A New Approach for the Design of FIR Analysis-Synthesis Filter Banks with Short Reconstruction Delays

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Abstract

A new approach for the design of two-band FIR filter banks with short system delays is proposed. The method is based on the perfect recontruction condition and takes into consideration the characteristics of the individual filters. It involves minimizing an objective function by optimization. Two design examples are presented to illustrated the design procedure.

I. Introduction

The two-band filter bank shown in Fig. 1 has been a subject of study for more than decade. Among the existing two-band FIR filter banks, the quatrature-mirror filter bank (QMF) is among the most widely used. In this system, by properly designing the lowpass and highpass analysis and synthesis filters such that certain symmetry relationships are satisfied, the aliasing term at the output can be eliminated completely. Several approaches have been proposed to the design of QMF banks, see for example [1] [2] [3]. In all the conventional QMF design approaches, symmetry is imposed on the prototype filter to guarantee linear phase in the filters. The linearity of the phase responses improves the performance of the filter bank. The overall delay of such a system is determined by the length of the filters. In a tree-structure multirate system [4], where a series of filters are cascaded to split the signal into several bands, this will cause a long system delay. In order to achieve reduced delay, filter banks with short reconstruction delays are desired. In [5] a time-domain method was proposed, which can be used to design low-delay filter banks.

In this paper, we propose a new approach for designing low-delay two-band QMF filter banks. The method is based on the perfect reconstruction condition in the frequency domain and takes into consideration the characteristics of the individual filters. The paper is organised as follows: In section II, the proposed design technique is described. In section III, two design examples are described and are then compared with corresponding designs obtained by using the method in [5].

II. Description of Proposed Method

The two-band analysis-synthesis filter bank illustrated in Fig. 1 can be represented by

$$\begin{aligned}\hat{X}(z) &= 1/2[H_0(z)G_0(z) + H_1(z)G_1(z)]X(z)\\ &\quad +1/2[H_0(-z)G_0(z) + H_1(-z)G_1(z)]X(-z)\end{aligned} \tag{1}$$

where $X(z)$ and $\hat{X}(z)$ are the z transforms of the system input and output, respectively, and $H_i(z)$, $i = 0, 1$ are the transfer functions of the analysis filters, denoted by $H_i, i = 0, 1$, while $G_i(z)$, $i = 0, 1$ are the transfer functions of the synthesis filters, denoted by $G_i, i = 0, 1$. From Eqn. (1), the output of the system contains two terms. One is the desired translation of the input to the output and the other is the undesirable aliasing term due to the decimation operation. To eliminate aliasing and obtain perfect reconstruction of the input, two equations, i.e.,

$$H_0(z)G_0(z) + H_1(z)G_1(z) = z^{-k} \tag{2}$$

$$H_0(-z)G_0(z) + H_1(-z)G_1(z) = 0 \tag{3}$$

must be satisfied. The system delay is kT where T is the sampling period. Without loss of generality, we assume a sampling frequency of 2π rad/s, so that $T = 1\,\mathrm{s}$, in the following discussion. Eqn. (2) deals with perfect reconstruction and Eqn. (3) deals with aliasing cancellation. If we assume that

$$\begin{aligned}H_1(z) &= G_0(-z)\\ G_1(z) &= -H_0(-z)\end{aligned} \tag{4}$$

then Eqn. (3) is automatically satisfied, and (2) becomes

$$H_0(z)G_0(z) - H_0(-z)G_0(-z) = z^{-k} \tag{5}$$

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If we further assume that

$$G_0(z) = H_0(z) \tag{6}$$

then Eqn. (5) becomes

$$H_0^2(z) - H_0^2(-z) = z^{-k} \tag{7}$$

Under these circumstances, only one lowpass filter, H_0, needs to be designed.

In conventional QMF designs, the impulse responses of the filters are assumed to be symmetric so that they are of linear phase with group delay $(N-1)/2$, where N is the length of the filter. This results in a system with reconstruction delay fixed at $N-1$. Unlike the conventional QMF design, the proposed design method imposes no symmetry constraints on the impulse responses of the filters. From [6] we know that optimization methods can be used to design an FIR filter with group dealy less than $(N-1)/2$. Supposed that we have designed two lowpass linear-phase filters H_0 and G_0 with group delays k_1 and k_2, respectively, where k_1 and k_2 are less than $(N-1)/2$ and $k_1 + k_2$ is assumed to be an odd integer, then their frequency responses can be expressed as

$$\begin{aligned} H_0(e^{j\omega}) &= |H_0(e^{j\omega})|e^{-j\omega k_1} \\ G_0(e^{j\omega}) &= |G_0(e^{j\omega})|e^{-j\omega k_2} \end{aligned} \tag{8}$$

By replacing z in Eqn. (5) by $e^{j\omega}$ and substituting Eqn. (8) into Eqn. (5), we obtain

$$\begin{aligned} &[|H_0(e^{j\omega})||G_0(e^{j\omega})| + |H_0(e^{j(\omega+\pi)})||G_0(e^{j(\omega+\pi)})|] \\ &\cdot e^{-j\omega(k_1+k_2)} = e^{-j\omega k} \end{aligned} \tag{9}$$

Therefore, if the condition

$$|H_0(e^{j\omega})||G_0(e^{j\omega})| + |H_0(e^{j(\omega+\pi)})||G_0(e^{j(\omega+\pi)})| = 1 \tag{10}$$

is satisfied for all ω, the perfect reconstruction condition will be satisfied with system delay $k = k_1 + k_2 < N - 1$. If $G_0(z)$ is chosen to be the same as $H_0(z)$, then the condition in Eqn. (10) becomes

$$|H_0(e^{j\omega})|^2 + |H_0(e^{j(\omega+\pi)})|^2 = 1 \tag{11}$$

and the system delay is $2k_1 < N - 1$. In the above two cases, it is obvious that the system delay is less than that in a conventional QMF system, and it can be made *small* if both k_1 and k_2 are *small*.

We first consider a design method, where two filters H_0 and G_0 are designed. An objective function is formed as

$$E = \sum_{i=1}^{3} w_i E_i \tag{12}$$

where w_i, $i = 1, 2, 3$ are weights and E_i are error components. The first error component is

$$E_1 = \sum_{\omega} |H_0(e^{j\omega})G_0(e^{j\omega}) - H_0(e^{j(\omega+\pi)})G_0(e^{j(\omega+\pi)}) - e^{-j\omega k}|^2$$

which deals with the perfect reconstruction condition. The other two error components are given by

$$E_2 = \sum_{\omega} |H_0(e^{j\omega}) - H_I(e^{j\omega})|^2$$

$$E_3 = \sum_{\omega} |G_0(e^{j\omega}) - G_I(e^{j\omega})|^2$$

where $H_I(e^{j\omega})$ and $G_I(e^{j\omega})$ are the frequency responses of two ideal lowpass filters with group delays k_1 and k_2, respectively. The lowpass filters H_0 and G_0 are designed by minimizing E with respect to their coefficients. The transfer functions of two highpass analysis and synthesis filters, $H_1(z)$ and $G_1(z)$, can then be obtained from Eqn. (4) once $H_0(z)$ and $G_0(z)$ are determined.

An alterative possibility is to design only one filter H_0. An objective function can be formed as

$$E = \sum_{i=1}^{2} w_i E_i \tag{13}$$

with

$$\begin{aligned} E_1 &= \sum_{\omega} |H_0^2(e^{j\omega}) - H_0^2(e^{j(\omega+\pi)}) - e^{-j\omega k}|^2 \\ E_2 &= \sum_{\omega} |H_0(e^{j\omega}) - H_I(e^{j\omega})|^2 \end{aligned}$$

where $H_I(e^{j\omega})$ is the frequency response of an ideal lowpass filter with group delay k_1. Note that the second design case is a special case of the first one.

The minimization of the objective functions in the above design procedures can be achieved by optimization. We have employed a quasi-Newton optimization algorithm based on the BFGS updating formula and the inexact line search described in [6] to perform the optimization. The number of parameters is $2N$ for the first design method and N for the second design, where N is the length of the filter. A fortran program has been written to implement the algorithm.

III. Design Examples and Comparisons

The proposed method has been used to design two-band filter banks with short reconstruction delays. Here we describe two design examples. The first example is to demonstrate the first design method where two lowpass

TABLE I
SIGNAL-TO-RECONSTRUCTION NOISE RATIOS

	Sys. delay	SNR_{r1}(dB)	SNR_{r2}(dB)
Exp. 1	15	86.10	79.78
Exp. 2	9	78.87	75.99

filters are designed. A filter bank was designed with filter length $N = 32$ and system delay $k = 15$. First, one low-pass analysis H_0 and one synthesis filter G_0 are designed by minimizing the objective function defined in Eqn. (12). The highpass analysis and synthesis filters are obtained from the Eqn. (4). The amplitude responses of two analysis filters are illustrated in Fig. 2. The second example is to demonstrate the second design method , in which only one filter is designed. A filter bank was designed with filter length $N = 32$ and system delay $k = 9$. First the coefficients of filter H_0 are determined by minimizing the objective function given in Eqn. (13) and then three other analysis and synthesis filters are obtain through (4) and (6). The amplitude response and group delay characteristic of filter H_0 are shown in Fig. 3 and Fig. 4, respectively. It is observed that in the passband the delay characteristic is flat.

To check the perfect reconstruction quality of the designed filter bank, the signal-to-reconstruction noise ratio (SNR_r) in decibels which is defined as

$$\begin{aligned} SNR_r &= 10\log_{10}\left(\frac{\text{signal energy}}{\text{reconstruction noise energy}}\right) \\ &= 10\log_{10}\left\{\frac{\Sigma_n x^2(n)}{\Sigma_n[x(n)-\hat{x}(n+k)]^2}\right\} \qquad (14) \end{aligned}$$

was used as a measure of reconstruction performance, where $x(n)$ and $\hat{x}(n)$ are the input and output signals, and k is the system delay. The reconstruction performance is examined by calculating the SNR_r for two types of input signals, namely, a step input and a random input, which are denoted by SNR_{r1} and SNR_{r2}, respectively. The SNR_r ratios for the designed filter banks are listed in Table I. In the proposed design method, the perfect reconstrcution condition is achieved approximately by optimization. Note that from Table I, all the SNR_r ratios are over 75 dB, which should be considered good enough for many applications.

In [5], a time domain design algorithm is described for the design of FIR filter bank systems. The method can also be used to design low-delay filter banks. However, the design process involves calculating the pseudo inverse of large matrices, which is time-consuming. In addition the coefficients of the four analysis and synthesis filters are independent to each other, which results in more multiplications and additions in the implementation. In the design methods porposed here there is no need to calculate inverses of matrices. In addition, in the second design method only one filter which has independent coefficients is needed. This leads to improvement in the computational efficiency during the implementation of the system.

IV. CONCLUSION

A new approach for the design of two-band filter banks with low system delays has been presented. Two design examples have been included to illustrate the design procedure. Some comparisons have shown that improved designs realtive to those in [5] can be achieved. The results presented are preliminary and the method is expected to be improved as the efficiency of the optimization algorithm is improved.

Acknowledgement

The authors are grateful to Micronet, Networks of Centres of Excellence Program, for supporting this work.

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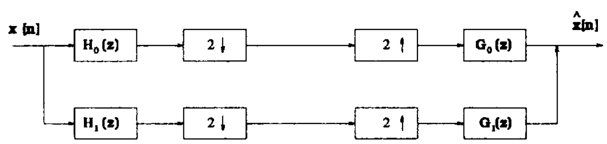

FIGURE 1
DIAGRAM OF TWO-BAND FIR FILTER BANK.

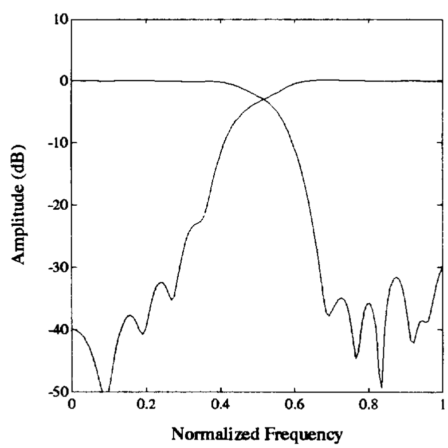

FIGURE 2
AMPLITUDE RESPONSES OF ANALYSIS FILTERS WITH $k = 15$.

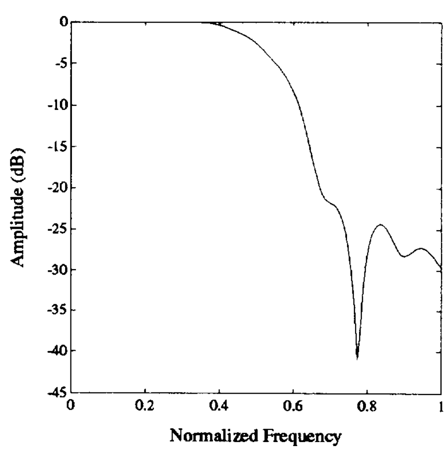

FIGURE 3
AMPLITUDE RESPONSE OF H_0 WITH $k = 9$.

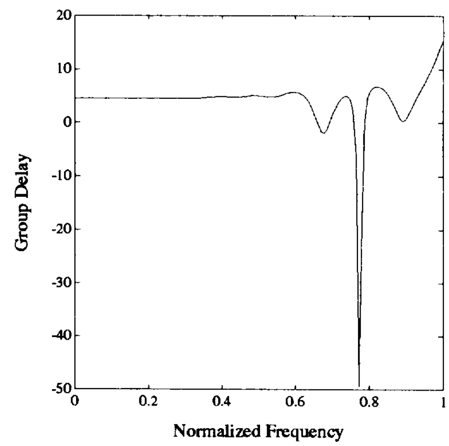

FIGURE 4
GROUP DELAY PLOT OF H_0 WITH $k = 9$.