A Two-Step Regularization Approach for Image Noise Removal

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Abstract—A two-step regularization algorithm is proposed for image noise removal. In the first step of the algorithm a smoothing filter such as lowpass or Wiener filter is applied to reduce the noise level in the image and a new image model is formulated where the smoothing filter is treated as a degradation operator. In the second step of the algorithm a generalized regularization filter is applied to deblur the image. Motivated by this idea, a two-step regularization algorithm for image restoration is also proposed. Experimental results are included to demonstrate the superiority of the proposed methods over the conventional approaches.

I. INTRODUCTION

Images are often contaminated by noise introduced possibly by transmission medium, recording medium, measurement errors, or quantization of the data for digital storage. The noise is usually shown as an additive random process which is often assumed to be uncorrelated with the image. Lowpass filtering has been one of the conventional tools for smoothing noisy images. Although a lowpass filter is able to reduce high-frequency components of the noise, it degrades sharp details such as edges and lines. Another popular approach for noise removal is the Wiener filtering. As the ratio of the power spectrum of the noise to the power spectrum of the original image can only be estimated with limited accuracy, the outcome image from Wiener filtering of a noisy image often needs to be further improved.

In this paper we propose a two-step regularization method for noise removal. The underlying idea here can be described as follows. First we use a lowpass filter or a Wiener filter to reduce the noise contained in a given image, and treat the filter used as a *degradation operator*. In doing so we obtain a new image model with a reduced noise term and a precisely-known degradation mechanism. Based on this new model, the second step of the method uses a regularization filter to deblur the image. As is expected, this two-step approach allows smoothing without introducing undesirable blurring.

In the case where the noise-contaminated image has already been blurred, the goal of image restoration is to reduce as much as possible the degradations from the recorded image so that the resulting image will best approximate the original scene subject to some criteria. The standard method for the treatment of this typical ill-posed problem is the regularization technique first proposed by Tikhonov [1]. Following the same idea adopted in the two-step method for noise removal, we incorporate a pre-smoothing step into the conventional Tikhonov regularization technique so as to reduce the noise level before the conventional regularization begins. Experimental results are included to demonstrate that the new approach gives better results than those obtained from the conventional Tikhonov approach.

II. A TWO-STEP APPROACH FOR NOISE REMOVAL

A. *Disadvantage of Lowpass Filters in Noise Removal*

If an image is degraded by additive noise only, the discrete image model is simply given by

$$g(k,l) = f(k,l) + n(k,l) \tag{1}$$

where $f(k,l)$ and $g(k,l)$ are the light intensity (gray value) of the original and noisy image at point (k,l), respectively, and $n(k,l)$ is the additive noise at (k,l) with zero mean and variance σ_n^2. Image noise removal is a process that attempts to recover the image from its noise-contaminated version. Typically the frequency spectrum of the noise is distributed in a high frequency range, while the frequency spectrum of the image is well concentrated in a relatively low frequency range due to high correlation among neighboring pixels. Hence, lowpass filtering can be employed for reducing the noise. A linear lowpass FIR filter is characterized by an $M \times M$ impulse response matrix L, where M is usually an odd integer. In order to preserve the gray level in the images's uniform zones, the sum of all the entries in L should be one which means that in average $L(p,q)$ is in the magnitude of $1/M^2$. The output image from the filter is given by

$$\begin{aligned}(L*g)(k,l) &= \sum_{q=-m}^{q=m}\sum_{p=-m}^{p=m} L(p,q)g(k-p,l-q) \\ &= \sum_{q=-m}^{q=m}\sum_{p=-m}^{p=m} L(p,q)f(k-p,l-q) \\ &+ \sum_{q=-m}^{q=m}\sum_{p=-m}^{p=m} L(p,q)n(k-p,l-q)\end{aligned} \tag{2}$$

where $m = (M-1)/2$. Note that the second term in the right side of (2) can be regarded as a random variable with zero mean and variance $(\sum\sum L(p,q)^2)\sigma_n^2$. One can argue that $(\sum\sum L(p,q)^2)\sigma_n^2 \approx \sigma_n^2/M^2$, Thus, by smoothing the amplitude of the noise fluctuations is reduced approximately to one Mth of the original amount. However, the first term in the right side of (2) indicates that the image itself is also being filtered, leading to blurred edges etc. Similarly, it can be argued that the well known Wiener filter also blurs sharp details when it is used for noise removal.

B. *The Two-Step Approach*

In order to substantially reduce the blurring effect occurred in noise-removal process when a lowpass or a Wiener filter is used, we propose a two-step regularization method. Again we consider the discrete image model

$$g = f + n \tag{3}$$

In the spatial domain the terms in (3) are $N^2 \times 1$ vectors with N the number of pixels along each side of the image. It is well known that when it applies to the image g, a linear filter which could for example be a lowpass or a Wiener filter can be characterized by a constant matrix H of dimension $N^2 \times N^2$ [2]. The first step of the approach is to apply such a H to (3), yielding

$$\hat{g} = Hf + \hat{n} \tag{4}$$

where $\hat{g} = Hg$ is the filtered image and $\hat{n} = Hn$ is the reduced noise term. The second step begins with this "new" image model and its goal is to obtain a good estimation of f based on the available image $\hat{g}$. To this end the generalized regularization filter proposed in [3] fits into (4) quite nicely as the degradation mechanism (i.e., operator H) is precisely known. The best estimate in the spatial domain, $\hat{f}$, is given by

$$\hat{f} = (H^T H + \lambda C^T C)^{-1} H^T \hat{g} \tag{5}$$

where C is a stabilizing operator and λ is a regularization parameter controlling the degree of solution smoothness [2][3]. It is worthwhile to note that (5) can be implemented in the frequency domain efficiently by the fast discrete Fourier transform [2] and methods exist for determining an "optimal" parameter λ [4][5].

C. *Simulation Results*

The example presented here uses the 256×256 Lena shown in Fig. 1(a) as the original image f. White noise is added to f with the signal to noise ratio (SNR) equal to 10 dB and the resulting noisy image is shown in Fig. 1(b). The mean square error (MSE) of the noisy image is equal to 496.6. Here the SNR is defined by [6]

$$\text{SNR} = 10\log\frac{\text{variance of the degraded image}}{\text{variance of the noise}} \tag{6}$$

and the MSE is defined by

$$\text{MSE} = \text{variance of } \hat{f}(k,l) - f(k,l) \tag{7}$$

where $\hat{f}$ is an estimate of f. Two popular smoothing filters, namely the Gaussian filter and the Wiener filter, are separately used in the pre-smoothing step. The result of a smoothing 7×7 Gaussian filter is depicted in Fig. 1(c) with MSE = 1534.6. As can be observed, although the noise was reduced in the filtered image, it was also severely blurred. After applying a regularization filter with $\lambda = 0.005$ to the image in Fig. 1(c), we obtain the image shown in Fig. 1(d) which reduces the MSE to 199.8. In the second experiment the result of the noise removal Wiener filtering is shown in Fig. 1(e) with MSE = 478.0. After applying a regularization filter with $\lambda = 0.1$ to the Wiener-filtered image, we obtain the image shown in Fig. 1(f) with MSE = 156.6.

III. Deblurring with a Pre-Smoothing Step

A. *The Algorithm*

Motivated by the method of Sec. II, in this section we propose a two-step method for image restoration, in which smoothing the blurred image is the first step and applying the standard regularization method to estimate the original image is the second step. To describe this method analytically, we consider the standard image model

$$g = Hf + n \tag{8}$$

Applying a smoothing operator S to (8) which corresponds to a noise-removal filter, we obtain

$$\hat{g} = \hat{H}f + \hat{n} \tag{9}$$

where $\hat{g} = Sg$ is the smooth version of the originally blurred image, $\hat{H} = SH$ is the new blurring operator, and $\hat{n} = Sn$ is the reduced noise term. It is now obvious that using model (9) a good estimation of f based on the available image $\hat{g}$ can be obtained by the standard regularization methods.

B. *Simulation Results*

To demonstrate the effectiveness of the proposed pre-smoothing restoration algorithm, the Airplane image in Fig. 2(a) is linearly blurred with $L = 7$ units and color noise is added with SNR = 15 dB (Fig. 2(b)). Taking the optimal $\lambda = 0.002$ in the Tikhonov regularization process, the resored image without pre-smoothing is shown in Fig. 2(c) with MSE = 90.0 and ISNR = 8.72 dB. To apply the proposed approach, the degraded image ((Fig. 2(b))) is first smoothed by a Gaussian filter with standard deviation = 2 pixels. Then the degraded image is restored by the proposed pre-smoothing restoration algorithm with $\lambda = 2 \times 10^{-8}$. The resulting image is shown in Fig. 2(d). The restored image in this case gives MSE = 52.3 and ISNR = 11.0 dB, a significant improvement over the conventional regularization method.

IV. Conclusions

In this paper a two-step method for image noise removal and a pre-smoothing deblurring method for image restoration has been proposed. The two-step noise-removal method smooths a noise-contaminated image by using a lowpass filter as usual and then applies a restoration filter to deblur the known degradation due to the lowpass filtering. In essence, the pre-smoothing-and-deblurring method first extracts a "cleaner" version of the image from the noisy blurred image, and followed by a regularized-restoration filter treating the cascade of the smoothing filter and the original blurring operator as the new blurring operator. As has been demonstrated through examples each of the methods considerably improves the conventional methods,

Acknowledgment

The authors are grateful to Micronet and NSERC for supporting this work.

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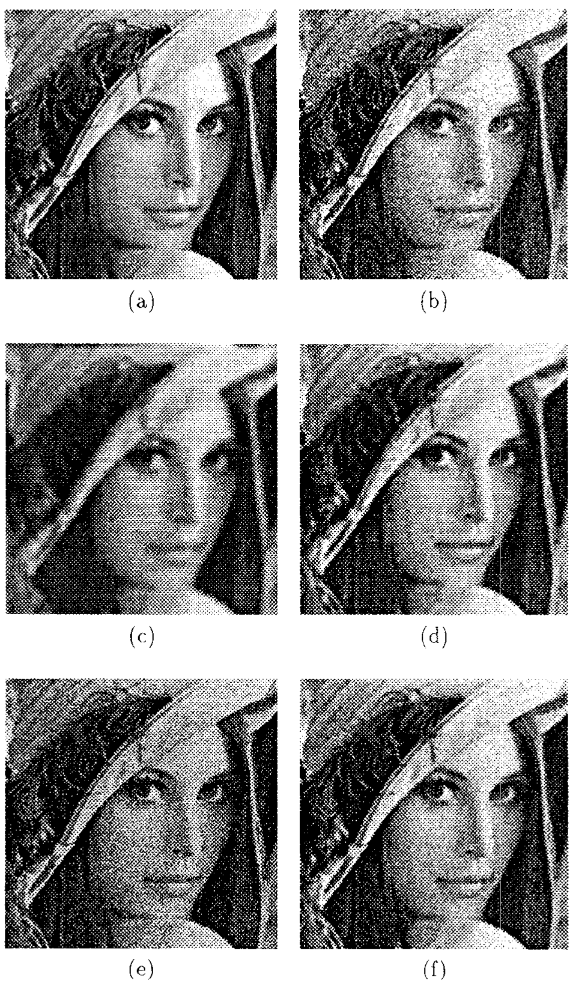
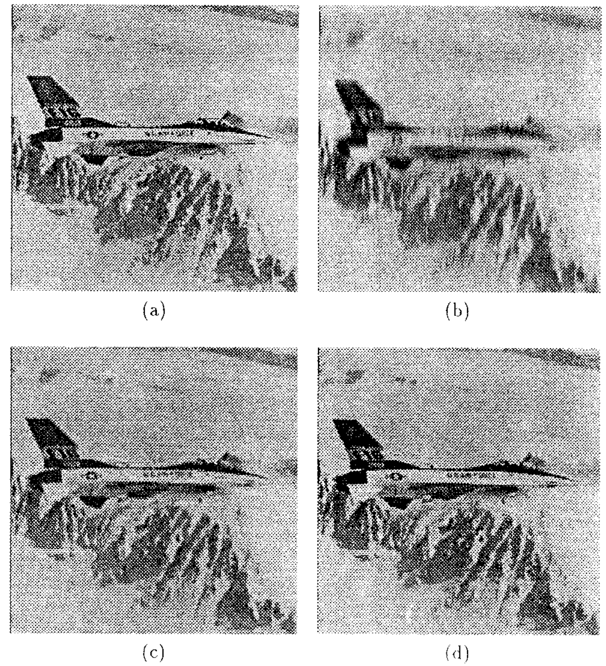


(a) (b) (c) (d) (e) (f)

Fig. 1



(a) (b) (c) (d)

Fig. 2