SOME NEW RESULTS ON STABILITY AND STABILITY ROBUSTNESS OF FORNASINI-MARCHESINI STATE-SPACE 2-D DIGITAL FILTERS

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***Abstract*- This paper describes a Lyapunov approach to stability and stability robustness analysis for 2-D digital filters in the Fornasini-Machesini local state-space setting. A class of new constant 2-D Lyapunov equations, that generalize the 2-D Lyapunov equation proposed recently by T. Hinamoto, is described. It is shown that the generalized 2-D Lyapunov equation can be used to derive a new stability condition that narrows the gap between the "necessity" and "sufficiency" that occurs in Hinamoto's stability theorem. The generalized 2-D Lyapunov equations are then utilized to derive two lower bounds of unstructured, stable perturbations of a given stable filter. Numerical techniques for solving the proposed 2-D Lyapunov equations are also presented.**

I. INTRODUCTION

Various Lyapunov equations have been used as effective tool for stability analysis of both one-dimensional (1-D) and two-dimensional (2-D) systems, see e.g. [1]-[8] and the references cited there. Unlike its 1-D counterpart, however, the stability theorems based on the constant 2-D Lyapunov equation investigated in [4][7] for the Roesser model and the constant 2-D Lyapunov equation proposed and studied in [8] for the Fornasini-Marchesini model provide only *sufficient* conditions for stability of the state-space digital filters considered. In particularly, as can be seen from [8] and the analysis presented in Sec. II of this paper, there are many 2-D filters that are stable but do not satisfy the Lyapunov equation proposed in [8]. One of the chief objectives of this paper is to propose a generalized constant 2-D Lyapunov equation and to apply it in the stability analysis of the 2-D digital filters in the Fornasini-Marchesini local state-space setting. Again the stability theorem obtained here gives a sufficient condition, but it includes Hinamoto's stability theorem [8] as a special case and narrows the gap between "sufficiency" and "necessity" for a state-space digital filter to be stable, which occurs in Hinamoto's Lyapunov theorem. In addition, we make use of the generalized 2-D Lyapunov equation to analyze the robust stability of a F-M 2-D model, leading to two lower bounds of the unstructured, stable perturbations.

II. PRELIMINARIES

The 2-D digital filters considered here are those modeled in Fornasini and Marchesini's local state space by

$$\begin{aligned} \mathbf{x}(i+1,j+1) &= \mathbf{A}_1 x(i,j+1) + \mathbf{A}_2 x(i+1,j) \\ &\quad + \mathbf{b}_1 u(i,j+1) + \mathbf{b}_2 u(i+1,j) \quad (1\text{a}) \\ y(i,j) &= \mathbf{c}\mathbf{x}(i,j) + du(i,j) \quad (1\text{b}) \end{aligned}$$

where $\mathbf{A}_1$, $\mathbf{A}_2 \in R^{n\times n}$. It is known [9] that (1) is asymptotically stable if and only if

$$p(z_1,z_2) \equiv \det(\mathbf{I} - z_1\mathbf{A}_1 - z_2\mathbf{A}_2) \neq 0 \quad \text{for } (z_1,z_2) \in \bar{U} \quad (2)$$

where $\bar{U} = \{(z_1,z_2) : |z_1| \le 1, |z_2| \le 1\}$, In [8] Hinamoto proved the following theorem:

Theorem 1 [8]: (1) is asymptotically stable if there exists a positive definite symmetric $\mathbf{P} \in R^{n\times n}$ such that

$$\mathbf{Q} = \begin{bmatrix} \alpha\mathbf{P} & \mathbf{0} \\ \mathbf{0} & \beta\mathbf{P} \end{bmatrix} - \mathbf{A}^T\mathbf{P}\mathbf{A} \quad (3)$$

is positive definite where $\mathbf{A} = [\mathbf{A}_1 \quad \mathbf{A}_2]$, and

$$\alpha, \beta > 0, \quad \alpha + \beta = 1 \quad (4)$$

A graphic illustration of Theorem 1 was also given in [8] as follows. If filter (1) satisfies (3) and (4), then all zeros of characteristic polynomial $p(z_1,z_2)$ correspond to

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locations outside the ellipse in the first quadrant shown in Fig. 1, where the ellipse is described by

$$\frac{|z_1|^2}{1/\alpha} + \frac{|z_2|^2}{1/\beta} = 1 \tag{5}$$

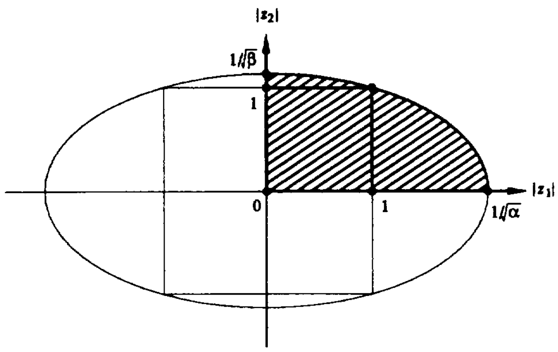

Figure 1: A graphic illustration of the conditions in Theorem 1 [8].

From this illustration it follows that stable filters with poles (i.e. zeros of $p(z_1, z_2)$) corresponding to the locations in *both* shadowed areas S_1 and S_2 shown in Fig. 2, for example, do not satisfy (3) and (4). This is to say, no α and β can be found to meet (4) such that zeros of $p(z_1, z_2)$ in both areas S_1 and S_2 are left outside the corresponding ellipse. Thus we see that there is a large number of filters that are stable but their stability cannot be confirmed using Theorem 1.

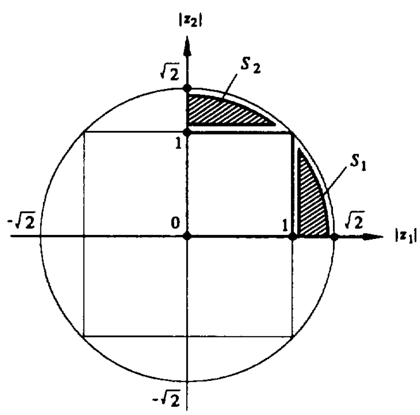

Figure 2: A graphic illustration of the sufficiency of Theorem 1.

Note that (3) can be written as

$$\mathbf{Q} = \begin{bmatrix} \mathbf{P}^{T/2}(\alpha\mathbf{I})\mathbf{P}^{1/2} & \mathbf{0} \\ \mathbf{0} & \mathbf{P}^{T/2}(\beta\mathbf{I})\mathbf{P}^{1/2} \end{bmatrix} - \mathbf{A}^T\mathbf{P}^{T/2}\mathbf{I}\mathbf{P}^{1/2}\mathbf{A} \tag{6}$$

In the rest of the paper we shall consider the Lyapunov equation

$$\mathbf{Q} = \begin{bmatrix} \mathbf{P}^{T/2}\mathbf{W}_1\mathbf{P}^{1/2} & \mathbf{0} \\ \mathbf{0} & \mathbf{P}^{T/2}\mathbf{W}_2\mathbf{P}^{1/2} \end{bmatrix} - \mathbf{A}^T\mathbf{P}^{T/2}\mathbf{R}\mathbf{P}^{1/2}\mathbf{A} \tag{7}$$

where matrices $\mathbf{P}$, $\mathbf{W}_1$, $\mathbf{W}_2$ and $\mathbf{R}$ are all positive definite. Clearly (7) becomes (3) if $\mathbf{W}_1 = \alpha\mathbf{I}$, $\mathbf{W}_2 = \beta\mathbf{I}$ and $\mathbf{R} = \mathbf{I}$. For this reason we shall call (7) the *generalized* 2-D Lyapunov equation associated with state-space model (1).

III. A GENERALIZED 2-D LYAPUNOV THEORY

The main theorem, stated and proved below, describes the use of the generalized Lyapunov equation (7) to confirm stability of a 2-D state-space digital filter.

Theorem 2: (1) is asymptotically stable if there are positive definite matrices $\mathbf{P}$, $\mathbf{W}_1$, $\mathbf{W}_2$, and $\mathbf{R}$, such that matrix $\mathbf{Q}$ given by (7) is positive difinite and that

$$\mathbf{R} - \mathbf{W}_1 - \mathbf{W}_2 \geq 0 \tag{8}$$

Proof: We prove the theorem by contradiction. Suppose the conditions of the theorem are satisfied but (1) is unstable. Then there is $(z_1, z_2) \in \bar{U}$ such that

$$\det(\mathbf{I} - z_1\mathbf{A}_1 - z_2\mathbf{A}_2) = 0 \tag{9}$$

Hence there exists $\mathbf{v} \neq 0$ such that

$$\mathbf{v} = \mathbf{A}\begin{bmatrix} z_1\mathbf{I} \\ z_2\mathbf{I} \end{bmatrix}\mathbf{v} \tag{10}$$

where $\mathbf{A} = [\mathbf{A}_1 \quad \mathbf{A}_2]$. We now use (7) and (10) to compute

$$\begin{aligned} &\mathbf{v}^*\mathbf{P}^{T/2}\mathbf{R}\mathbf{P}^{1/2}\mathbf{v} \\ &= \mathbf{v}^*[\bar{z}_1\mathbf{I} \quad \bar{z}_2\mathbf{I}]\begin{bmatrix} \mathbf{P}^{T/2}\mathbf{W}_1\mathbf{P}^{1/2} & \mathbf{0} \\ \mathbf{0} & \mathbf{P}^{T/2}\mathbf{W}_2\mathbf{P}^{1/2} \end{bmatrix} \\ &\quad \cdot \begin{bmatrix} z_1\mathbf{I} \\ z_2\mathbf{I} \end{bmatrix}\mathbf{v} - \mathbf{v}_z^*\mathbf{Q}\mathbf{v}_z \end{aligned}$$

where $\mathbf{v}^*$ denotes the complex conjugate transposition of $\mathbf{v}$, $\bar{z}_i$ is the complex conjugate of z_i, and $\mathbf{v}_z^* = \mathbf{v}^*[\bar{z}_1\mathbf{I} \ \ \bar{z}_2\mathbf{I}]$. It follows that

$$\mathbf{v}^*\mathbf{P}^{T/2}(\mathbf{R} - |z_1|^2\mathbf{W}_1 - |z_2|^2\mathbf{W}_2)\mathbf{P}^{1/2}\mathbf{v} = -\mathbf{v}_z^*\mathbf{Q}\mathbf{v}_z \tag{11}$$

By (9) we note that $(z_1, z_2) \neq (0, 0)$, hence $\mathbf{v}_z \neq 0$. So $\mathbf{Q} > \mathbf{0}$ means that the right-hand side of (11) is negative. On the other hand, $|z_1| \leq 1$, $|z_2| \leq 1$ and the positive semi-definiteness of $\mathbf{R} - \mathbf{W}_1 - \mathbf{W}_2$ imply that $\mathbf{R} - |z_1|^2\mathbf{W}_1 - |z_2|^2\mathbf{W}_2 \geq \mathbf{0}$, therefore the left-hand side of (11) is nonnegative, leading to a contradiction. This completes the proof. □

Several consequences can be drawn from Theorem 2 immediately.

Corollary 2.1: (1) is asymptotically stable if there is a matrix $\mathbf{P} > 0$ such that

$$\mathbf{Q} = \begin{bmatrix} \mathbf{P}^{T/2}\mathbf{W}_1\mathbf{P}^{1/2} & 0 \\ 0 & \mathbf{P}^{T/2}\mathbf{W}_2\mathbf{P}^{1/2} \end{bmatrix} - \mathbf{A}^T\mathbf{P}\mathbf{A} \quad (12)$$

is positive definite, where $\mathbf{W}_1 = \mathbf{U}^T\mathbf{\Sigma}_1\mathbf{U}$, $\mathbf{W}_2 = \mathbf{U}^T\mathbf{\Sigma}_2\mathbf{U}$ with $\mathbf{U}$ orthogonal, $\mathbf{\Sigma}_1 = \text{diag}\{\sigma_{11}, \ldots, \sigma_{1n}\}$, $\mathbf{\Sigma}_2 = \text{diag}\{\sigma_{21}, \ldots, \sigma_{2n}\}$,

$$\sigma_{1k} > 0,\ \sigma_{2k} > 0 \ \text{ and } \ \sigma_{1k} + \sigma_{2k} = 1 \qquad 1 \leq k \leq n \quad (13)$$

Corollary 2.2: (1) is asymptotically stable if there is a matrix $\mathbf{P} > 0$ such that

$$\mathbf{Q} = \begin{bmatrix} \mathbf{P}^{T/2}\mathbf{W}_1\mathbf{P}^{1/2} & 0 \\ 0 & \mathbf{P}^{T/2}\mathbf{W}_2\mathbf{P}^{1/2} \end{bmatrix} - \mathbf{A}^T\mathbf{P}\mathbf{A} \quad (14)$$

is positive definite, where $\mathbf{W}_i = \text{diag}\{\sigma_{i1}, \ldots, \sigma_{in}\}$ $(i = 1, 2)$ satisfy (13).

Corollary 2.3 (Hinamoto): (1) is asymptotically stable if there is a matrix $\mathbf{P} > 0$ such that

$$\mathbf{Q} = \begin{bmatrix} \alpha\mathbf{P} & \mathbf{0} \\ \mathbf{0} & \beta\mathbf{P} \end{bmatrix} - \mathbf{A}^T\mathbf{P}\mathbf{A} \quad (15)$$

is positive definite, where

$$\alpha > 0,\ \beta > 0 \ \text{ and } \ \alpha + \beta = 1 \quad (16)$$

Note that in all three corollaries matrix $\mathbf{R}$ is assigned to be the identity matrix. As will be seen from the next theorem, this assignment does not lose the generality of Theorem 2.

Theorem 3: The sufficient conditions stated in Theorem 2 for (1) to be asymptotically stable and the conditions stated below are equivalent: There exist matrices $\mathbf{P} > \mathbf{0}$, $\mathbf{W}_1 > 0$, and $\mathbf{W}_2 > 0$ such that

$$\mathbf{Q} = \begin{bmatrix} \mathbf{P}^{T/2}\mathbf{W}_1\mathbf{P}^{1/2} & 0 \\ 0 & \mathbf{P}^{T/2}\mathbf{W}_2\mathbf{P}^{1/2} \end{bmatrix} - \mathbf{A}^T\mathbf{P}\mathbf{A} \quad (17)$$

is positive definite and

$$\mathbf{I} - \mathbf{W}_1 - \mathbf{W}_2 \geq \mathbf{0} \quad (18)$$

The proof of this theorem can be found in [10].

IV. TWO LOWER BOUNDS OF ν_1

Consider a stable system (1a) with $\mathbf{b}_1 = \mathbf{0}$, $\mathbf{b}_2 = \mathbf{0}$ that satisfies (17) and (18). Let $\mathbf{A} = [\mathbf{A}_1 \ \ \mathbf{A}_2]$ and denote by S_u the set of all perturbations $\Delta\mathbf{A} = [\Delta\mathbf{A}_1 \ \ \Delta\mathbf{A}_2] \in C^{n\times 2n}$ with $\mathbf{A} + \Delta\mathbf{A}$ unstable, i.e.,

$$S_u = \{\Delta\mathbf{A} : \ [\Delta\mathbf{A}_1 \ \ \Delta\mathbf{A}_2] \in C^{n\times 2n},\ A + \Delta\mathbf{A} \text{ unstable}\}$$

A measure for robust stability of system (1) is defined as

$$\nu_1 = \inf_{\Delta\mathbf{A}\in S_u} ||\Delta\mathbf{A}||$$

Let the perturbed system be given by

$$x(i+1, j+1) = (\mathbf{A}_1 + \Delta\mathbf{A}_1)x(i, j+1) + (\mathbf{A}_2 + \Delta\mathbf{A}_2)x(i+1, j) \quad (19)$$

The matrices $\mathbf{P}$, $\mathbf{W}_1$ and $\mathbf{W}_2$ are employed to construct the Lyapunov function

$$\phi(i,j) = \phi_1(i,j) + \phi_2(i,j) \quad (20)$$

where

$$\begin{aligned} \phi_1(i,j) &= x^T(i, j+1)\mathbf{P}^{T/2}\mathbf{W}_1\mathbf{P}^{1/2}x(i, j+1) \quad (21) \\ \phi_2(i,j) &= x^T(i+1, j)\mathbf{P}^{T/2}\mathbf{W}_2\mathbf{P}^{1/2}x(i+1, j) \quad (22) \end{aligned}$$

Define

$$\phi_{11}(i,j) = \phi_1(i+1, j) + \phi_2(i, j+1) \quad (23)$$

By (18),

$$\begin{aligned} \phi_{11}(i,j) &= x^T(i+1, j+1) \\ &\quad \cdot\mathbf{P}^{T/2}(\mathbf{W}_1 + \mathbf{W}_2)\mathbf{P}^{1/2}x(i+1, j+1) \\ &\leq x^T(i+1, j+1)\mathbf{P}x(i+1, j+1) \quad (24) \end{aligned}$$

Now using (17) and (19)–(24) we compute

$$\begin{aligned} \Delta\phi(i,j) &= \phi_{11}(i,j) - \phi(i,j) \\ &\leq -[\underline{\sigma}(\mathbf{Q}) - 2\bar{\sigma}^{1/2}(\tilde{\mathbf{P}} - \mathbf{Q})\bar{\sigma}^{1/2}(\mathbf{P})||\Delta\mathbf{A}|| \\ &\quad -\bar{\sigma}(\mathbf{P})||\Delta\mathbf{A}||^2]\, ||\tilde{x}(i,j)||^2 \end{aligned}$$

where

$$\tilde{x}(i,j) = \begin{bmatrix} x(i, j+1) \\ x(i+1. j) \end{bmatrix}$$

$$\text{and } \ \tilde{\mathbf{P}} = \begin{bmatrix} \mathbf{P}^{T/2}\mathbf{W}_1\mathbf{P}^{T/2} & 0 \\ \mathbf{0} & \mathbf{P}^{T/2}\mathbf{W}_2\mathbf{P}^{1/2} \end{bmatrix} \quad (25)$$

Hence if

$$||\Delta\mathbf{A}|| < \frac{[\bar{\sigma}(\tilde{\mathbf{P}} - \mathbf{Q}) + \underline{\sigma}(\mathbf{Q})]^{1/2} - \bar{\sigma}^{1/2}(\tilde{\mathbf{P}} - \mathbf{Q})}{\bar{\sigma}^{1/2}(\mathbf{P})} \quad (26)$$

then $\Delta\phi(i,j) < 0$ for all $i,\ j$, and a typical argument (see e.g., [4]) applies to show the stability of the perturbed system (19).

An alternative Lyapunov function may be defined as

$$\psi(i,j) = [\phi_1(i,j) + \phi_2(i,j)]^{1/2} \tag{27}$$

Similarly we define

$$\psi_{11}(i,j) = [\phi_1(i+1,j) + \phi_2(i,j+1)]^{1/2} \tag{28}$$

and use (18) to write

$$\psi_{11}(i,j) \le [x^T(i+1,j+1)\mathbf{P}x(i+1,j+1)]^{1/2} \tag{29}$$

Now (17), (19), (27) and (29) imply that

$$\begin{aligned}\Delta\psi(i,j) &= \psi_{11}(i,j) - \psi(i,j)\\ &\le -\left[\frac{\underline{\sigma}(\mathbf{Q})}{\bar\sigma^{1/2}(\tilde{\mathbf{P}}-\mathbf{Q}) + \bar\sigma^{1/2}(\tilde{\mathbf{P}})} - \bar\sigma^{1/2}(\mathbf{P})||\Delta\mathbf{A}||\right]\\ &\quad ||\tilde{x}(i,j)||\end{aligned}$$

Thus if

$$||\Delta\mathbf{A}|| < \frac{\underline{\sigma}(\mathbf{Q})}{\bar\sigma^{\frac{1}{2}}(\mathbf{P})[\bar\sigma^{\frac{1}{2}}(\tilde{\mathbf{P}}-\mathbf{Q}) + \bar\sigma^{\frac{1}{2}}(\tilde{\mathbf{P}})]} \tag{30}$$

then $\Delta\psi(i,j) < 0$ for all i,j, and therefore, the perturbed system (19) will remain stable.

Although our numerical experience indicates that the lower bounds of ν_1 given in (26) and (30) usually stay quite close each other, the bound given in (26) is always better than that in (30) [11].

V. NUMERICAL SOLUTION TO THE GENERALIZED LYAPUNOV EQUATION

In this section we consider the problem of numerically solving Lyapunov equation (17), (18) in the following sense: Given $\mathbf{A} = [\mathbf{A}_1\ \ \mathbf{A}_2]$, find $\mathbf{P} > 0$, $\mathbf{W}_1 > 0$, and $\mathbf{W}_2 > 0$ such that $\mathbf{Q}$ defined by (17) is positive definite and (18) holds. In what follows, we first present a result that relates the existence of such positive definite $\mathbf{P}$, $\mathbf{W}_1$, $\mathbf{W}_2$ to a norm minimization problem. Discussions then follow to consider numerical solution of this norm minimization problem for two special cases where both matrices $\mathbf{W}_1$, $\mathbf{W}_2$ are diagonal.

Theorem 4: There exist $\mathbf{P} > 0$, $\mathbf{W}_1 > 0$ and $\mathbf{W}_2 > 0$ such that (18) holds and $\mathbf{Q}$ defined in (17) is positive definite if and only if

$$\underset{\substack{\mathbf{T},\mathbf{V}_1,\mathbf{V}_2 \text{ nonsingular}\\ \mathbf{I}-\mathbf{V}_1^{-T}\mathbf{V}_1^{-1}-\mathbf{V}_2^{-T}\mathbf{V}_2^{-1}\ge 0}}{\text{minimize}} \left|\left|\tilde{\mathbf{A}}\begin{bmatrix}\mathbf{V}_1 & 0\\ 0 & \mathbf{V}_2\end{bmatrix}\right|\right| < 1 \tag{31}$$

where $\tilde{\mathbf{A}} = [\tilde{\mathbf{A}}_1\ \ \tilde{\mathbf{A}}_2]$, $\tilde{\mathbf{A}}_1 = \mathbf{T}^{-1}\mathbf{A}_1\mathbf{T}$, $\tilde{\mathbf{A}}_2 = \mathbf{T}^{-1}\mathbf{A}_2\mathbf{T}$, and $||\cdot||$ denotes the induced 2-norm of the matrix involved.

Proof: see [10].

There is a variety of efficient methods that can be used to solve the optimization problem (31) [12][13]. In what follows we discuss two special cases where $\mathbf{V}_1$ and $\mathbf{V}_2$ assume diagonal forms, leading to much simplified unconstained minimization problems.

Case A: In this case $\mathbf{V}_1$ and $\mathbf{V}_2$ assume the form of

$$\mathbf{V}_1 = \begin{bmatrix}1/v_{11} & & 0\\ & \ddots & \\ 0 & & 1/v_{1n}\end{bmatrix},\quad \mathbf{V}_2 = \begin{bmatrix}1/v_{21} & & 0\\ & \ddots & \\ 0 & & 1/v_{2n}\end{bmatrix}$$

with $v_{1i} > 0$ and $v_{2i} > 0$ $(1 \le i \le n)$. To meet constraint (28), it is required that

$$v_{1i}^2 + v_{2i}^2 = 1 \qquad 1 \le i \le n$$

which implies that

$$\begin{aligned}0 < v_{1i} < 1\\ v_{2i} = (1 - v_{1i})^{1/2}\end{aligned} \tag{32}$$

The hyperbolic tangent transformation

$$v_{1i} = \frac{1+\tanh\nu_i}{2}, \qquad 1 \le i \le n$$

can be used to meet contraint (32) with $\nu_i \in (-\infty,\ \infty)$. The minimization problem is now simplified as

$$\underset{\substack{\mathbf{T}\text{ nonsingular}\\ -\infty<\nu_1,\ldots,\nu_n<\infty}}{\text{minimize}} \left|\left|\tilde{\mathbf{A}}\begin{bmatrix}\mathbf{V}_1 & 0\\ 0 & \mathbf{V}_2\end{bmatrix}\right|\right| \tag{33}$$

where

$$\mathbf{V}_1 = \begin{bmatrix}\dfrac{2}{1+\tanh\nu_1} & & 0\\ & \ddots & \\ 0 & & \dfrac{2}{1+\tanh\nu_n}\end{bmatrix}$$

$$\mathbf{V}_2 = \begin{bmatrix}\dfrac{2}{[4-(1+\tanh\nu_1)^2]^{1/2}} & & 0\\ & \ddots & \\ 0 & & \dfrac{2}{[4-(1+\tanh\nu_n)^2]^{1/2}}\end{bmatrix}$$

Although the state-variable transformation matrix $\mathbf{T}$ in problem (33) needs to be nonsingular, (33) can be considered as an *unconstrained* optimization problem due to the following two reasons: First, the measure of the set of points in the parameter space that make $\mathbf{T}$ singular is zero from a classic measure-theoretic point of view [14] meaning that the chance to meet a point in the parameter space that corresponds to a singular matrix $\mathbf{T}$ in a search process during minimization is extremely rare; second, during the minimization, updating a parameter vector to a new one that would be closer to a singular $\mathbf{T}$ is always discouraged as a nearly singular $\mathbf{T}$ tends to increase the value of the norm in (33) due to the presence of $\mathbf{T}^{-1}$ in $\tilde{\mathbf{A}}$. In other words, in this norm minimization problem the use of a good optimization algorithm would very unlikely lead to a solution that corresponds to a nearly singular $\mathbf{T}$.

Case B: In this case $\mathbf{V}_1$ and $\mathbf{V}_2$ assume the form of

$$\mathbf{V}_1 = \frac{1}{v_1}\mathbf{I} \text{ and } \mathbf{V}_2 = \frac{1}{v_2}\mathbf{I} \tag{34}$$

with $v_i > 0$ $(i = 1, 2)$. To satisfy the constraint in (31), it is required that

$$v_1^2 + v_2^2 = 1$$

Hence

$$0 < v_1 < 1 \text{ and } v_2 = (1 - v_1^2)^{1/2}$$

Similar to Case A, hyperbolic tangent transformations

$$v_1 = \frac{1 + \tanh\nu}{2}, \qquad -\infty < \nu < \infty$$

and

$$v_2 = \frac{1}{2}[4 - (1 + \tanh\nu)^2]^{1/2}, \qquad -\infty < \nu < \infty$$

can be used in (34) with $-\infty < \nu < \infty$ so that the minimization problem

$$\underset{\substack{\mathbf{T}\ \text{nonsingular} \\ -\infty<\nu<\infty}}{\text{minimize}} \left\| \tilde{\mathbf{A}} \begin{bmatrix} \mathbf{V}_1 & 0 \\ 0 & \mathbf{V}_2 \end{bmatrix} \right\| \tag{35}$$

with $\mathbf{V}_i$ $(i = 1, 2)$ specified above is essentially unconstrained. Finally, it is easy to see that as $\mathbf{W}_1 = \mathbf{V}_1^{-T}\mathbf{V}_1^{-1} = v_1^2\mathbf{I}$ and $\mathbf{W}_2 = \mathbf{V}_2^{-T}\mathbf{V}_2^{-1} = v_2^2\mathbf{I}$, (35) provides a feasible approach to numerically solving Hinamoto's Lyapunov equation (3), (4).

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