AN IMPROVED PEAK-TO-AVERAGE POWER-RATIO REDUCTION ALGORITHM FOR MULTICARRIER COMMUNICATIONS

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ABSTRACT

An improved peak-to-average power ratio reduction algorithm for discrete multitone systems is deduced by simultaneous optimization of the quadrature-amplitude-modulation constellation and the unused discrete multitone subsymbols. The proposed method is also applicable to multicarrier systems where all subchannels are active. Our simulations demonstrate that considerable performance improvement can be achieved over several existing methods at the cost of increased computational complexity.

1. INTRODUCTION

Multicarrier modulation finds applications in both wired and wireless communications [1]. Well known examples of multicarrier-modulation-based systems include digital subscriber lines (DSLs) using discrete multitone (DMT) over wired media, digital audio broadcasting (DAB), and digital video broadcasting (DVB) using orthogonal frequency-division multiplexing (OFDM) for wireless communications. Figure 1 illustrates a typical xDSL transmitter, where S/P and P/S represent serial-to-parallel and parallel-to-serial conversions, respectively.

A major problem of these modulation schemes is that multicarrier signals usually have large envelope fluctuations. This means that the peak-to-average power ratio (PAPR) of the multicarrier signals can be high. Since higher peak power requires greater dynamic range for the power amplifiers, large PAPR is undesireable. Recently, several PAPR-reduction algorithms have been proposed. In [2][3], PAPR reduction is achieved by modifying the frequency-domain signal $\mathbf{X}$ (see Fig. 1) in unused subchannels in an optimal way using linear programming (LP). In [4], the modulation constellation is modified in a way that will not degrade the bit-error rate (BER) performance while reducing the PAPR.

In this paper, it is demonstrated that significant PAPR reduction can be achieved by modifying both the modulation constellation in active subchannels and the modulation symbols in unused subchannels.

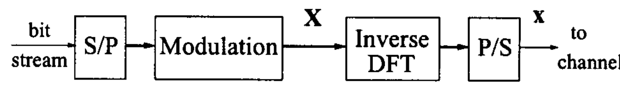

Fig. 1. A typical xDSL transmitter.

2. PROBLEM FORMULATION

Typically, a DMT/OFDM transmit signal can be expressed as a sum of N independent quadrature amplitude modulation (QAM) subsignals. In practice, the multicarrier transmit signal is generated by using the inverse discrete Fourier transform (IDFT) as

$$x_n = \frac{1}{N}\sum_{k=1}^{N} X_k e^{j2\pi(k-1)(n-1)/N} \tag{1}$$

where x_k and X_k represent the kth element of vectors $\mathbf{x}$ and $\mathbf{X}$, respectively. In the literature, modulation symbol X_k is refered to as the *subsymbol* in the kth subchannel [3]. In matrix form, (1) can be expressed as

$$\mathbf{x} = \mathbf{Q}\mathbf{X} \tag{2}$$

where $\mathbf{Q}$ is the IDFT matrix with elements $q_{n,k} = (1/N)$ $e^{j2\pi(k-1)(n-1)/N}$ for $n,\ k = 1, 2, \ldots, N$. In the DMT-based transmitter illustrated in Fig. 1, the frequency-domain signal $\mathbf{X}$ is symmetric and, consequently, the time-domain signal $\mathbf{x}$ is always real-valued.

The PAPR of signal $\mathbf{x}$ is defined as

$$\text{PAPR}_0 = \frac{\|\mathbf{x}\|_\infty^2}{\mathcal{E}[\|\mathbf{x}\|_2^2]/N} \tag{3}$$

where $\mathcal{E}[\cdot]$ denotes expectation, and $\|\mathbf{x}\|_\infty$ and $\|\mathbf{x}\|_2$ represent the infinity-norm and 2-norm of vector $\mathbf{x}$, respectively.

The basic idea behind the methods proposed in [2]-[4] is that the PAPR can be effectively reduced if the time-domain signal $\mathbf{x}$ (equivalently, the frequency-domain signal $\mathbf{X}$) is appropriately modified. Analytically, one seeks to find a vector $\mathbf{c}$ (equivalently, vector $\mathbf{C}$) that modifies $\mathbf{x}$ to $\mathbf{x}+\mathbf{c}$ (equivalently, $\mathbf{X}$ to $\mathbf{X}+\mathbf{C}$) such that the PAPR for the modified signal, i.e.,

$$\mathrm{PAPR}(\mathbf{c}) = \frac{\|\mathbf{x}+\mathbf{c}\|_\infty^2}{\mathcal{E}[\|\mathbf{x}\|_2^2]/N} \tag{4}$$

is minimized [2][3] or reduced [4] compared with PAPR_0. Note that vectors $\mathbf{c}$ and $\mathbf{C}$ are related to each other by

$$\mathbf{x}+\mathbf{c} = \mathbf{Q}(\mathbf{X}+\mathbf{C}). \tag{5}$$

It follows from (2) and (5) that

$$\mathbf{c} = \mathbf{QC}. \tag{6}$$

Since vector $\mathbf{c}$ must be real, $\mathbf{C}$ must have the same symmetry as $\mathbf{X}$. In the rest of the paper, the vectors $\mathbf{c}$ and $\mathbf{C}$ in (6) will be referred to as the time-domain and frequency-domain peak-reduction vectors, respectively.

The peak-reduction vector $\mathbf{C}$ used in [2][3] modifies the subsymbols in the unused subchannels. On the other hand, vector $\mathbf{C}$ used in [4] modifies the modulation constellation in active subchannels. Evidently, improved PAPR reduction may be achieved by carefully designing a peak-reduction vector $\mathbf{C}$ that modifies both the constellation modulation in active subchannels and the subsymbols in unused subchannels. This approach will be described in detail in Sec. 4 after a brief review of the methods proposed in [2]-[4] in Sec. 3.

3. REVIEW OF PREVIOUS WORK

3.1. LP-based method

It follows from (4) and (6) that for a given signal $\mathbf{x}$, the vector $\mathbf{c}$ that minimizes $\mathrm{PAPR}(\mathbf{x}+\mathbf{c})$ is the solution of the optimization problem

$$\underset{\mathbf{c}}{\text{minimize}}\ \|\mathbf{x}+\mathbf{c}\|_\infty = \underset{\mathbf{C}}{\text{minimize}}\ \|\mathbf{x}+\mathbf{QC}\|_\infty. \tag{7}$$

In the methods proposed in [2][3], both the transmitter and the receiver agree on reserving a small subset of subchannels for generating the PAPR reduction signals. Analytically, this means that if there are L unused subchannels whose indicies form the set $\mathcal{I} = \{i_1, i_2, \ldots, i_L\}$, then the components X_k for $k \in \mathcal{I}$ are set to zero and the components C_k are nonzero only if $k \in \mathcal{I}$.

Let $\hat{\mathbf{C}} = [C_{i_1}\ C_{i_2}\ \cdots\ C_{i_L}]^T$ and $\hat{\mathbf{Q}} = [\mathbf{q}_{i_1}\ \mathbf{q}_{i_2}\ \cdots\ \mathbf{q}_{i_L}]$ where $\mathbf{q}_{i_k}$ is the i_kth column of $\mathbf{Q}$. We can write $\mathbf{c} = \mathbf{QC} = \hat{\mathbf{Q}}\hat{\mathbf{C}}$ and the problem in (7) can be converted to

$$\underset{\hat{\mathbf{C}}}{\text{minimize}}\ \|\mathbf{x}+\hat{\mathbf{Q}}\hat{\mathbf{C}}\|_\infty \tag{8}$$

where the dimension of vector $\hat{\mathbf{C}}$ is L. The above problem can be cast as an LP problem with $2L+1$ variables [2].

3.2. Method in [4]

In the method of [4], it is assumed that all subchannels are active and the peak-reduction vector $\mathbf{C}$ is generated by certain reassignment of the constellation points as illustrated below. Let us consider a specific case of OFDM with 4-QAM modulation in each subchannel. As shown in Fig. 2, the conventional 4-QAM constellation points are located at the corners of the shaded regions. Each of these regions is called a *feasible region* for the reason that if a conventional constellation point is reassigned to a point inside the corresponding shaded region, the BER performance will not be degraded because the minimum distance between the newly assigned constellaton point and any constellation points located in other feasible regions is guaranteed not to be less than the minimum distance between the conventional constellation points. In the rest of the paper, a constellation point is said to be *feasible* if it is within the associated feasible region.

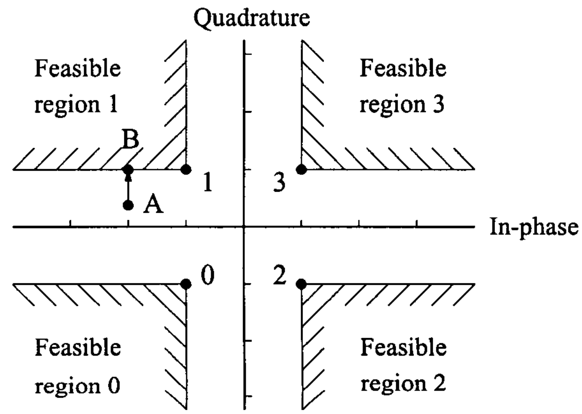

Fig. 2. Modification of a constellation point in an active subchannel.

Let $\mathbf{X}^{(0)}$ be the original DMT symbol obtained by applying 4-QAM modulation to a given data stream. The time-domain signal $\mathbf{x}^{(0)}$ is obtained as the IDFT of $\mathbf{X}^{(0)}$. For the components of $\mathbf{x}^{(0)}$ whose magnitudes exceed a certain target peak level, a clipping operation is used to limit their magnitudes to the target peak level. Denoting the modified time-domain signal by $\mathbf{x}^{(1)}$, we perform the DFT of $\mathbf{x}^{(1)}$ to obtain the DMT symbol $\mathbf{X}^{(1)}$. Due to the clipping operation, some subsymbols of $\mathbf{X}^{(1)}$ may lie outside their feasible regions. In order to avoid BER degradation, these subsymbols need to be modified properly. Suppose subsymbol $X_k^{(0)}$ is at corner point 1 as illustrated in Fig. 2, which becomes $X_k^{(1)}$ at point A after the application of IDFT, clipping, and DFT. Denoting the point that is feasible and nearest to point A by B, we modify $X_k^{(1)}$ such that it is represented by point B. If necessary, this IDFT/clipping/DFT/reassignment procedure is repeated for another $K-1$ times until the maximum magnitude of $\mathbf{x}^{(K)}$ is not larger than the target peak value.

4. PAPR REDUCTION BY OPTIMIZATION

4.1. Algorithm

For the sake of simplicity, we consider a specific DMT system with N subchannels where the L subchannels in the highest frequency range are unused. For each active subchannel, 4-QAM is chosen as the modulation scheme.

Define the index sets for active and unused subchannels as $\mathcal{I}_a = \{1, 2, \ldots, N-L\}$ and $\mathcal{I}_u = \{N-L+1, \ldots, N\}$, respectively. In the proposed method, the peak-reduction vector $\mathbf{C}$ is modified in the entire set of subchannels so as to minimize $\|\mathbf{x}+\mathbf{QC}\|_\infty$ with constraints that the modified subsymbols in the active subchannels, i.e., $X_k + C_k$ for $k \in \mathcal{I}_a$ remain feasible. This optimization problem can be formulated as

$$\underset{\mathbf{C}}{\text{minimize}} \quad \|\mathbf{x}+\mathbf{QC}\|_\infty \tag{9a}$$
$$\text{subject to:} \quad X_k + C_k \text{ feasible for } k \in \mathcal{I}_a. \tag{9b}$$

It can be readily shown that the problem in (9) can be cast as the LP problem

$$\text{minimize} \quad \tau \tag{10a}$$
$$\text{subject to:} \quad |x_k + \mathbf{q}_k^T\mathbf{C}| \le \tau \quad \text{for } k = 1, \ldots, N \tag{10b}$$
$$X_k + C_k \text{ feasible} \quad \text{for } k \in \mathcal{I}_a \tag{10c}$$

where $\mathbf{q}_k$ denotes the kth column of $\mathbf{Q}$. If we let $\mathbf{Q} = \mathbf{Q}_r + j\mathbf{Q}_i$ and $\mathbf{C} = \mathbf{C}_r + j\mathbf{C}_i$, then the constraints in (10b) can be expressed in a matrix form as

$$\begin{bmatrix} -\mathbf{Q}_r & \mathbf{Q}_i & \mathbf{e} \\ \mathbf{Q}_r & -\mathbf{Q}_i & \mathbf{e} \end{bmatrix} \begin{bmatrix} \mathbf{C}_r \\ \mathbf{C}_i \\ \tau \end{bmatrix} \ge \begin{bmatrix} \mathbf{x} \\ \mathbf{x} \end{bmatrix} \tag{11}$$

where $\mathbf{e} = [1\ 1\ \cdots\ 1]^T \in \mathcal{R}^{N-L}$. Furthermore, if we denote the original DMT symbol as $\mathbf{X}$, then the constraints in (10c) can be quantified as

$$S_{rk}C_{rk} \ \ge\ 0 \quad \text{for } k \in \mathcal{I}_a \tag{12a}$$
$$S_{ik}C_{ik} \ \ge\ 0 \quad \text{for } k \in \mathcal{I}_a \tag{12b}$$

where $S_{rk} = \text{sgn}[\text{real}(X_k)]$, $S_{ik} = \text{sgn}[\text{imag}(X_k)]$, and C_{rk} and C_{ik} are the kth componets of $\mathbf{C}_r$ and $\mathbf{C}_i$, respectively. The problem in (10) can now be expressed in a standard LP form as

$$\text{minimize} \quad \mathbf{d}^T\mathbf{y} \tag{13a}$$
$$\text{subject to:} \quad \mathbf{Ay} \ge \mathbf{b} \tag{13b}$$

where $\mathbf{d} = [0 \cdots 0\ 1]^T \in \mathcal{R}^{2(N-L)+1}$, $\mathbf{y} = [\mathbf{C}_r^T\ \mathbf{C}_i^T\ \tau]^T$, $\mathbf{b} = [\mathbf{x}^T\ \mathbf{x}^T\ \mathbf{0}\ \mathbf{0}]^T$, and

$$\mathbf{A} = \begin{bmatrix} -\mathbf{Q}_r & \mathbf{Q}_i & \mathbf{e} \\ \mathbf{Q}_r & -\mathbf{Q}_i & \mathbf{e} \\ \mathbf{S}_r & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_i & \mathbf{0} \end{bmatrix} \tag{14}$$

with $\mathbf{S}_r = [\mathbf{0}, \text{diag}\{S_{r1}, S_{r2}, \ldots, S_{r,N-L}\}]$ and $\mathbf{S}_i = [\mathbf{0}, \text{diag}\{S_{i1}, S_{i2}, \ldots, S_{i,N-L}\}]$.

It is worthwhile to note that the LP problem in (10) can be viewed as a generalized version of the LP-based optimization proposed in [2][3] where the peak-reduction vector $\mathbf{C}$ is the solution of the reduced-scale LP problem

$$\text{minimize} \quad \tau \tag{15a}$$
$$\text{subject to:} \quad |x_k + \mathbf{q}_k^T\mathbf{C}| \le \tau \quad \text{for } k = 1, \ldots, N \tag{15b}$$
$$C_k = 0 \quad \text{for } k = 1, \ldots, N-L. \tag{15c}$$

4.2. Simulations

The proposed LP-based PAPR reduction algorithm was applied to a DMT system that has 256 subchannels with 13 subchannels in the higheset frequency range unused. The subsymbols in the active subchannels were obtained by 16-QAM modulation. A total of 10^4 randomly generated DMT symbols were used for transmission. A commonly used performance measure for a PAPR reduction algorithm is the clipping probability for a given PAPR threshold, PAPR_0, which is defined as the probability that the PAPR of the DMT symbol exceeds PAPR_0 [2]. With the simulation setting described above, the clipping probability versus various PAPR threshold values for the proposed algorithm was evaluated and is plotted in Fig. 3 (solid line). For comparison, the clipping probabilities for the LP-based method in [2], for the method in [4] for various numbers of iterations, and for the original DMT symbols are plotted in the figure as dot-dashed, dashed, and dotted curves, respectively. From the figure, it is evident that compared with the PAPR reduction performance of [2]-[4], significant improvement can be achieved by the proposed algorithm. For example, at a clipping probability of 10^{-3}, the proposed algorithm offers a 2.3-dB improvement over the method in [2], and a 1.2-dB improvement over the method in [4] with 300 iterations. It can also be observed that for the method in [4], little improvement can be achived after 60 iterations.

For multicarrier communication systems where all subchannels are active, the methods in [2][3] cannot be used while the method in [4] and the proposed algorithm are still applicable. In our simulations, we considered an OFDM transmitter with 256 active subchannels with 16-QAM modulation in each subchannel. A total of 10^4 randomly generated OFDM symbols were used for transmission. The clipping probability for the proposed algorithm versus various

PAPRs is plotted in Fig. 4 (solid line). For the sake of comparison, the clipping probabilities for the method in [4] with various numbers of iterations and for the original OFDM symbols are also plotted in the figure as dashed and dotted curves, respectively. It is observed that significant improvement can be achieved by the proposed algorithm over that achieved with the method in [4]. For example, at a clipping probability of 10^{-3}, the proposed algorithm offers a 1.5-dB improvement over the method in [4] with 300 iterations.

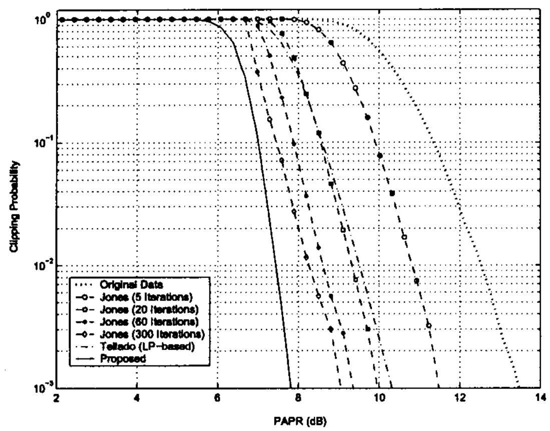

Fig. 3. Performance comparison of different PAPR reduction algorithms for a DMT transmitter.

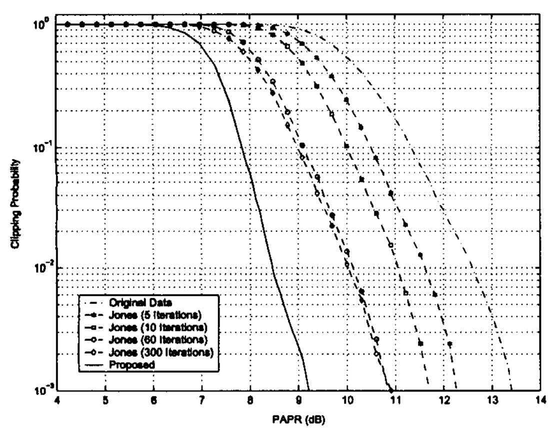

Fig. 4. Performance comparison of different PAPR reduction algorithms for a OFDM transmitter.

It is of interest to compare the computational complexity of the proposed algorithm with those of the methods in [2]-[4]. In the first set of simulations described above that involved a DMT transmitter, it was found that the amount of computation required by the LP-based method in [2][3] and the method in [4] with 60 iterations were approximately 4% and 25%, respectively, of that required by the proposed algorithm. In our second set of simulations of an OFDM transmitter, the computational complexity of the method in [4] with 60 iterations was about 25% of that of the proposed algorithm. As can be observed from Figs. 3 and 4, the convergence of the method in [4] is relatively fast in the first several iterations and becomes gradually slower, and after 60 iterations little improvement can be achieved. Therefore, for the method in [4], a tradeoff can be made between performance and computational complexity. Based on these observations, the following conclusions can be drawn:

• The LP-based method in [2][3] achieves a reasonable performance with a smaller amount of computation than the method in [4].

• The method in [4] offers some flexibility for performance/complexity and it can achieve significantly better performance than that of the method in [2][3] at the cost of increased computational complexity.

• Considerable performance improvement can be achieved by the proposed method over the methods in [2]-[4] but the computational complexity is increased.

5. CONCLUSION

A new PAPR-reduction algorithm that optimizes simultaneously the QAM constellation and the unused subsymbols has been proposed. The new algorithm is based on an LP approach where the QAM constellation in the active subchannels and the DMT subsymbols in unused subchannels are optimized simultaneously. The proposed method is also applicable to multicarrier systems where all subchannels are active. Our simulations have demonstrated that considerable performance improvement can be achieved over several existing methods but at the cost of increased computational complexity.

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