Low-Delay QMF Banks with Equiripple Complex Reconstruction Error

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Abstract— A method for the design of two-channel low-delay QMF banks with equiripple complex reconstruction error is proposed. A weighted objective function in which the reconstruction error is defined in terms of the difference between the frequency response of the filter bank and that of an ideal allpass channel is constructed. Based on this objective function an iterative algorithm is developed which forces the magnitude of the complex reconstruction error to become equiripple. The paper concludes with design examples which illustrate the efficiency of the algorithm and the performance of the filter banks designed.

I. INTRODUCTION

Since late 1970's, quadrature mirror filter (QMF) banks have been widely used in one-dimensional (1-D) and two-dimensional (2-D) signal processing [1][2]. In the design of QMF filter banks, it is required that the perfect reconstruction condition be satisfied while the intra-band aliasing be eliminated or minimized. Most design methods developed [3][4] involve minimizing a highly nonlinear objective function directly, which leads to a difficult optimization problem.

In [5], Chen and Lee introduced an iterative algorithm for the design of two-channel linear-phase QMF banks. The method modifies the objective function into a quadratic function whose minimum can be obtained analytically. As demonstrated in [5], the method is also suitable for the design of linear-phase QMF banks with equiripple amplitude response. Compared with other QMF design methods, this method reduces the computational complexity significantly. In [6] the iterative algorithmin in [5] was extended to design QMF banks with low reconstruction delays, which are highly desirable in real-time applications.

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In this paper, we propose an iterative algorithm based the methods in [5][6] for the design of low-delay QMF banks with equiripple complex reconstruction error. The algorithm leads to designs that include linear-phase QMF banks with equiripple amplitude response as a particular case. The paper is organized as follows: In Sec. II, the algorithm is proposed and the design procedure is described. In Sec. III, two design examples are given to illustrate the proposed algorithm.

II. PROPOSED METHOD AND DESIGN PROCEDURE

A two-channel QMF bank is illustrated in Fig. 1, where $H_1(z) = H_0(-z)$, $G_0(z) = H_0(z)$, $G_1(z) = -H_0(-z)$ in order to cancel aliasing. It is known [7] that the design of QMF banks with low reconstruction delays can be achieved by minimizing an objective function

$$E = E_1 + \alpha E_2 \tag{1a}$$

where $\alpha > 0$ is a weighting and

$$E_1 = \int_0^\pi |H_0^2(e^{j\omega}) - H_0^2(e^{j(\omega+\pi)}) - e^{-j\omega k_d}|^2 d\omega \tag{1b}$$

$$E_2 = \int_{\omega_s}^\pi |H_0(e^{j\omega})|^2 \, d\omega \tag{1c}$$

In (1c) ω_s is the stopband edge and the reconstruction delay k_d is less than $N-1$ where N is the filter length. In this case, filter H_0 usually possesses an *asymmetrical* impulse response. The complex reconstruction error is defined as

$$e_r(\omega) = H_0^2(e^{j\omega}) - H_0^2(e^{j(\omega+\pi)}) - e^{-j\omega k_d} \tag{2}$$

In what follows it will be shown that if a weighted objective function of the form

$$E_w = E_{1w} + \alpha E_{2w} \tag{3a}$$

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$$E_{1w} = \sum_{0\le\omega\le\pi} W(\omega)\,|H_0^2(e^{j\omega}) - H_0^2(e^{j(\omega+\pi)}) - e^{-j\omega k_d}|^2 \tag{3b}$$

$$E_{2w} = \sum_{\omega_s\le\omega\le\pi} |H_0(e^{j\omega})|^2 \tag{3c}$$

is adopted and the weighting $W(\omega)$ is chosen appropriately, equiripple complex reconstruction error can be achieved.

First, the objective function in (3) is modified as

$$E'_w = E'_{1w} + \alpha E'_{2w} \tag{4a}$$

$$E'_{1w} = \sum_{0\le\omega\le\pi} W(\omega)\,|H_0(e^{j\omega})F_0(e^{j\omega}) - H_0(e^{j(\omega+\pi)}) \cdot F_0(e^{j(\omega+\pi)}) - e^{-j\omega k_d}|^2 \tag{4b}$$

$$E'_{2w} = \sum_{\omega_s\le\omega\le\pi} |F_0(e^{j\omega})|^2 \tag{4c}$$

where $F_0(e^{j\omega})$ is the frequency response of a lowpass filter F_0 whose coefficient vector is $\mathbf{f}$, i.e.,

$$F_0(e^{j\omega}) = \mathbf{f}^T\mathbf{c}$$
$$\mathbf{f} = [f_0(0)\ f_0(1)\ \cdots\ f_0(N-1)]^T$$
$$\mathbf{c} = [1\ e^{-j\omega}\ \cdots\ e^{-j(N-1)\omega}]^T$$

The iteration starts with designing a lowpass FIR filter H_0 with a group delay of $k_d/2$. Then E'_w in (4a) can be rewritten in the form

$$E'_w = (\mathbf{Uf}-\mathbf{I})^H\hat{\mathbf{W}}(\mathbf{Uf}-\mathbf{I}) + \alpha(\mathbf{U_s f})^H(\mathbf{U_s f}) \tag{5}$$

where superscript H denotes complex conjugate transposition and

$$\mathbf{U_t} = \begin{bmatrix} 1 & e^{-j\omega_1} & \cdots & e^{-j\omega_1(N-1)} \\ \vdots & & & \vdots \\ 1 & e^{-j\omega_s} & \cdots & e^{-j\omega_s(N-1)} \\ \vdots & & & \vdots \\ 1 & e^{-j\omega_L} & \cdots & e^{-j\omega_L(N-1)} \end{bmatrix} \tag{6a}$$

$$\mathbf{U_s} = \begin{bmatrix} 1 & e^{-j\omega_s} & \cdots & e^{-j\omega_s(N-1)} \\ \vdots & & & \vdots \\ 1 & e^{-j\omega_L} & \cdots & e^{-j\omega_L(N-1)} \end{bmatrix} \tag{6b}$$

$$\mathbf{H} = \mathrm{diag}[H_0(e^{j\omega_1}),\ \ldots,\ H_0(e^{j\omega_s}),\ \ldots,\ H_0(e^{j\omega_L})] \tag{6c}$$

$$\hat{\mathbf{W}} = \mathrm{diag}[W(\omega_1),\ \ldots,\ W(\omega_s),\ \ldots,\ W(\omega_L)] \tag{6d}$$

$$\mathbf{U} = \mathbf{HU_t} - \mathbf{HU_t}|_{\omega_i:=\omega_i+\pi, 1\le i\le L} \tag{6e}$$

$$\mathbf{I} = [e^{-j\omega_1 k_d}\ e^{-j\omega_2 k_d}\ \ldots\ e^{-j\omega_L k_d}]^T \tag{6f}$$

$$0 = \omega_1 < \omega_2 \ldots < \omega_s < \ldots < \omega_L = \pi \tag{6g}$$

Note that E'_w in (5) is a quadratic function of $\mathbf{f}$ with matrix Re$[\mathbf{U}^H\hat{\mathbf{W}}\mathbf{U} + \alpha\mathbf{U_s}^H\mathbf{U_s}]$ positive definite. Hence

$$\mathbf{f} = (\mathrm{Re}\ [\mathbf{U}^H\hat{\mathbf{W}}\mathbf{U} + \alpha\mathbf{U_s}^H\mathbf{U_s}])^{-1}\cdot \mathrm{Re}\ ([\mathbf{U}^H\hat{\mathbf{W}}\mathbf{I}]) \tag{7}$$

gives the minimum of E'_w, where Re $[\cdot]$ is the real part of $[\cdot]$. Having obtained $\mathbf{f}$, a linear formula is used to update the coefficient vector of H_0, $\mathbf{h} = [h_0(0)\ h_0(1)\ \cdots\ h_0(N-1)]^T$, as

$$\mathbf{h} := (1-\tau)\mathbf{h} + \tau\mathbf{f} \tag{8}$$

where τ, $0 < \tau < 1$, is a smoothing parameter. The above procedure is repeated until $\|\ \mathbf{h}-\mathbf{f}\ \|_2$ is less than a prescribed tolerance.

To obtain equiripple complex reconstruction error, the weighting $W(\omega)$ should be updated based on the weighted least-squares (WLS) algorithm proposed in [8]. Suppose that $W_k(\omega)$ is the weighting function used in the kth iteration, then the weighting used in the $(k+1)$th iteration is expressed as

$$W_{k+1}(\omega) = W_k(\omega)v_k(\omega) \tag{9}$$

where $v_k(\omega) > 0$ is selected such that $v_k(\omega_i) > v_k(\omega_j)$ if $|e_{rk}(\omega_i)| > |e_{rk}(\omega_j)|$ where $e_{rk}(\omega)$ denotes the values of $e_r(\omega)$ defined in (2) at the kth iteration. This ensures that in the next iteration $|e_r(\omega_i)|$ will decrease at the expense of increasing $|e_r(\omega_j)|$. The criterion of choosing $v_k(\omega)$ is as follows: Compute the magnitude of the complex reconstruction error $e_{rk}(\omega)$ at the kth iteration. Let the Jth extremal value of $|e_{rk}(\omega)|$ be $V_k(J) = |e_{rk}(\omega_J)|$ if $|e_{rk}(\omega_{J+1})| < |e_{rk}(\omega_J)|$ and $|e_{rk}(\omega_{J-1})| < |e_{rk}(\omega_J)|$ are satisfied where ω_J is defined as the Jth extremal frequency of the reconstruction error. For any non-band-edge extremal point, let $V_k(J) = 0.1\mathrm{MIN}[V_k(J-1), V_k(J+1)]$ if $V_k(J) < 0.1\mathrm{MIN}[V_k(J-1), V_k(J+1)]$ where $\mathrm{MIN}(u,v)$ denotes the smaller of u and v. An envelop function $B_k(\omega)$ is then formed by joining together all the extremal points of the same frequency band of interest with straight lines, i.e., for $\omega_J < \omega < \omega_{J+1}$

$$B_k(\omega) = [(\omega-\omega_J)/(\omega_{J+1}-\omega_J)]V_k(J+1) + [(\omega_{J+1}-\omega)/(\omega_{J+1}-\omega_J)]V_k(J) \tag{10}$$

Finally, $v_k(\omega)$ is constructed as

$$v_k(\omega) = \frac{L\{B_k(\omega)\}^\theta}{\sum_{i=1}^{L}[W_k(\omega_i)\{B_k(\omega_i)\}^\theta]} \tag{11}$$

where parameter θ affects the convergence rate and is chosen to be 1.5 in our designs.

Since the purpose of the design procedure is to achieve equiripple magnitude of the complex reconstruction error, the process could be terminated if

$$\frac{[\max(V) - \min(V)]}{\max(V)} \le \kappa \tag{12}$$

where κ is a prescribed positive constant and $\max(V)$ and $\min(V)$ are the maximum and minimum values of the magnitude of the complex reconstruction error at all extremal frequencies.

The steps that need to be taken to accomplish the design of low-delay QMF banks with equiripple reconstruction error are as follows:

Algorithm

Step 1 Use a least-squares approach to design a lowpass, FIR filter of length N with stopband edge ω_s and group delay $k_d/2$, and use the coefficient vector of the filter obtained to initialize $\mathbf{h}$. The initial weighting is set to $W(\omega) = 1$.

Step 2 Use (6b) to compute matrix $\mathbf{U}_s$.

Step 3 Form matrix $\mathbf{U}$ using (6e), and compute $\mathbf{f}$ using (7).

Step 4 If $\| \mathbf{h} - \mathbf{f} \|_2 < \epsilon$ where ϵ is a prescribed tolerance, go to Step 5. Otherwise, update $\mathbf{h}$ using (8) with a τ close to 0.5 and repeat from Step 3.

Step 5 If termination condition (12) is satisfied, output $\mathbf{h}$ as the design result and stop. Otherwise, update $W(\omega)$ using equations (9)-(11) and repeat from Step 3.

It should be pointed out that the algorithm leads to designs of low-delay QMF banks with equiripple complex reconstruction error as well as linear-phase QMF banks with equiripple amplitude response if $k_d = N - 1$ and initial $\mathbf{h}$ is symmetrical.

III. DESIGN EXAMPLES

Two low-delay QMF banks with equiripple complex reconstruction error were designed by using the proposed algorithm. The performance of the designs are evaluated in terms of

- the number of iterations (NI)
- the peak-to-peak passband ripple

$$A_p = \max_{0\le\omega\le\omega_p} \left[20\log_{10}|H_0(e^{j\omega})|\right] - \min_{0\le\omega\le\omega_p} \left[20\log_{10}|H_0(e^{j\omega})|\right]$$

  where ω_p is the passband edge.

- the peak reconstruction error

$$\text{PRE} = \max_{\omega} |e_r(\omega)|$$

- the signal-to-noise ratio

$$\text{SNR} = 10\log_{10}\left(\frac{\text{energy of the signal}}{\text{energy of the reconstruction noise}}\right) = 10\log_{10}\left\{\frac{\Sigma\, x^2(n)}{\Sigma\, [x(n) - \hat{x}(n+k_d)]^2}\right\}$$

where the input is a random signal where amplitude is uniformly distributed between 0 and 1.

In Example 1, a filter bank with the specifications $N = 32$, $k_d = 15$, $\alpha = 0.01$, $\omega_s = 0.7\pi$, $\tau = 0.5$, $\kappa = 0.02$, $\epsilon = 10^{-4}$ was designed. The results are summarized in Table I. The amplitude responses of the analysis filters obtained are depicted in Fig. 2. Fig. 3 shows the magnitude of the complex reconstruction error.

Example 2, a filter bank with the specifications $N = 44$, $k_d = 21$, $\alpha = 0.1$, $\omega_s = 0.65\pi$, $\tau = 0.5$, $\kappa = 0.02$ $\epsilon = 10^{-4}$ was designed. The results are listed in Table I. The amplitude responses of the analysis filters and the magnitude of the complex reconstruction error are shown in Figs. 4 and 5, respectively.

TABLE I
EVALUATION RESULTS FOR EXAMPLES 1 AND 2.

	Example 1	Example 2
NI	23	11
A_p (dB)	7.95×10^{-2}	7.12×10^{-2}
PRE	2.4223×10^{-4}	$1.4627\times^{-4}$
SNR (dB)	73.20	77.32

From Table I and the figures, it is observed the filters obtained have satisfactory frequency responses, and that the magnitude of the complex, reconstruction error is equiripple.

IV. CONCLUSION

An iterative algorithm for the design of low-delay QMF banks with equiripple complex reconstruction error has been proposed. The algorithm includes linear-phase QMF banks with equiripple amplitude response as a particular case if the reconstruction delay $k_d = N - 1$ and filter H_0 has a symmetrical impulse response. Two design examples were given to illustrate the performance of the filter banks designed.

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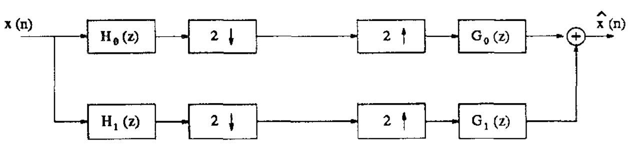

FIGURE 1
TWO-BAND FIR FILTER BANK

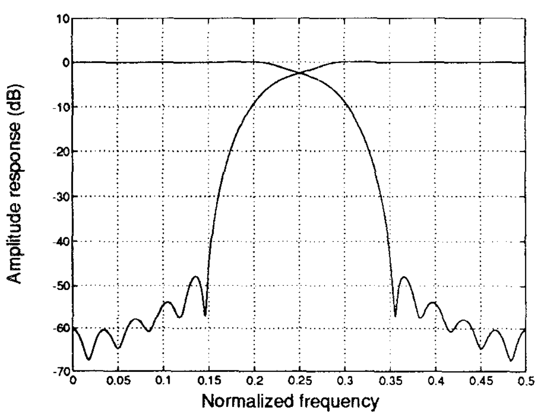

FIGURE 2
EXAMPLE 1: AMPLITUDE RESPONSES OF THE ANALYSIS FILTERS.

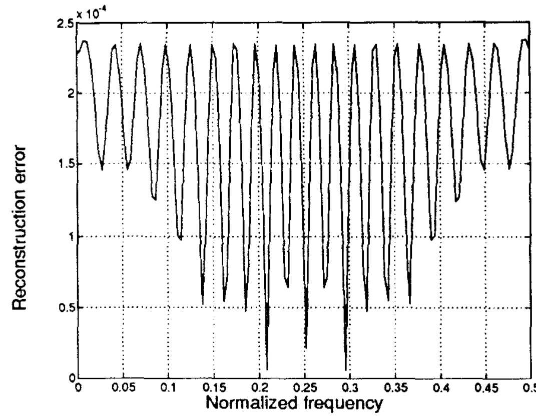

FIGURE 3
EXAMPLE 1: RECONSTRUCTION ERROR.

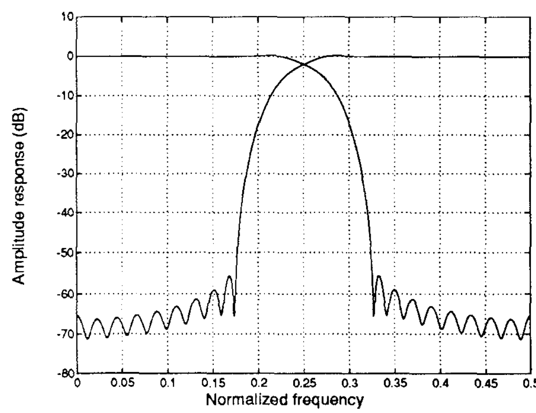

FIGURE 4
EXAMPLE 2: AMPLITUDE RESPONSES OF THE ANALYSIS FILTERS.

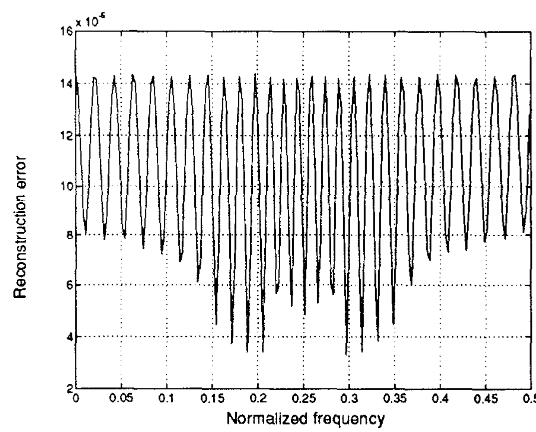

FIGURE 5
EXAMPLE 2: RECONSTRUCTION ERROR.