A New Approach for the Design of Low-delay Quadrature Mirror Filter Banks

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Abstract — An iterative approach is proposed for the design of two-channel QMF banks with low reconstruction delays. The perfect reconstruction condition is formed in the time-domain and the design efficiency is significantly improved by formulating a quadratic objective function in each step of an iterative process whose minimum point can be obtained in closed form. The proposed method is demonstrated by a design example.

I. INTRODUCTION

Quadrature mirror filter (QMF) banks are among the most widely used filter banks, and many methods have been developed for their design [1]-[3]. Usually QMF banks are designed with symmetrical impulse responses in order to achieve linear phase response. As a result, the reconstruction delay is equal to $N-1$, where N is the length of the filter used. If N is large, or if a tree structure is employed, the system delay can be too large for some applications.

Recently several approaches have been proposed for the design of low-delay filter banks. These include a time-domain approach [4] and an optimization method [5]. In this paper, a new approach is proposed for the design of two-channel QMF banks with low reconstruction delays. The perfect reconstruction condition is formulated in the time-domain. To avoid constrained optimization, an iterative procedure is adopted in which the objective function in each step is formed as a quadratic function whose minimum can be obtained in closed form. Compared with the approach in [4], the proposed approach has shown improved design efficiency, and improved stopband attenuation. Moreover, the filter banks obtained possess a quadrature-mirror structure which is amendable to efficient polyphase implementation. A case study is included to illustrate the proposed method.

II. THE PROPOSED METHOD AND DESIGN PROCEDURE

A. Review of Two-Channel QMF Banks

The input-output relation of a two-channel filter bank shown in Fig. 1 is given by

$$\hat{X}(z) = \tfrac{1}{2}[H_0(z)G_0(z) + H_1(z)G_1(z)]X(z) + \tfrac{1}{2}[H_0(-z)G_0(z) + H_1(-z)G_1(z)]X(-z) \quad (1)$$

By assuming that $H_1(z) = H_0(-z), G_1(z) = -H_0(-z)$ and $G_0(z) = H_0(z)$, the aliasing term on the right-hand side of (1) is cancelled and hence

$$\hat{X}(z) = \tfrac{1}{2}[H_0^2(z) - H_0^2(-z)]X(z) \quad (2)$$

In conventional QMF banks, filter H_0 has a symmetrical impulse response which guarantees a linear-phase response, and perfect reconstruction requires that

$$H_0^2(z) - H_0^2(-z) = z^{-(N-1)} \quad (3)$$

where N is the filter length of H_0 and is assumed to be even in this paper. Hence the system delay, often referred to as the reconstruction delay, is $N-1$.

If N is large or if the filter bank is to be used in a tree structure system, the reconstruction delay will be quite large, which is highly undesirable in real-time applications. In [5] a method for the design of two-channel QMF banks with low reconstruction delays has been proposed, in which the analysis and synthesis filters have the same mirror-image relationship as in conventional QMF banks and the perfect reconstruction condition becomes

$$H_0^2(z) - H_0^2(-z) = z^{-k_d} \quad (4)$$

where $k_d < N-1$ is the system delay. Equation (4) can be expressed in the time domain as

$$\mathbf{Bh} = \mathbf{m} \quad (5a)$$

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where $\mathbf{h} = [\ h(0)\ h(1)\ \cdots\ h(N-1)\ ]^T$ is the coefficient vector of H_0, and

$$\mathbf{m} = [\ 0\ \cdots\ 0\ 1\ 0\ \cdots\ 0\ ]^T \tag{5b}$$

where the $[(k_d+1)/2]th$ entry is unity, and $\mathbf{B}$ is a $(N-1) \times N$ matrix defined by

$$\mathbf{B}=2\begin{bmatrix} h(1) & h(0) & 0 & \cdots & 0 \\ h(3) & h(2) & h(1) & \cdots & \vdots \\ \vdots & & & & \\ h(N-1) & h(N-2) & \cdots & & h(0) \\ 0 & 0 & h(N-1) & \cdots & h(2) \\ \vdots & & & & \\ 0 & \cdots & & h(N-1) & h(N-2) \end{bmatrix} \tag{5c}$$

To meet the design specifications for the frequency response of H_0, the L_2 objective function

$$\Psi = \int_0^{\omega_p} |H_0(e^{j\omega}) - e^{-jk_d\omega/2}|^2 d\omega + \int_{\omega_s}^{\pi} |H_0(e^{j\omega})|^2 d\omega \tag{6}$$

can be constructed where ω_p and ω_s are the passband and stopband edges of H_0, respectively.

Taking the perfect reconstruction condition in (5a) into account, the design problem is reduced to minimizing Ψ in (6) subject to the constraints in (5a), which is a constrained optimization problem.

B. The Iterative Approach

To directly solve the above-mentioned constrained optimization problem is time-consuming in general. Instead an iterative approach is proposed in this paper in which the objective function is formed as

$$E = E_1 + \alpha E_2 \tag{7a}$$

where $\alpha > 0$ is a weighting constant,

$$E_1 = (\mathbf{Bf} - \mathbf{m})^T(\mathbf{Bf} - \mathbf{m}) \tag{7b}$$

deals with the prefect reconstruction condition with $\mathbf{B}$ and $\mathbf{m}$ defined in (5b) and (5c), respectively, and $\mathbf{f} = [\ f(0)\ f(1)\ \cdots\ f(N-1)\ ]^T$. The function

$$E_2 = \int_0^{\omega_p} |F_0(e^{j\omega}) - e^{-jk_d\omega/2}|^2 d\omega + \int_{\omega_s}^{\pi} |F_0(e^{j\omega})|^2 d\omega \tag{7c}$$

deals with frequency-response requirements where $F_0(e^{j\omega}) = \mathbf{f}^T\mathbf{c}(\omega)$ and $\mathbf{c}(\omega) = [\ 1\ e^{-j\omega}\ \cdots\ e^{-j\omega(N-1)}\ ]^T$.

The iteration procedure starts by first designing a lowpass filter H_0 with a group delay $k_d/2$, where k_d is the desired system delay, and taking its coefficient vector as the initial $\mathbf{h}$. Then error E in (7a) can be formulated as a quadratic function in $\mathbf{f}$, i.e.,

$$E = \mathbf{f}^T(\mathbf{B}^T\mathbf{B} + \alpha\mathbf{Q})\,\mathbf{f} + (\alpha\mathbf{b}^T - 2\mathbf{m}^T\mathbf{B})\,\mathbf{f} + \omega_p + 1 \tag{8}$$

where

$$\mathbf{Q} = \mathrm{Re}\left[\int_0^{\omega_p} \mathbf{c}(\omega)\mathbf{c}^H(\omega)\,d\omega + \int_{\omega_s}^{\pi} \mathbf{c}(\omega)\mathbf{c}^H(\omega)\,d\omega\right] = \mathbf{U}_p + \mathbf{U}_s \tag{9a}$$

with $\mathbf{U}_p$ and $\mathbf{U}_s$ being two Toeplitz matrices defined by

$$\mathbf{U}_p = \begin{bmatrix} \omega_p & \sin\omega_p & \cdots & \frac{1}{N-1}\sin(N-1)\omega_p \\ \sin\omega_p & \omega_p & \cdots & \frac{1}{N-2}\sin(N-2)\omega_p \\ \vdots & & \ddots & \vdots \\ \frac{1}{N-1}\sin(N-1)\omega_p & \cdots & \cdots & \omega_p \end{bmatrix}$$

$$\mathbf{U}_s = \begin{bmatrix} \pi-\omega_s & -\sin\omega_s & \cdots & -\frac{1}{N-1}\sin(N-1)\omega_s \\ -\sin\omega_s & \pi-\omega_s & \cdots & -\frac{1}{N-2}\sin(N-2)\omega_s \\ \vdots & & \ddots & \vdots \\ -\frac{1}{N-1}\sin(N-1)\omega_s & \cdots & \cdots & \pi-\omega_s \end{bmatrix}$$

and

$$\mathbf{b} = -2\,\mathrm{Re}\left[\int_0^{\omega_p} \mathbf{c}(\omega)e^{jk_d\omega/2}\,d\omega\right] = -2\begin{bmatrix} \frac{2}{k_d}\sin\left(\frac{k_d\omega_p}{2}\right) \\ \frac{2}{k_d-2}\sin\left[\left(\frac{k_d}{2}-1\right)\omega_p\right] \\ \vdots \\ \frac{2}{k_d-2N+2}\sin\left[\left(\frac{k_d}{2}-N+1\right)\omega_p\right] \end{bmatrix} \tag{9b}$$

Since $\mathbf{B}^T\mathbf{B} + \alpha\mathbf{Q}$ is positive definite, the global minimum of E is achieved if

$$\mathbf{f} = -\tfrac{1}{2}(\mathbf{B}^T\mathbf{B} + \alpha\mathbf{Q})^{-1}(\alpha\mathbf{b} - 2\mathbf{B}^T\mathbf{m}) \tag{10}$$

Having obtained $\mathbf{f}$, a linear formula is used to update $\mathbf{h}$

$$\mathbf{h} := (1-\tau)\mathbf{h} + \tau\mathbf{f} \tag{11}$$

where $\tau \in (0,1)$ is called the smoothing parameter. The above process is repeated until $\|\ \mathbf{h} - \mathbf{f}\ \|$ is less than a prescribed tolerance.

The design procedure can be summarized as follows:

Algorithm

Step 1 Use a least-squares approach to design a lowpass FIR filter of length N with a group delay $k_d/2$, and passband and stopband edges ω_p and ω_s, respectively; then use the coefficient vector of the filter obtained as the initial $\mathbf{h}$.

Step 2 Calculate $\mathbf{Q}$ and $\mathbf{b}$ using (9a) and (9b), respectively.

Step 3 Use (5b) and (5c) to form $\mathbf{B}$ and $\mathbf{m}$, and compute $\mathbf{f}$ in (10).

Step 4 If $\| \mathbf{h} - \mathbf{f} \| < \epsilon$, where ϵ is a prescribed tolerance, output $\mathbf{f}$ as the design result and stop; otherwise, update $\mathbf{h}$ using (11) with a τ close to 0.5 and repeat from Step 3.

Several comments on the proposed design method are now in order.

- In the design of two-channel QMF with low delays, the filter coefficients obtained are not symmetrical in general; hence linear phase response is achieved only with respect to the passband.

- The proposed method can be used to design low-delay QMF filter banks whose lowpass and highpass filters have mirror-symmetric relationship, which leads to efficient polyphase implementation. The numbers of multiplications and additions per input in a polyphase implementation can be reduced to 25% of that required by direct implementation.

- When a low reconstruction delay k_d is required, artifacts may occur in the transition band of the obtained analysis and synthesis filters, which were also observed in [4]. This problem can be eliminated by modifying the objective function to include an additional term $\alpha_1 E_3$, i.e.,

$$E = E_1 + \alpha E_2 + \alpha_1 E_3 \tag{12}$$

where

$$E_3 = \int_{\omega_{t1}}^{\omega_{t2}} |F_0(e^{j\omega}) - e^{-jk_d\omega/2}|^2 \, d\omega \tag{13}$$

and $[\omega_{t1}, \omega_{t2}]$ is an interval in the transition band where the artifacts occur. It can be readily shown that with this additional error term, the proposed algorithm can still be used after modifying (8) as

$$\begin{aligned} E = \; & \mathbf{f}^T(\mathbf{B}^T\mathbf{B} + \alpha\mathbf{Q} + \alpha_1\mathbf{Q}_t)\,\mathbf{f} \\ & +(\alpha\mathbf{b}^T + \alpha_1\mathbf{b}_t^T - 2\mathbf{m}^T\mathbf{B})\,\mathbf{f} + K \end{aligned} \tag{14}$$

where K is a constant,

$$\mathbf{Q}_t = \begin{bmatrix} \omega_{t2}-\omega_{t1} & \phi(\omega_{t2},\omega_{t1},1) & \cdots & \phi(\omega_{t2},\omega_{t1},N-1) \\ \phi(\omega_{t2},\omega_{t1},1) & \omega_{t2}-\omega_{t1} & & \\ \vdots & & & \\ \phi(\omega_{t2},\omega_{t1},N-1) & \cdots & \cdots & \omega_{t2}-\omega_{t1} \end{bmatrix}$$

$$\phi(\omega_{t2},\, \omega_{t1},\, k) = \frac{1}{k}\,[\sin k\omega_{t2} - \sin k\omega_{t1}]$$

and

$$\begin{aligned} \mathbf{b}_t &= -2\,\mathrm{Re}\left[\int_{\omega_{t1}}^{\omega_{t2}} \mathbf{c}(\omega)e^{jk_d\omega/2}\,d\omega\right] \\ &= -2\begin{bmatrix} \phi(\omega_{t2},\omega_{t1},\frac{k_d}{2}) \\ \phi(\omega_{t2},\omega_{t1},\frac{k_d}{2}-1) \\ \vdots \\ \phi(\omega_{t2},\omega_{t1},\frac{k_d}{2}-N+1) \end{bmatrix} \end{aligned}$$

The global minimum of E is now given by

$$\mathbf{f} = -\tfrac{1}{2}(\mathbf{B}^T\mathbf{B}+\alpha\mathbf{Q}+\alpha_1\mathbf{Q}_t)^{-1}(\alpha\mathbf{b}+\alpha_1\mathbf{b}_t-2\mathbf{B}^T\mathbf{m}) \tag{15}$$

III. A CASE STUDY

We now present a design example to illustrate the proposed iterative approach. The design parameters are $N = 32$, $k_d = 15$, $\alpha = 5\times10^{-3}$, $\alpha_1 = 3\times10^{-5}$, $\omega_p = 0.35\pi$, $\omega_s = 0.7\pi$, $\omega_{t1} = 0.35\pi$, $\omega_{t2} = 0.45\pi$, $\tau = 0.5$ and $\epsilon = 10^{-3}$. The design is achieved after 8 iterations.

For comparison purposes, we refer to example 6.6.1 presented in [4], which is designed by a different time-domain approach. The comparisons are made in terms of the number of floating-point operations in millions (MFLOPS), average stopband attenuation (ASA), peak reconstruction error (PRE) defined as $\mathrm{PRE} = \max_{\omega}|20\log_{10}[\,|H_0(\omega)G_0(\omega)+H_1(\omega)G_1(\omega)|\,]|$, and signal-to-noise ratio (SNR) defined as

$$\begin{aligned} \mathrm{SNR} &= 10\log_{10}\left(\frac{\text{energy of the input signal}}{\text{energy of the reconstruction noise}}\right) \\ &= 10\log_{10}\left\{\frac{\Sigma\, x^2(n)}{\Sigma\,[x(n)-\hat{x}(n+k_d)]^2}\right\} \end{aligned}$$

The results are summarized in Table I, where SNR_{r1} and SNR_{r2} denote the SNR with a ramp input and a random input signal with uniformly distributed amplitude between 0 and 100, respectively. The amplitude responses of the lowpass analysis filters designed by the proposed method and the method of [4] are depicted in Fig. 2. The group delay of filter H_0 obtained by the proposed method is shown in Fig. 3 where it can be noticed that the filter has approximate linear phase in the passband.

TABLE I
Comparisons between the proposed method and that of [4]

	Proposed	Method of [4]
MFLOPS	1.26	-
ASA (dB)	-65.2	-43.7
PRE (dB)	1.8×10^{-3}	1.2×10^{-3}
SNR_{r1} (dB)	78.5	75.8
SNR_{r2} (dB)	77.6	77.4

As can be seen in Table I, the proposed method has resulted in a large increase in the stopband attenuation relative to the design approach in [4]. The number of floating point operations needed for the design was only 1.26×10^6, which should be substantially lower than that required by the method of [4], which is not known. In addition, unlike the low-delay systems proposed in [4], the quadrature mirror structure obtained by the proposed design is amenable to efficient polyphase-type implementation [2] which requires only $N/2$ additions and $N/2$ multiplications per input, as compared to $2N$ additions and $2N$ multiplications per input for the low-delay filter banks of [4].

IV. CONCLUSION

A new time-domain iterative approach for the design of low-delay QMF banks has been proposed. The design example in the case study shows that the proposed method is computationally efficient and leads to low-delay QMF banks with improved stopband attenuation and polyphase implementation.

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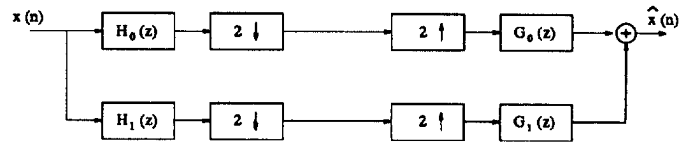

Figure 1. Two-channel FIR filter bank.

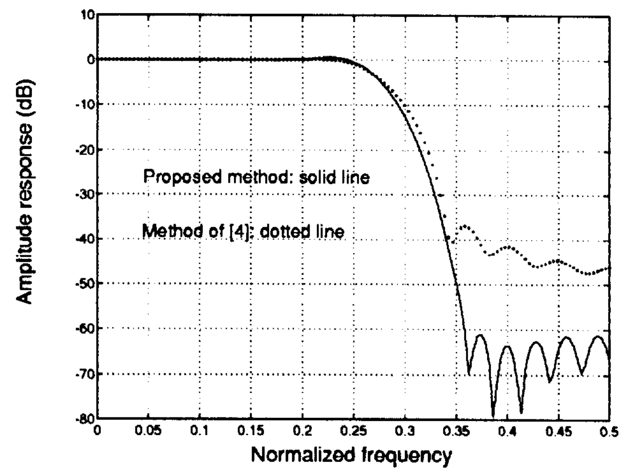

Figure 2. Amplitude responses of H_0 in the example.

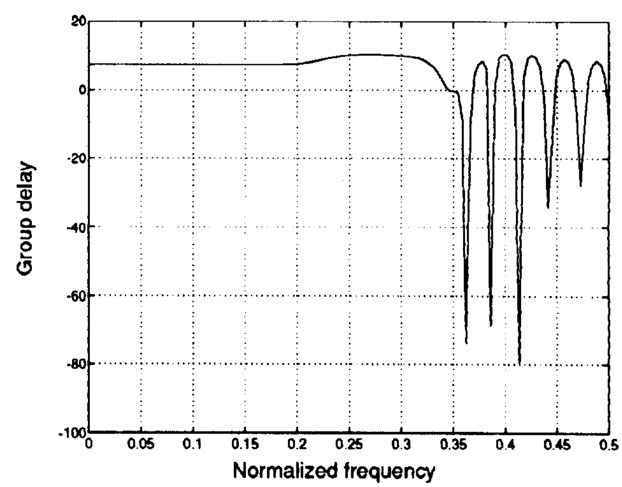

Figure 3. Group delay of filter H_0 in the example.