha\mathbf{I})^{-1}\mathbf{S}_1$ associated with its least eigenvalue. Using a similar analysis method to that presented in [5], it can be shown that the channel impulse response estimate is related to the noise power and constant α in terms of the relation

$$\hat{\mathbf{g}}_1 = \frac{\mathbf{g}_1}{||\mathbf{g}_1||} - (\gamma_0 - \alpha)\mathbf{A}_0^\dagger\mathbf{A}_1\frac{\mathbf{g}_1}{||\mathbf{g}_1||} + O[(\gamma_0 - \alpha)^2] \quad (28)$$

where $\dagger$ denotes the pseudo-inverse, and $\mathbf{A}_0$ and $\mathbf{A}_1$ are given by

$$\begin{aligned}\mathbf{A}_0 &= \mathbf{S}_1^H\hat{\mathbf{U}}_n\hat{\mathbf{U}}_n^H\mathbf{S}_1\\ \mathbf{A}_1 &= \mathbf{S}_1^H\hat{\mathbf{U}}_s\hat{\mathbf{\Lambda}}_s^{-1}\hat{\mathbf{U}}_s^H\mathbf{S}_1\end{aligned}$$

where matrices $\hat{\mathbf{\Lambda}}_s$ and $\hat{\mathbf{U}}_s$ are the counterparts of $\mathbf{\Lambda}_s$ and $\mathbf{U}_s$ in (10), respectively. After $\mathbf{g}_1$ is obtained, the detection vector $\mathbf{w}_1$ can be obtained using (17). The proposed algorithm can be

summarized as follows:
i) Obtain the estimated data correlation matrix $\hat{\mathbf{R}}$ using (12).
ii) Compute the least eigenvalue of $\hat{\mathbf{R}}$ denoted as γ_0.
iii) Choose α close to but less than γ_0 and compute $(\hat{\mathbf{R}} - \alpha\mathbf{I})^{-1}$.
iv) Obtain $\mathbf{g}_1$ by solving the optimization problem in (27).
v) Form detection vector $\mathbf{w}_1$ using (17).

5. SIMULATIONS

Computer simulations were performed to examine the performance of the proposed subspace estimation-based constrained optimization method (SECOM), and to compare it with that of the subspace methods (SMs) described in [3][4] and the constrained optimization methods (COMs) described in [5-7]. Gold sequences of length 31 were used as spreading sequences. The impulse responses of the channels for all users were randomly generated and assumed to have a maximum order of 4. The performance of these methods is measured in terms of the averaged mean-squared-error (MSE) of the estimated channel impulse responses and their true values.

In the first example, we consider a 10-user CDMA system where each interference user was 40 dB stronger than the desired user and the signal-to-noise ratio varied from -20 dB to 20 dB. The constant α in (27a) was assumed to be $0.999\gamma_0$ where γ_0 is the lowest eigenvalue of $\hat{\mathbf{R}}$. The MSE (averaged over 10^3 runs) of the estimation of the channel impulse response is plotted in Fig. 1. As expected, the methods offer almost the same performance when the SNR of the channel is very high (about 20 dB) because the noise subspace can be accurately obtained by these methods. When the SNR of the channel is low, however, the performance of SECOM approaches that of the SMs, which is much better than that of the COMs.

In the second example, the role of parameter α in (27a) was investigated. The simulation environment was the same as in the first example except that each interference user was 10 dB stronger than the desired user. The parameter α assumed the values $\alpha_m = 1 - 10^{-0.5m}$ for $m = 1, 2, \ldots, 7$. The MSE obtained (averaged over 10^2 runs) is plotted in Fig. 2. It is obvious that the closer α is to 1, the lower is the MSE in the estimation. A good value of α can be determined based on information about the approximation accuracy of matrix $\hat{\mathbf{R}}$.

6. CONCLUSIONS

A subspace estimation-based constrained optimization algorithm for the design of multiuser detectors in a multipath DS-CDMA channel has been proposed. By using the new method, the noise subspace can be well approximated. The algorithm has been shown to be robust in both high and low SNR white Gaussian noise channels. It has been demonstrated that the performance of the proposed method can approach that of some known subspace methods while requiring a much reduced computational complexity. Relative to some known constrained optimization methods, our method offers a significantly improved performance while requiring a comparable amount of computation.

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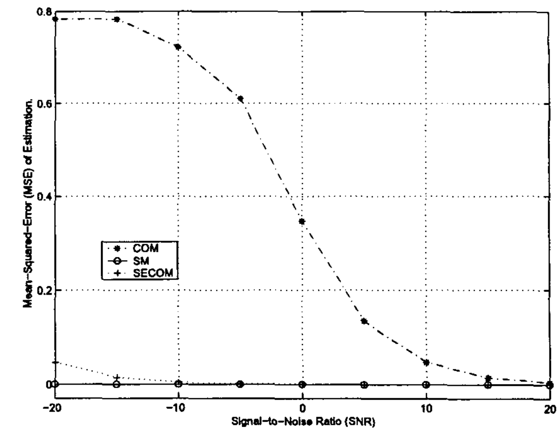

Fig. 1. MSE of the estimation of the channel impulse response in Example 1.

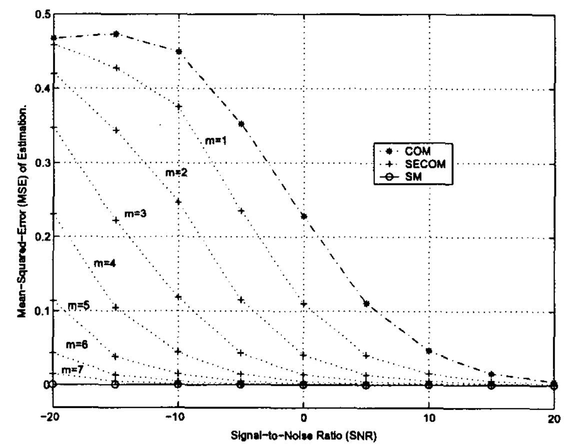

Fig. 2. MSE of the estimation of the channel impulse response in Example 2.

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