Determination of the Transfer-Function Matrix from the Two-Dimensional Fornasini-Marchesini State-Space Model

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Abstract—**Two efficient algorithms for the determination of the transfer-function matrix of a two-dimensional (2-D) discrete system represented by the Fornasini-Marchesini state-space model are proposed. The development of the algorithms involves two distinct steps. First, the 2-D transfer-function matrix is reformulated in terms of the characteristic polynomials of the coefficient matrices involved. Second, the coefficient matrices are determined by using an efficient algorithm for the determination of 1-D polynomial coefficients. The simplicity and efficiency of the proposed algorithms are illustrate by examples.**

I. Introduction

The representation of the transfer-function matrix of a two-dimensional (2-D) discrete systems by a state-space model and vice versa are two basic problems of great importance in system analysis and design. One of the commonly used state-space models for 2-D discrete systems is the Roesser model [1]. Several algorithms for the derivation of the 2-D transfer-function matrix from the Roesser state-space model have been proposed [2]-[6]. Another popular state-space representation for 2-D discrete systems is the Fornasini-Marchesini model [7]. To date, no efficient algorithms for the determination of the 2-D transfer-function matrix from the Fornasini-Marchesini state-space representation have been reported.

This paper proposes new algorithms for the determination of the 2-D transfer-function matrix from the Fornasini-Marchesini state-space model. The algorithms are extensions of efficient algorithms for the determination of the 2-D transfer-function matrix from the Roesser state-space model [6]. First, the transfer-function matrix is reformulated in terms of the characteristic polynomials of several matrices that depend on one complex variable. Second, algorithms are proposed that identify the coefficients of a 1-D polynomial of order n when its values at $(n+1)$ points on the unit circle are known. Our algorithms entail solving a system of linear equations whose coefficient matrix is an unitary Vandermonde matrix. Examples are given to illustrate the efficiency of the proposed algorithms.

II. Algorithms for SISO Systems

Consider a single-input, single-output (SISO), linear, shift-invariant, discrete 2-D system represented by the Fornasini-Marchesini state-space model given by

$$\boldsymbol{x}(k+1,\, l+1) = \boldsymbol{A}_1\boldsymbol{x}(k,\, l+1)+\boldsymbol{A}_2\boldsymbol{x}(k+1,\, l) + \boldsymbol{b}_1u(k,\, l+1)+\boldsymbol{b}_2u(k+1,\, l) \tag{1a}$$

$$y(k,\, l) = \boldsymbol{c}\,\boldsymbol{x}(k,\, l)+d\,u(k,\, l) \tag{1b}$$

where $\boldsymbol{x} \in \Re^n$ is the state vector, and u and y are the input and output, respectively. The transfer function of the system is given by

$$H(z_1,\, z_2) = \boldsymbol{c}\,(z_1z_2\mathbf{I}-z_2\boldsymbol{A}_1-z_1\boldsymbol{A}_2)^{-1}(z_2\boldsymbol{b}_1+z_1\boldsymbol{b}_2)+d$$

A. Reformulation of the Transfer Function

We first reformulate the transfer function $H(z_1,\, z_2)$ in terms of the characteristic polynomials of the matrices involved. By using a well-known formula for the transfer function of a 1-D SISO state-space model (see Appendix A.13 of [8]), $H(z_1,\, z_2)$ can be rewritten as

$$\begin{aligned} H(z_1,\, z_2) &= \frac{\det(z_2\mathbf{I}-\boldsymbol{A}_2+\boldsymbol{b}_2\,\boldsymbol{c})\det[z_1\mathbf{I}-\boldsymbol{F}(z_2)]}{\det(z_2\mathbf{I}-\boldsymbol{A}_2)\det[z_1\mathbf{I}-\boldsymbol{E}(z_2)]}+d-1 \\ &= \frac{P(z_2,\, \boldsymbol{A}_2-\boldsymbol{b}_2\,\boldsymbol{c})\,P[z_1,\, \boldsymbol{F}(z_2)]}{P(z_2,\, \boldsymbol{A}_2)\,P[z_1,\, \boldsymbol{E}(z_2)]}+d-1 \qquad (2) \\ &= \frac{\sum_{k=0}^{n} q_k(z_2)\, z_1^k}{\sum_{k=0}^{n} p_k(z_2)\, z_1^k} \qquad (3) \end{aligned}$$

where $p_k(z_2)$ and $q_k(z_2)$ are polynomials in z_2 of order not greater than n, and

$$\boldsymbol{E}(z_2) = z_2\,\boldsymbol{A}_1\,(z_2\mathbf{I}-\boldsymbol{A}_2)^{-1} \tag{4a}$$

$$\boldsymbol{F}(z_2) = z_2(\boldsymbol{A}_1-\boldsymbol{b}_1\,\boldsymbol{c})\,(z_2\mathbf{I}-\boldsymbol{A}_2+\boldsymbol{b}_2\,\boldsymbol{c})^{-1} \tag{4b}$$

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In (2), $P(z_2, \boldsymbol{A}_2)$, $P(z_2, \boldsymbol{A}_2 - \boldsymbol{b}_2 \boldsymbol{c})$, $P[z_1, \boldsymbol{E}(z_2)]$, $P[z_1, \boldsymbol{F}(z_2)]$ are the characteristic polynomials of $\boldsymbol{A}_2$, $\boldsymbol{A}_2 - \boldsymbol{b}_2 \boldsymbol{c}$, $\boldsymbol{E}(z_2)$, and $\boldsymbol{F}(z_2)$, respectively. From (2) and (3), it follows that

$$\sum_{k=0}^{n} q_k(z_2)\, z_1^k = P(z_2, \boldsymbol{A}_2 - \boldsymbol{b}_2 \boldsymbol{c})\, P[z_1, \boldsymbol{F}(z_2)] + (d-1) P(z_2, \boldsymbol{A}_2) P[z_1, \boldsymbol{E}(z_2)] \quad (5a)$$

$$\sum_{k=0}^{n} p_k(z_2)\, z_1^k = P(z_2, \boldsymbol{A}_2)\, P[z_1, \boldsymbol{E}(z_2)] \quad (5b)$$

B. Determination of the Coefficients of a 1-D Polynomial

An efficient method for the determination of the coefficients of a 1-D polynomial will now be examined. Let

$$p(z_2) = \alpha_n z_2^n + \cdots + \alpha_1 z_2 + \alpha_0$$

be a polynomial of order n with coefficients $\alpha_n, \ldots, \alpha_1, \alpha_0$. Also let $\{z_2(l),\ 0 \le l \le n\}$ be $(n+1)$ points that are uniformly distributed on the unit circle of the complex z_2 plane, i.e.,

$$z_2(l) = e^{j2\pi l/(n+1)}, \quad 0 \le l \le n \quad (6)$$

If the values $\{p_l = p[z_2(l)],\ 0 \le l \le n\}$ are known, then the coefficients $\{\alpha_l,\ 0 \le l \le n\}$ can be determined as

$$\boldsymbol{\alpha} = \mathbf{V}^{-1}(\mathbf{z}_2)\, \mathbf{q} \quad (7)$$

where

$$\boldsymbol{\alpha} = [\alpha_n \cdots \alpha_1\ \alpha_0]^T, \qquad \mathbf{q} = [p_0\ p_1 \cdots p_n]^T,$$

and $\mathbf{V}(\mathbf{z}_2)$ is the $(n+1) \times (n+1)$ Vandermonde matrix whose second to last column is

$$\mathbf{z}_2 = \begin{bmatrix} z_2(0) & z_2(1) & \cdots & z_2(n) \end{bmatrix}^T$$

that is,

$$\mathbf{V}(\mathbf{z}_2) = \begin{bmatrix} z_2(0)^n & \cdots & z_2(0) & 1 \\ \vdots & & \vdots & \vdots \\ z_2(n)^n & \cdots & z_2(n) & 1 \end{bmatrix}$$

Since $z_2(l)$, $0 \le l \le n$, are distinct, $\mathbf{V}(\mathbf{z}_2)$ is always nonsingular. More important, it follows from (6) that

$$\mathbf{V}^H(\mathbf{z}_2)\, \mathbf{V}(\mathbf{z}_2) = (n+1)\, \mathbf{I}$$

where $\mathbf{V}^H(\mathbf{z}_2)$ denotes the complex-conjugate transpose of $\mathbf{V}(\mathbf{z}_2)$. Therefore, (7) can be written as

$$\boldsymbol{\alpha} = \frac{1}{n+1} \mathbf{V}^H(\mathbf{z}_2)\, \mathbf{q} \quad (8)$$

Equation (8) provides an efficient formula for the determination of 1-D polynomial $p(z_2)$.

C. New Algorithms

The algorithms are based on (5a), (5b), and the efficient method for the determination of the coefficients of a 1-D polynomial. It is assumed that matrices $\boldsymbol{A}_2$ and $\boldsymbol{A}_2 - \boldsymbol{b}_2 \boldsymbol{c}$ have no eigenvalues on the unit circle. The case where the matrices $\boldsymbol{A}_2$ and/or $\boldsymbol{A}_2 - \boldsymbol{b}_2 \boldsymbol{c}$ have eigenvalues on the unit circle will be considered in subsection D.

Algorithm 1

Step 1: Use (4a) and (4b) to evaluate $\boldsymbol{E}(z_2)$ and $\boldsymbol{F}(z_2)$ over the set of points defined in (6).

Step 2: Compute the determinants of $z_2\mathbf{I} - \boldsymbol{A}_2$ and $z_2\mathbf{I} - \boldsymbol{A}_2 + \boldsymbol{b}_2 \boldsymbol{c}$, and the characteristic equations of $\boldsymbol{E}(z_2)$ and $\boldsymbol{F}(z_2)$ for $z_2 = z_2(l)$, $0 \le l \le n$.

Step 3: Use (5a) and (5b) to obtain the values of $p_k[z_2(l)]$ and $q_k[z_2(l)]$ for $0 \le l \le n$, $0 \le k \le n$.

Step 4: For each k, $0 \le k \le n$, form vectors $\mathbf{q} = [p_0 \cdots p_n]^T$ and $\mathbf{q} = [q_0 \cdots q_n]^T$, and determine polynomials $p_k(z_2)$ and $q_k(z_2)$ by using (8).

D. Unstable and Special Cases

If $\boldsymbol{A}_2$ has eigenvalues on the unit circle (the system is unstable) or the special case where $\boldsymbol{A}_2 - \boldsymbol{b}_2 \boldsymbol{c}$ has eigenvalues on the unit circle, the $n+1$ points defined by (6) need to be modified to

$$z_2(l) = r\, e^{j2\pi l/(n+1)}, \quad 0 \le l \le n \quad (9)$$

where $r > 0$ denotes the radius of a circle in the z_2 plane where $\boldsymbol{A}_2$ and $\boldsymbol{A}_2 - \boldsymbol{b}_2 \boldsymbol{c}$ have no eigenvalues on the circle. Consequently, (8) is modified to

$$\boldsymbol{\alpha} = \frac{1}{n+1} \operatorname{diag}\{r^{-n}, \ldots, r^{-1}, 1\}\, \mathbf{V}^H(\mathbf{z}_2)\, \mathbf{q} \quad (10)$$

Note that (8) is a special case of (10) with $r = 1$, as may be expected.

III. Dual Algorithms

A dual algorithm to Algorithm 1 can be obtained if the roles of variables z_1 and z_2 are interchanged. By representing $H(z_1, z_2)$ in (3) as

$$H(z_1, z_2) = \frac{\sum_{l=0}^{n} \hat{q}_l(z_1)\, z_2^l}{\sum_{l=0}^{n} \hat{p}_l(z_1)\, z_2^l}$$

where $\hat{p}_l(z_1)$ and $\hat{q}_l(z_1)$ are polynomials in z_1, it can be readily shown that

$$\sum_{l=0}^{n} \hat{q}_l(z_1)\, z_2^l = P(z_1, \boldsymbol{A}_1 - \boldsymbol{b}_1 \boldsymbol{c})\, P[z_2, \hat{\boldsymbol{F}}(z_1)]$$

$$+(d-1)P(z_1,\, \boldsymbol{A}_1)P[z_2,\, \hat{\boldsymbol{E}}(z_1)] \quad (11a)$$

$$\sum_{l=0}^{n} \hat{p}_l(z_1)\, z_2^l = P(z_1,\, \boldsymbol{A}_1)\, P[\, z_2,\, \hat{\boldsymbol{E}}(z_1)] \quad (11b)$$

where

$$\hat{\boldsymbol{E}}(z_1) = z_1 \boldsymbol{A}_2\, (z_1 \mathbf{I} - \boldsymbol{A}_1)^{-1} \quad (12a)$$

$$\hat{\boldsymbol{F}}(z_1) = z_1 (\boldsymbol{A}_2 - \boldsymbol{b}_2\, \boldsymbol{c})\, (z_1 \mathbf{I} - \boldsymbol{A}_1 + \boldsymbol{b}_1\, \boldsymbol{c})^{-1} \quad (12b)$$

Further, (8) needs to be modified as

$$\boldsymbol{\alpha} = \frac{1}{n+1} \mathbf{V}^H(\boldsymbol{z}_1)\, \mathbf{q} \quad (13)$$

where $\boldsymbol{z}_1 = [\, z_1(0) \quad z_1(1) \quad \cdots \quad z_1(n)\,]^T$ with

$$z_1(k) = e^{j2\pi k/(n+1)}, \qquad 0 \le k \le n \quad (14)$$

The dual algorithm is as follows:

Algorithm 2

Step 1: Use (12a) and (12b) to evaluate $\hat{\boldsymbol{E}}(z_1)$ and $\hat{\boldsymbol{F}}(z_1)$ over the set of points defined by (14).

Step 2: Compute the characteristic equations of $\boldsymbol{A}_1$, $\boldsymbol{A}_1 - \boldsymbol{b}_1\, \boldsymbol{c}$, $\hat{\boldsymbol{E}}(z_1)$, and $\hat{\boldsymbol{F}}(z_1)$ for $z_1 = z_1(k)$, $0 \le k \le n$.

Step 3: Use (11a) and (11b) to obtain the values of $\hat{q}_l[\, z_1(k)]$ and $\hat{p}_l[\, z_1(k)]$ for $0 \le l \le n$, $0 \le k \le n$.

Step 4: For each l, $0 \le l \le n$, form vectors $\mathbf{q} = [\hat{p}_0 \cdots \hat{p}_n]^T$ and $\mathbf{q} = [\hat{q}_0 \cdots \hat{q}_n]^T$, and determine polynomials $\hat{p}_l(z_1)$ and $\hat{q}_l(z_1)$ by using (13).

Obviously, Algorithm 2 can be used to evaluate $H(z_1,\, z_2)$ only if matrices $\boldsymbol{A}_1$ and $\boldsymbol{A}_1 - \boldsymbol{b}_1 \boldsymbol{c}$ have no eigenvalues on the unit circle. If matrix $\boldsymbol{A}_1$ or $\boldsymbol{A}_1 - \boldsymbol{b}_1 \boldsymbol{c}$ has eigenvalues on the unit circle, then modifications similar to (9) and (10) should be made.

III. Algorithms for MIMO Systems

Consider a multi-input, multi-output (MIMO), linear, shift-invariant, 2-D discrete system represented by the Fornasini-Marchesini state-space model given by

$$\begin{aligned} \boldsymbol{x}(k+1,\, l+1) &= \boldsymbol{A}_1\, \boldsymbol{x}(k,\, l+1) + \boldsymbol{A}_2\, \boldsymbol{x}(k+1,\, l) \\ &\quad + \boldsymbol{B}_1 \mathbf{u}(k,\, l+1) + \boldsymbol{B}_2 \mathbf{u}(k+1,\, l) \quad (15a) \\ \mathbf{y}(k,\, l) &= \boldsymbol{C}\, \boldsymbol{x}(k,\, l) + \mathbf{D}\, \mathbf{u}(k,\, l) \quad (15b) \end{aligned}$$

where $\mathbf{u} \in \Re^t$, $\mathbf{y} \in \Re^s$, and $\mathbf{D} \in \Re^{s \times t}$. The $s \times t$ transfer-function matrix of the system can be expressed as

$$\begin{aligned} \mathbf{H}(z_1,\, z_2) &= \boldsymbol{C}\, (z_1 z_2 \mathbf{I} - z_2 \boldsymbol{A}_1 - z_1 \boldsymbol{A}_2)^{-1} (z_2 \boldsymbol{B}_1 \\ &\quad + z_1 \boldsymbol{B}_2) + \mathbf{D} \quad (16) \end{aligned}$$

whose entry $(k,\, l)$ is a scalar rational function given by

$$\begin{aligned} H_{kl}(z_1,\, z_2) &= \boldsymbol{C}_k\, (z_1 z_2 \mathbf{I} - z_2 \boldsymbol{A}_1 - z_1 \boldsymbol{A}_2)^{-1}\, (z_2 \boldsymbol{B}_{1l} \\ &\quad + z_1 \boldsymbol{B}_{2l}) + D_{kl} \quad (17) \end{aligned}$$

where $\boldsymbol{C}_k$, $\boldsymbol{B}_{1l}$, and $\boldsymbol{B}_{2l}$ are the kth row of $\boldsymbol{C}$ and the lth column of $\boldsymbol{B}_1$ and $\boldsymbol{B}_2$, respectively. Therefore, the transfer-function matrix $\mathbf{H}(z_1,\, z_2)$ given by (16) can be evaluated entry by entry and each entry can be treated as an SISO transfer function. Hence, (5a) associated with $H_{kl}(z_1,\, z_2)$ in (17) becomes

$$\begin{aligned} \sum_{k=0}^{n} q_k(z_2)\, z_1^k &= P(z_2,\, \boldsymbol{A}_2 - \boldsymbol{B}_{2l}\, \boldsymbol{C}_k)\, P[\, z_1,\, \tilde{\boldsymbol{F}}(z_2)] \\ &\quad + (D_{kl} - 1) P(z_2,\, \boldsymbol{A}_2) P[z_1,\, \boldsymbol{E}(z_2)] \quad (18) \end{aligned}$$

where

$$\tilde{\boldsymbol{F}}(z_2) = z_2 (\boldsymbol{A}_1 - \boldsymbol{B}_{1l}\, \boldsymbol{C}_k)\, (z_2 \mathbf{I} - \boldsymbol{A}_2 + \boldsymbol{B}_{2l}\, \boldsymbol{C}_k)^{-1} \quad (19)$$

Therefore, Algorithm 1 can be extended to deal with the MIMO case by substituting (18) and (19) into (5a) and (4b), respectively.

Similarly, Algorithm 2 can be extended to deal with the MIMO case by modifying (11a) and (12b).

IV. Examples

Example 1 is a two-input two-output system of order (2, 2), which was used to illustrate the algorithm in [3]. The system is represented by the Roesser model with

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 \\ \mathbf{A}_3 & \mathbf{A}_4 \end{bmatrix} = \left[\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ \hline 1 & 0 & -2 & 0 \\ 0 & 1 & 0 & -2 \end{array}\right]$$

$$\mathbf{B} = [\, \mathbf{B}_1^T \quad \mathbf{B}_2^T\,]^T = \left[\begin{array}{cc|cc} 1 & -1 & 2 & 1 \\ 1 & 0 & 1 & 0 \end{array}\right]^T$$

$$\mathbf{C} = [\, \mathbf{C}_1 \quad \mathbf{C}_2\,] = \left[\begin{array}{cc|cc} 1 & 1 & 0 & -1 \\ 0 & -1 & 1 & 1 \end{array}\right]$$

It can be represented by the Fornasini-Marchesini model [7] with

$$\boldsymbol{A}_1 = \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \boldsymbol{B}_1 = \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{0} \end{bmatrix}$$

$$\boldsymbol{A}_2 = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{A}_3 & \mathbf{A}_4 \end{bmatrix}, \quad \boldsymbol{B}_2 = \begin{bmatrix} \mathbf{0} \\ \mathbf{B}_2 \end{bmatrix}, \quad \boldsymbol{C} = \mathbf{C}$$

The transfer-function matrix obtained by using Algorithms 1 and 2 is

$$\mathbf{H}(z_1,\, z_2) = \begin{bmatrix} H_1(z_1,\, z_2) & H_2(z_1,\, z_2) \\ H_3(z_1,\, z_2) & H_4(z_1,\, z_2) \end{bmatrix}$$

Table 1: Performance of the Algorithms

Algorithm	Computational Complexity, Flops		
	Example 1	Example 2	Example 3
1	6.195×10^4	4.140×10^7	1.189×10^8
2	6.044×10^4	2.674×10^7	1.877×10^8

where the denominator is given by the matrix

$$\mathbf{D}_t = \begin{bmatrix} 1 & -2 & -1 \\ 4 & -10 & -2 \\ 4 & -12 & 1 \end{bmatrix}$$

and the numerators are specified by $\mathbf{N}_{t1}$, $\mathbf{N}_{t2}$, $\mathbf{N}_{t3}$, and $\mathbf{N}_{t4}$ as follows:

$$\mathbf{N}_t = \begin{bmatrix} \mathbf{N}_{t1} & \mathbf{N}_{t2} \\ \mathbf{N}_{t3} & \mathbf{N}_{t4} \end{bmatrix} = \left[\begin{array}{ccc|ccc} 0 & 0 & 2 & 0 & 1 & 1 \\ -1 & 6 & 7 & 0 & 5 & 3 \\ -2 & 13 & 0 & 0 & 6 & 0 \\ \hline 0 & 1 & -3 & 0 & 0 & -1 \\ 3 & -3 & -14 & 1 & -1 & -5 \\ 6 & -13 & -8 & 2 & -3 & -4 \end{array}\right]$$

Example 2 is an SISO 2-D discrete system of order (16, 8), represented by the Fornasini-Marchesini state-space model given in (1a) and (1b), where each entry of $\{\boldsymbol{A}_1, \boldsymbol{A}_2, \boldsymbol{b}_1, \boldsymbol{b}_2, \boldsymbol{c}, d\}$ is a random number chosen from a normal distribution with zero mean and unit variance.

Example 3 is a four-input two-output 2-D discrete system of order (8, 16) represented by the Fornasini-Marchesini state-space model given in (15a) and (15b), where each entry of $\{\boldsymbol{A}_1, \boldsymbol{A}_2, \boldsymbol{B}_1, \boldsymbol{B}_2, \boldsymbol{C}, \mathbf{D}\}$ is a random number chosen from a normal distribution with zero mean and unit variance.

The amounts of computation required by the algorithms for the three examples are listed in Table 1. It is evident that Algorithms 1 and 2 require different amounts of computation if the order of the system $n_1 \neq n_2$ (n_1, $n_2 \leq n$). Extensive results with $1 \leq n_1 \leq 30$ and $1 \leq n_2 \leq 30$ have shown that Algorithm 1 requires less computation than Algorithm 2 when $n_1 < n_2$ (see Example 3), and Algorithm 2 requires less computation when $n_1 > n_2$ (see Example 2).

V. Conclusion

Two algorithms for the determination of the transfer-function matrices of 2-D discrete systems using the Fornasini-Marchesini state-space model have been proposed. These are based on a 1-D polynomial determination technique. The algorithms are efficient and reliable, and can be used for multi-input, multi-output two-dimensional discrete systems.

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