Improved methods for the design of 1-D and 2-D QMF banks

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ABSTRACT

An iterative procedure proposed by Chen and Lee for the design of quadrature mirror filter (QMF) banks is extended to the design of two types of filter banks, i.e, 1-D QMF banks with low reconstruction delay and 2-D nonseparable hexagonal QMF banks. Our simulations show that the extended methods are very efficient and yield good designs.

INTRODUCTION

Filter banks have been widely used in one-dimensional (1-D) and two-dimensional (2-D) signal processing [1][2]. In the design of filter banks, it is required that the perfect reconstruction condition be satisfied while the intra-band aliasing be eliminated or minimized. Design methods developed so far [3][4] involve minimizing an error function directly in the frequency domain to achieve the design requirements, which leads to a difficult optimization problem.

In [5], Chen and Lee introduced an iterative procedure to replace the conventional direct minimization of the error function in the design of quadrature mirror filter (QMF) banks. The method is based on a linearization of the error function associated with the design. After linearizing the least-square error function, which is a fourth-order function of the filter coefficients, the resulting error function to be minimized becomes a quadratic function of the filter coefficients. As a result, the optimal filter coefficients can be obtained by an analytic solution. Compared with conventional QMF design techniques, this iterative design method needs much less computation and leads to fairly good results.

In this paper, the procedure in [5] is extended to the design of two types of filter banks, namely, 1-D QMF banks with low reconstruction delay and 2-D nonseparable hexagonal QMF banks. Our simulations show that the design methods yield good results while reducing the computation load significantly. The paper is organized as follows: In the second section, the proposed methods are described and the design procedure is presented. In the third section several design examples are supplied to illustrate the methods.

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DESCRIPTION OF THE EXTENDED ITERATIVE METHODS

Design of FIR Filter Banks with Low Reconstruction Delay

A two-channel QMF bank is illustrated in Fig. 1, where $H_1(z) = H_0(-z), G_0(z) = H_0(z), G_1(z) = -H_0(-z)$ in order to achieve aliasing cancellation. In [6], a method has been proposed for the design of QMF banks with low reconstruction delays. The design is accomplished by minimizing an objective function of the form

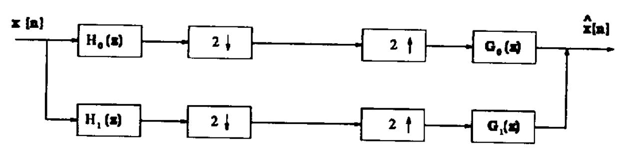

FIGURE 1
DIAGRAM OF TWO BAND FIR FILTER BANK

$$E = E_1 + \alpha E_2 \tag{1}$$

where

$$E_1 = \sum_{0 \le \omega \le \pi} |H_0^2(e^{j\omega}) - H_0^2(e^{j(\omega+\pi)}) - e^{-j\omega k}|^2$$
$$E_2 = \sum_{\omega_s \le \omega \le \pi} |H_0(e^{j\omega})|^2$$

and α is a weighting constant in the range $0 < \alpha < 1$. Term E_1 deals with the perfect reconstrucion condition and k is the system delay. Term E_2 deals with the intra-band aliasing where ω_s is the stopband edge. In general, the coefficients of H_0 have no symmetry and k is not fixed to $N-1$ as in conventional QMF designs, where N is the filter length. By letting k be an integer less than $N-1$ and minimizing the error function defined in (1), the coefficients of H_0 can be obtained. Then a filter bank can be constructed which has a lower reconstruction delay than a conventional QMF bank.

Instead of minimizing directly the above objective function with respect to the coefficients of H_0, an iterative method, which is an extension of that in [5], is adopted. The error components E_1 and E_2 in (1) are changed to

$$\begin{aligned} E' &= E_1' + \alpha E_2' \quad (2)\\ E_1' &= \sum_{0\le\omega\le\pi} |H_0(e^{j\omega})Q_0(e^{j\omega}) - H_0(e^{j(\omega+\pi)}) \\ &\qquad Q_0(e^{j(\omega+\pi)}) - e^{-j\omega k}|^2 \\ E_2' &= \sum_{\omega_s\le\omega\le\pi} |Q_0(e^{j\omega})|^2 \end{aligned}$$

where

$$Q_0(e^{j\omega}) = \sum_{l=0}^{N-1} q_0(l)e^{-j\omega l}$$

is the transfer function of a lowpass filter Q_0, whose coefficient vector is $\mathbf{v} = [q_0(0)\ q_0(1)\ \cdots\ q_0(N-1)]^T$. It is assumed that at the start of optimization the coefficient vector of H_0, $\mathbf{u} = [h_0(0)\ h_0(1)\ \cdots\ h_0(N-1)]^T$, is known and so $H_0(e^{j\omega})$ is known. Let $\mathbf{\Omega} = \{\omega_1,\ \omega_2,\ \ldots,\ \omega_s,\ \ldots,\ \omega_m\}$ be the set of sampling points and construct the matrices

$$\mathbf{U_t}(\mathbf{\Omega}) = \begin{bmatrix} 1 & e^{-j\omega_1} & \cdots & e^{-j\omega_1(N-1)} \\ \vdots & & & \vdots \\ 1 & e^{-j\omega_s} & \cdots & e^{-j\omega_s(N-1)} \\ \vdots & & & \vdots \\ 1 & e^{-j\omega_m} & \cdots & e^{-j\omega_m(N-1)} \end{bmatrix} \quad (3)$$

$$\mathbf{U_s} = \begin{bmatrix} 1 & e^{-j\omega_s} & \cdots & e^{-j\omega_s(N-1)} \\ \vdots & & & \vdots \\ 1 & e^{-j\omega_m} & \cdots & e^{-j\omega_m(N-1)} \end{bmatrix} \quad (4)$$

$$\mathbf{H}(\mathbf{\Omega}) = \mathrm{diag}[H_0(e^{j\omega_1}),\ \ldots,\ H_0(e^{j\omega_s}),\ \ldots,\ H_0(e^{j\omega_m})] \quad (5)$$

$$\mathbf{U} = \mathbf{H}(\mathbf{\Omega})\mathbf{U_t}(\mathbf{\Omega}) - \mathbf{H}(\mathbf{\Omega}+\pi)\mathbf{U_t}(\mathbf{\Omega}+\pi) \quad (6)$$

E' in (2) can be expressed in the form

$$E' = (\mathbf{U}\mathbf{v} - \mathbf{I})^H(\mathbf{U}\mathbf{v} - \mathbf{I}) + \alpha(\mathbf{U_s}\mathbf{v})^H(\mathbf{U_s}\mathbf{v}) \quad (7)$$

where $\mathbf{I} = [e^{-j\omega_1 k}\ e^{-j\omega_2 k}\ \cdots\ e^{-j\omega_m k}]^T$ and superscript H denotes complex conjugate transposition. E' in (7) is a quadratic function of the coefficients in $\mathbf{v}$. It can be shown that this function has a global minimum point given by the closed-form solution

$$\mathbf{v} = (\mathrm{Re}\,[\mathbf{U}^H\mathbf{U} + \alpha\mathbf{U_s}^H\mathbf{U_s}])^{-1} \cdot \mathrm{Re}\,([\mathbf{U}^H\mathbf{I}]) \quad (8)$$

where Re $[\cdot]$ is the real part of $[\cdot]$. After obtaining $\mathbf{v}$, a linear formula is adopted to update $\mathbf{u}$ as

$$\mathbf{u} := (1-\tau)\mathbf{u} + \tau\mathbf{v} \quad (9)$$

where τ, $0 < \tau < 1$, is the smoothing parameter. The above process is repeated until the distance between $\mathbf{u}$ and $\mathbf{v}$ is smaller than a specified tolerance.

Design of 2-D Nonseparable Hexagonal QMF Filter Banks

In [7], a four-channel 2-D nonseparable hexagonal QMF bank system, as illustrated in Fig. 2, was proposed. By forcing the analysis and synthesis filters to satisfy certain relationships, the aliasing terms in the system output are cancelled and the design is accomplished by minimizing an error function of the form

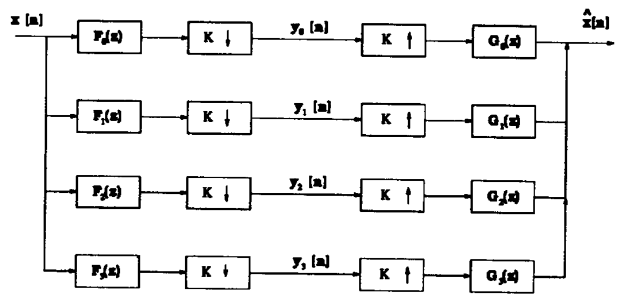

FIGURE 2
DIAGRAM OF A NONSEPARABLE HEXAGONAL QMF BANK

$$\begin{aligned} E &= E_1 + \alpha E_2 \quad (10)\\ E_1 &= \sum_{\mathbf{\Omega}} \left[\sum_{i=0}^{3} H(\omega + \tilde{\mathbf{k}}_i)^2 - 1\right]^2 \\ E_2 &= \sum_{\mathbf{\Omega}_s} H(\omega)^2 \end{aligned}$$

where α, $0 < \alpha < 1$, is a weighting constant. E_1[1] is to approximate the perfect reconstruction condition where $H(\omega)$ is the transfer function of a hexagonal lowpass filter, $\mathbf{\Omega} = \{\omega_1,\ \omega_2,\ \cdots,\ \omega_s,\ \ldots,\ \omega_m\}$ are sampling points and vectors $\mathbf{k}_i$, $i = 0,\ \ldots,\ 3$ are the four modulation vectors defined in [7]. Intra-band aliasing is reduced by minimizing E_2 where $\mathbf{\Omega}_s = \{\omega_s,\ \ldots,\ \omega_m\}$ are the sampling points which correspond to the stopband.

[1] E_1 differs from that in [7] by a constant multiplier of 4

With 12-fold symmetry in its impulse response, a hexagonal FIR filter will have zero-phase frequency response. Then $H(\boldsymbol{\omega})$ can be written as

$$H(\boldsymbol{\omega}) = \mathbf{b}(\boldsymbol{\omega})^T\mathbf{u}$$

where $\mathbf{u} = [h_a\ h_b\ h_c\ \cdots]^T$ is a column vector containing N independent filter coefficients and $\mathbf{b}(\boldsymbol{\omega}) = [T_0(\boldsymbol{\omega})\ T_1(\boldsymbol{\omega})\ \cdots\ T_{N-1}(\boldsymbol{\omega})]^T$ is a column vector with entries being real functions of $\boldsymbol{\omega}$.

As in the design of 1-D filter banks, an iterative method is adopted, which modifies the error function in (10) into

$$E' = E_1' + \alpha E_2' \tag{11}$$
$$E_1' = \sum_{\boldsymbol{\Omega}}\left[\sum_{i=0}^{3} H(\boldsymbol{\omega}+\tilde{\mathbf{k}}_i)G(\boldsymbol{\omega}+\tilde{\mathbf{k}}_i) - 1\right]^2$$
$$E_2' = \sum_{\boldsymbol{\Omega}_s} G(\boldsymbol{\omega})^2$$

Like $H(\boldsymbol{\omega})$, $G(\boldsymbol{\omega})$ is the transfer function of a lowpass hexagonal filter of the same length with 12-fold symmetry in its impulse response and

$$G(\boldsymbol{\omega}) = \mathbf{b}(\boldsymbol{\omega})^T\mathbf{v}$$

where $\mathbf{v} = [g_a\ g_b\ g_c\ \cdots]^T$ is a column coefficient vector. It is assumed that at the start of the optimization, the coefficients in $\mathbf{u}$ are known and so $H(\boldsymbol{\omega})$ is known. By defining

$$\mathbf{U_t}(\boldsymbol{\Omega}) = \begin{bmatrix} T_0(\boldsymbol{\omega}_1) & T_1(\boldsymbol{\omega}_1) & \cdots & T_{N-1}(\boldsymbol{\omega}_1) \\ \vdots & & & \vdots \\ T_0(\boldsymbol{\omega}_s) & T_1(\boldsymbol{\omega}_s) & \cdots & T_{N-1}(\boldsymbol{\omega}_s) \\ \vdots & & & \vdots \\ T_0(\boldsymbol{\omega}_m) & T_1(\boldsymbol{\omega}_m) & \cdots & T_{N-1}(\boldsymbol{\omega}_m) \end{bmatrix} \tag{12}$$

$$\mathbf{U_s} = \begin{bmatrix} T_0(\boldsymbol{\omega}_s) & T_1(\boldsymbol{\omega}_s) & \cdots & T_{N-1}(\boldsymbol{\omega}_s) \\ \vdots & & & \vdots \\ T_0(\boldsymbol{\omega}_m) & T_1(\boldsymbol{\omega}_m) & \cdots & T_{N-1}(\boldsymbol{\omega}_m) \end{bmatrix} \tag{13}$$

$$\mathbf{H}(\boldsymbol{\Omega}) = \mathrm{diag}[H(\boldsymbol{\omega}_1), \ldots, H(\boldsymbol{\omega}_s), \ldots, H(\boldsymbol{\omega}_m)] \tag{14}$$

$$\mathbf{U} = \sum_{i=0}^{3} \mathbf{H}(\boldsymbol{\Omega}+\tilde{\mathbf{k}}_i)\mathbf{U_t}(\boldsymbol{\Omega}+\tilde{\mathbf{k}}_i) \tag{15}$$

E' in (11) can be expressed as

$$E' = (\mathbf{U}\mathbf{v}-\mathbf{I})^T(\mathbf{U}\mathbf{v}-\mathbf{I}) + \alpha(\mathbf{U_S}\mathbf{v})^T(\mathbf{U_S}\mathbf{v}) \tag{16}$$

where $\mathbf{I}$ is a column vector with each entry being a 1. E' in (16) is a quadratic function of the coefficients in $\mathbf{v}$ and its minimum point can be obtained by an analytic solution as

$$\mathbf{v} = (\mathbf{U}^T\mathbf{U} + \alpha\mathbf{U_S}^T\mathbf{U_S})^{-1}\cdot(\mathbf{U}^T\mathbf{I}) \tag{17}$$

As in the design of 1-D filter banks, $\mathbf{u}$ is updated and the procedure is repeated.

Design Procedure

A step-by-step procedure based on the above methods is as follows:

1. Set the weigthing constant α, the smoothing parameter τ, and the stopping criterion ϵ.
2. Initialize the coefficients of vector $\mathbf{u}$.
3. Calculate the matrices expressed in (3), (4), (5), and (6) or (12), (13), (14), and (15).
4. Obtain coefficient vector $\mathbf{v}$ using (8) or (17).
5. If $|\mathbf{u}-\mathbf{v}| < \epsilon$, terminate the process; otherwise, update coefficient vector $\mathbf{u}$ using (9) and go to step 3.

DESIGN EXAMPLES

The proposed iterative methods have been used to design two-band filter banks with low recontruction delay as well as 2-D nonseparable hexagonal QMF filter banks. Four examples are illustrated.

In examples 1 and 2, filter banks with filter length $N = 32$ were designed with system delays of $k = 9$ and $k = 15$, respectively. The design results are summarized in Table I in terms of function evaluations in the design (FE), peak reconstruction error (PRE) defined as $\mathrm{PRE} = \max_{\omega}|20\log_{10}[|H_0^2(e^{j\omega}) - H_0^2(e^{j(\omega+\pi)})|]|$, passband ripple δ_p, stopband edge attenuation (AS), defined as $\mathrm{AS} = 20\log_{10}|H_0(e^{j\omega_s})|$. The amplitude response of H_0 in example 2 is plotted in Fig. 3. Compared with the design method proposed in [8], the iterative method described here reduces the computation by a considerable amount.

TABLE I
SIMULATION RESULTS FOR EXAMPLES 1 AND 2

k	ω_p	ω_s	FE	PRE(dB)	δ_p(dB)	AS(dB)
9	0.35π	0.64π	15	0.0025	0.0067	−15.56
15	0.35π	0.65π	7	0.0073	0.0025	−37.07

Examples 3 and 4 demonstrate the design of 2-D nonseparable hexagonal QMF banks with $N = 9$ and $N = 12$

coefficients, respectively. The simulation results are summarized in Table II in terms of function evaluations in the design (FE), $\text{PRE} = \max_{\boldsymbol{\omega}} |20\log_{10}[\sum_{i=0}^{3} H(\omega + \tilde{\mathbf{k}}_i)^2]|$, passband ripple δ_p and stopband ripple δ_s. Fig. 4 shows the 3-D plot of the amplitude response of the lowpass hexagonal filter with 9 coefficients. The designs by the iterative method are superior relative to those in [7] in terms of having better frequency responses and needing less computational effort.

TABLE II
SIMULATION RESULTS FOR EXAMPLES 3 AND 4

N	FE	PRE(dB)	δ_p(dB)	δ_s(dB)
9	6	0.0285	0.0039	−30.65
12	9	0.0178	0.0068	−30.42

CONCLUSION

The iterative method in [5] has been extended to the design of FIR filter banks with low reconstruction delays as well as the design of 2-D nonseparable hexagonal QMF banks. These methods are quite efficient in producing good designs while reducing the amount of computation.

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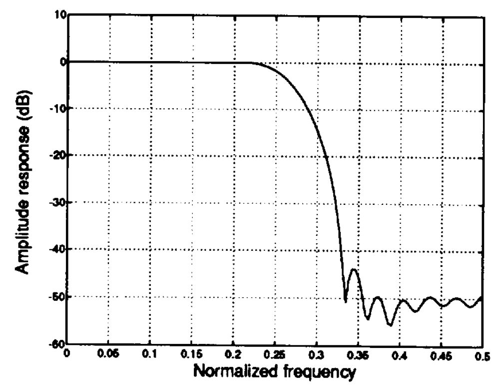

FIGURE 3
AMPLITUDE RESPONSE OF H_0 WITH $k = 15$

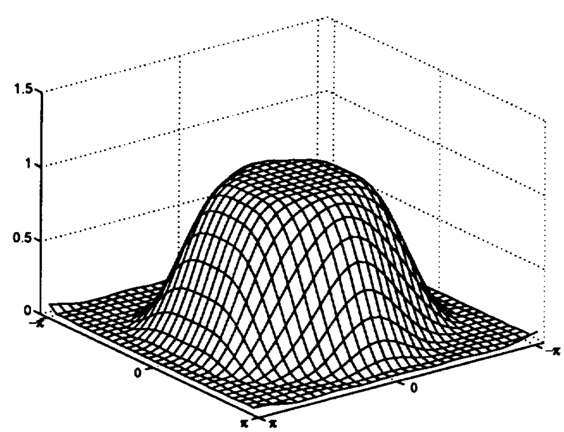

FIGURE 4
AMPLITUDE RESPONSE OF HEXAGONAL FILTER WITH 9 COEFS.