Integer QP relaxation-based algorithms for intercarrier-interference reduction in OFDM systems

Algorithmes basés sur la programmation de relaxation quadratique de nombres entiers pour la réduction d'interférence inter-porteuse dans les systèmes OFDM

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Orthogonal frequency-division multiplexing (OFDM) modulation can be utilized to deal with severe channel conditions without complex equalization. However, in a fast-fading channel, Doppler spread caused by user mobility destroys the orthogonality among subcarriers, prompting intercarrier interference (ICI). In this paper, the OFDM ICI reduction problem is formulated as a combinatorial optimization problem. Two relaxation methods are proposed to relax the maximum-likelihood detection problem into convex quadratic programming (QP) problems. To further reduce computational complexity, the QP problems are solved by limiting the search to the two-dimensional subspace. A low-bit descent search can also be employed to improve the system performance. The extension to higher-order quadrature amplitude modulation (QAM) OFDM systems is also addressed. Performance results are given which demonstrate that the integer QP relaxation-based algorithms provide excellent performance with reasonable computational complexity.

La modulation à multiplexage de division orthogonale de fréquences (OFDM) peut être utilisée pour affronter de conditions sévères dans les canaux sans égalisation complexe. Cependant, dans un canal à évanouissement rapide, la diffusion Doppler provoquée par la mobilité des utilisateurs détruit l'orthogonalité parmi les sous porteuses, ce qui entraîne l'interférence inter-porteuse (ICI). Dans cet article, le problème de réduction d'OFDM ICI est formulé comme problème d'optimisation combinatoire. On propose deux méthodes de relaxation pour convertir le problème de détection à vraisemblance maximale vers des problèmes quadratiques convexes de programmation (QP). Pour réduire encore plus la complexité opérationnelle, les problèmes QP sont résolus en limitant la recherche au sous espace bidimensionnel. Une recherche à descente à peu de bits peut également être utilisée pour améliorer la performance du système. L'extension vers les systèmes d'ordre supérieur de modulation à quadrature d'amplitude (QAM) OFDM est également considérée. Les résultats de performances sont donnés, ce qui démontre que les algorithmes basés sur la relaxation à nombres entiers QP fournissent d'excellentes performances avec une complexité opérationnelle raisonnable.

Keywords: convex optimization; intercarrier interference; orthogonal frequency-division multiplexing (OFDM); quadratic programming; time-varying channels

I. Introduction

Orthogonal frequency-division multiplexing (OFDM) modulation is widely used in communication systems to meet the demand for ever-increasing data rates. The major advantage of OFDM over single-carrier transmission is its ability to deal with severe channel conditions without complex equalization. The standards employing OFDM modulation include digital video broadcasting (DVB) [1], digital audio broadcasting (DAB) [2], IEEE 802.11a and 802.11g [3] for wireless local area networks (WLAN), and IEEE 802.16 [4] for wireless metropolitan area networks (WMAN).

In an OFDM system, the data stream is divided into N parallel lower-rate data streams and is multiplexed onto a number of subcarriers using an inverse fast Fourier transform (IFFT). These subcarriers are overlapped orthogonally to provide bandwidth-efficient transmission. A cyclic prefix (CP) is inserted at the beginning of each OFDM symbol before transmission and is removed before demodulation, where the length of the cyclic prefix is greater than or equal to that of the channel impulse response to eliminate intersymbol interference (ISI). Generally, one-tap equalizers can be utilized in the frequency domain to cancel multipath distortion effectively over time-invariant channels [5].

OFDM is sensitive to Doppler spread caused by user mobility, and to phase noise caused by the frequency difference between the transmitter and the receiver [5]–[10], both of which result in the loss of orthogonality among subcarriers. This in turn leads to intercarrier interference (ICI) and degrades system performance. While it is straightforward to estimate and reduce the ICI induced by phase noise, the ICI introduced by Doppler spread is a more challenging problem. Various algorithms have been proposed to mitigate the ICI and improve system performance over time-varying channels [7]–[10]. In [7], Li and Cimini provide universal bounds on the ICI in an OFDM system over time-varying fading channels which can be evaluated and compared with the exact ICI. In [8], a block decision feedback equalizer (DFE) algorithm is described which utilizes signals from several neighbouring subcarriers to eliminate the ICI for a certain subcarrier. An ICI suppression algorithm using parallel cancellation with frequency-domain equalization techniques is presented in [9]. Two-stage prefilters and ICI reduction filters are utilized to achieve minimum mean-square er-

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ror (MMSE) equalization. However, it is assumed that the channel varies linearly during one symbol duration. Furthermore, Stamoulis et al. [10] derived linear time-varying filters in multiple-input multiple-output (MIMO) OFDM systems that maximize the ratio of signal energy to ICI-plus-noise energy.

Based on the maximum-likelihood (ML) criterion, the OFDM ICI reduction problem can be formulated as a combinatorial optimization problem with integer constraints. It has been shown that in the multi-user detection of direct-sequence code-division multiple access (DS-CDMA) systems [11], such a problem can be solved more efficiently by using suboptimal detectors. In this paper, two relaxation methods are utilized to convert the ICI reduction problem into convex quadratic programming (QP) problems. To further reduce the computational complexity, the QP problems can be solved by limiting the search to the two-dimensional subspace spanned by its steepest-descent and Newton directions. A low-bit descent search (LBDS) can also be employed to improve the system performance. Furthermore, the proposed algorithms are extended to higher-order quadrature amplitude modulation (QAM) OFDM systems, and an iterative detection process is utilized. Performance results demonstrate that the integer QP relaxation-based algorithms provide excellent performance with reasonable computational complexity.

The rest of the paper is organized as follows. The OFDM system model is presented in Section II. Section III describes the proposed integer QP relaxation-based algorithms for ICI reduction in a 4-QAM OFDM system. The extension of the proposed algorithms to higher-order QAM OFDM systems is addressed in Section IV. Simulations are carried out and the results are described in Section V. Finally, some conclusions are given in Section VI.

II. System model

In an OFDM system, the system bandwidth is divided into N subchannels, and the data stream is modulated on the subcarriers using QAM or phase-shift keying (PSK). The transmitted signal is generated using an IFFT,

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k \exp\left(\frac{j2\pi kn}{N}\right) \quad \text{for} \quad n = 0, \ldots, \; N-1, \quad (1)$$

where x_n is the time-domain signal at the n-th sampling instant and X_k is the frequency-domain data symbol for the k-th subcarrier. Equation (1) can be written in vector form as

$$\mathbf{x} = \mathbf{F}\mathbf{X}, \quad (2)$$

where $\mathbf{x} = [x_0 \; x_1 \; \cdots \; x_{N-1}]^T$ and $\mathbf{X} = [X_0 \; X_1 \; \cdots \; X_{N-1}]^T$ represent the time-domain and frequency-domain OFDM symbols, respectively, and $\mathbf{F}$ is the IFFT matrix with elements $f_{n,k} = (1/\sqrt{N}) \exp(j2\pi kn/N)$. The OFDM symbol duration is denoted by T_s, so the chip duration of each subchannel is $T_c = T_s/N$. The basic structure of an OFDM transmitter is depicted in Fig. 1.

In this paper, we adopt a doubly frequency-selective fading channel model [8]. Thus, we have a wide-sense stationary uncorrelated scattering (WSSUS) channel with impulse response given by

$$h(t;\tau) = \sum_{d=1}^{D} h(t;\tau_d)\delta(\tau - \tau_d), \quad (3)$$

where τ_d is the d-th path delay with $\tau_1 < \tau_2 < \cdots < \tau_D$. In a rich scattering environment, the channel autocorrelation function is separable in terms of time and delay, i.e., $\phi_h(\triangle t;\tau) = \phi_t(\triangle t)\phi_\tau(\tau)$, where $\phi_t(\triangle t)$ is the time-correlation function based on Jakes' model

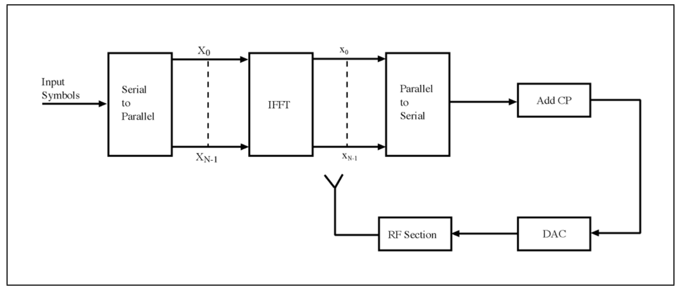

Figure 1: *The basic structure of an OFDM transmitter.*

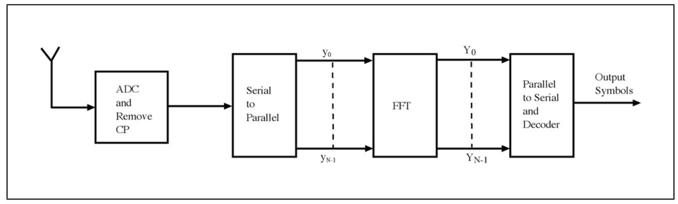

Figure 2: *The basic structure of an OFDM receiver.*

and $\phi_\tau(\tau)$ is the multipath intensity profile [12]. In (3), $h(t;\tau_d)$ is a complex Gaussian process with zero mean and variance $\sigma_d^2 \triangleq \phi_\tau(\tau_d)$.

A discrete version of the WSSUS channel in (3) can be modelled as a tapped delay line (TDL) [13]:

$$h(n;l) = \sum_{d=1}^{D} h(nT_c;\tau_d) \; \mathrm{sinc}\left(\frac{\tau_d}{T_c} - l\right), \quad (4)$$

where $h(n;l)$ denotes the channel coefficient for the l-th tap at the n-th sampling instant; $n = 0, \ldots, N-1$; $l = 0, \ldots, L-1$, where $L = \lfloor \tau_D/T_c \rfloor + 1$; and the delay between two taps is T_c.

Thus, the discrete received signal at the n-th sampling instant can be expressed as

$$y_n = \sum_{l=0}^{L-1} h(n,l)x(n-l) + w_n \quad \text{for} \quad n = -N_p, \ldots, N-1, \quad (5)$$

where N_p is the length of the CP added to the OFDM symbol and w_n denotes additive white Gaussian noise (AWGN) at the n-th sampling instant with zero mean and variance σ^2. Since the CP is only a copy of part of OFDM symbol $\mathbf{x}$, after the removal of the CP, (5) can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}, \quad (6)$$

where $\mathbf{y}$ and $\mathbf{w}$ denote the time-domain received signal and AWGN noise, respectively, and $\mathbf{H}$ is the channel matrix given by

$$\mathbf{H} = \begin{bmatrix} h(0,0) & 0 & \ldots & h(0,1) \\ h(1,1) & h(1,0) & \ldots & h(1,2) \\ \vdots & \vdots & \ddots & \vdots \\ h(L-1,L-1) & h(L-1,L-2) & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & h(N-1,0) \end{bmatrix}.$$

By performing a fast Fourier transform (FFT), we obtain

$$\mathbf{Y} = \mathbf{A}\mathbf{X} + \mathbf{W}, \quad (7)$$

where $\mathbf{Y} = [Y_0 \; \cdots \; Y_{N-1}]^T$ is the frequency-domain received signal, $\mathbf{A} = \mathbf{F}^H\mathbf{H}\mathbf{F}$, and $\mathbf{W} = \mathbf{F}^H\mathbf{w}$. The basic structure of an OFDM receiver is shown in Fig. 2.

If $h(t;\tau_d)$ in (3) remains constant within one OFDM symbol duration, then matrix $\mathbf{A}$ in (7) is a diagonal matrix, and no ICI will occur.

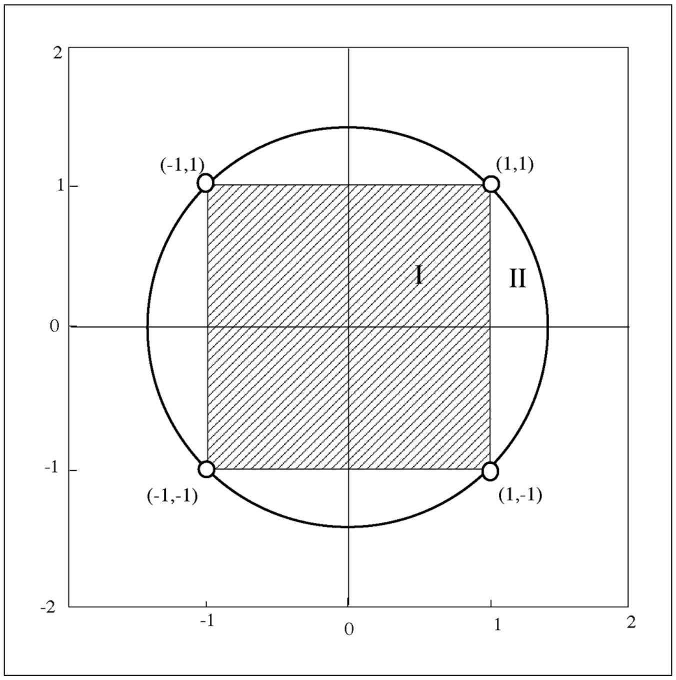

Figure 3: *The feasible set defined by (11.b) (points on the circle), the feasible region defined by (12.b) (I), and the feasible region defined by (13.b) (I+II).*

Conversely, if the channel varies within one OFDM symbol, the orthogonality of the subcarriers does not hold and the received signal at a particular subcarrier depends not only on the transmitted signal at that subcarrier, but also on the transmitted signals from other subcarriers [14]. In this case, the received signal on the k-th subcarrier is

$$Y_k = A_{k,k}X_k + \sum_{m=0,m\neq k}^{N-1} A_{k,m}X_m + W_k, \tag{8}$$

where $k = 0, \ldots, N-1$; $A_{k,m}$ denotes the (k,m)-th element of $\mathbf{A}$; and $\sum_{m=0,m\neq k}^{N-1} A_{k,m}X_m$ represents the ICI caused by other subcarriers.

III. Problem formulation and relaxation

Based on the ML detection criterion, the ICI reduction problem in OFDM systems can be formulated as the optimization problem

$$\text{minimize } \|\mathbf{Y} - \mathbf{AX}\|_2^2 \tag{9.a}$$
$$\text{subject to } X_k \in \mathcal{M} \quad \text{for} \quad k = 0, 1, \ldots, N-1, \tag{9.b}$$

where $\mathcal{M}$ contains the constellation points according to the modulation being used.

The variables in (9) are complex-valued. If we define $\mathbf{Y} = \mathbf{Y}_r + \mathbf{j}\mathbf{Y}_i$, $\mathbf{A} = \mathbf{A}_r + \mathbf{j}\mathbf{A}_i$, and $\mathbf{X} = \mathbf{X}_r + \mathbf{j}\mathbf{X}_i$, then (9) can be reformulated into an optimization problem with real-valued variables as

$$\text{minimize } \|\hat{\mathbf{Y}} - \hat{\mathbf{A}}\mathbf{z}\|_2^2 \tag{10.a}$$
$$\text{subject to } z_k \in \hat{\mathcal{M}} \quad \text{for} \quad k = 0, 1, \ldots, N-1, \tag{10.b}$$

where

$$\hat{\mathbf{Y}} = \begin{bmatrix} \mathbf{Y}_r \\ \mathbf{Y}_i \end{bmatrix}, \quad \mathbf{z} = \begin{bmatrix} \mathbf{X}_r \\ \mathbf{X}_i \end{bmatrix}, \quad \hat{\mathbf{A}} = \begin{bmatrix} \mathbf{A}_r & -\mathbf{A}_i \\ \mathbf{A}_i & \mathbf{A}_r \end{bmatrix}.$$

In this section, the OFDM system is assumed to employ 4-QAM modulation, which corresponds to $\hat{\mathcal{M}} = \{\pm 1\}$. Clearly, (10) is a quadratic optimization problem with discrete variables and can be expressed as

$$\text{minimize } \mathbf{z}^T\mathbf{Q}\mathbf{z} + \mathbf{q}^T\mathbf{z} \tag{11.a}$$
$$\text{subject to } z_k = \{-1,\ 1\} \quad \text{for} \quad k = 0, \ldots, 2N-1, \tag{11.b}$$

where $\mathbf{Q} = \hat{\mathbf{A}}^T\hat{\mathbf{A}}$ and $\mathbf{q} = -2\hat{\mathbf{A}}^T\hat{\mathbf{Y}}$.

A. Convex relaxation

Since the vector $\mathbf{z}$ in (11) is a discrete set, we have a combinatorial problem with exponential computational complexity that becomes prohibitive even for a moderate number of variables. It has been shown [11] that this type of ML detection problem can be solved more efficiently by expanding the discrete feasible set into a continuous and convex feasible region. In this paper, two convex relaxation methods are utilized that allow us to consider convex QP problems that admit a fast solution which yields good performance. The first QP problem minimizes a convex quadratic objective function, subject to the condition that the solution be contained within an n-dimensional box centred at the origin. The second QP problem minimizes the same objective function, subject to the condition that the solution be contained within an n-dimensional ball.

1. Bounded constraint relaxation

The discrete constraints in (11.b) imply that $-1 \leq z_k \leq 1$ for $k = 0, \ldots, 2N-1$. Thus the ICI reduction problem (11) can be relaxed into the bounded constraint optimization problem

$$\text{minimize } \mathbf{z}^T\mathbf{Q}\mathbf{z} + \mathbf{q}^T\mathbf{z} \tag{12.a}$$
$$\text{subject to } -1 \leq z_k \leq 1 \quad \text{for} \quad k = 0, \ldots, 2N-1. \tag{12.b}$$

The feasible region in (12.b) is an n-dimensional hypercube centred at the origin with linear constraints. Thus, (12) is a convex QP problem which can be solved efficiently to provide suboptimal performance to that of (11).

2. Quadratic convex relaxation

The constraints in (11.b) imply that $\mathbf{z}^T\mathbf{z} = 2N$, which is associated with the feasible region of a $2N$-dimensional ball centred at the origin with radius $\sqrt{2N}$. If we expand such a feasible region within the ball, the ICI reduction problem (11) is relaxed into the problem

$$\text{minimize } \mathbf{z}^T\mathbf{Q}\mathbf{z} + \mathbf{q}^T\mathbf{z} \tag{13.a}$$
$$\text{subject to } \mathbf{z}^T\mathbf{z} \leq 2N. \tag{13.b}$$

Problem (13) seeks to minimize a quadratic objective function over a convex feasible region. Thus, it is a convex QP minimization problem. A unique global solution can be obtained using efficient interior-point QP solvers with reduced computational complexity.

The feasible regions of (11), (12), and (13) are depicted in Fig. 3. Efficient optimization algorithms are available in the literature [15] to solve the minimization problems (12) and (13). Once the solution $\mathbf{z}^*$ of (12) or (13) is obtained, the solution of (11) can be approximated as $\text{sign}(\mathbf{z}^*)$.

B. Two-dimensional search method

To further reduce computational complexity, the solutions of (12) or (13) can be obtained by limiting the search to the two-dimensional subspace spanned by its steepest-descent direction (i.e., the negative gradient of the objective function) and the Newton direction. To do so, we set

$$\mathbf{z} = \eta_1\mathbf{v}_1 + \eta_2\mathbf{v}_2, \tag{14}$$

where $\mathbf{v}_1 = \mathbf{q}$, $\mathbf{v}_2 = \mathbf{Q}^{-1}\mathbf{q}$, and η_1, η_2 are two scalar variables. Then (12) is converted into the two-dimensional problem

$$\text{minimize } \boldsymbol{\eta}^T\mathbf{S}\boldsymbol{\eta} + \mathbf{p}^T\boldsymbol{\eta} \tag{15.a}$$
$$\text{subject to } -1 \leq V_k\boldsymbol{\eta} \leq 1, \tag{15.b}$$

where $\boldsymbol{\eta} = [\eta_1\eta_2]^T$, $\mathbf{S} = \mathbf{V}^T\mathbf{Q}\mathbf{V}$, $\mathbf{p} = \mathbf{V}^T\mathbf{q}$, V_k is the k-th row of the matrix $\mathbf{V}$, and $\mathbf{V} = [\mathbf{v}_1\mathbf{v}_2]$.

Similarly, (13) can be reformulated into the two-dimensional problem

$$\text{minimize } \boldsymbol{\eta}^T \mathbf{S}\boldsymbol{\eta} + \mathbf{p}^T\boldsymbol{\eta} \tag{16.a}$$
$$\text{subject to } \boldsymbol{\eta}^T\mathbf{R}\boldsymbol{\eta} \le 2N, \tag{16.b}$$

where $\mathbf{R} = \mathbf{V}^T\mathbf{V}$. If we denote the solution of (15) or (16) as $\boldsymbol{\eta}^*$, then the solution $\mathbf{z}^*$ of (12) or (13) can be calculated using (14) accordingly, and $\text{sign}(\mathbf{z}^*)$ is then taken as the solution of (11).

C. Performance enhancement by low-bit descent search

In LBDS, a given binary sequence is associated with an objective function to be minimized. The search process evaluates, compares, and determines the optimal sign switches of a relatively small number of sequence components to yield maximum reduction in the objective function in (10). LBDS has been applied recently to various problems [16]. As will be demonstrated, the performance of the proposed algorithm can be considerably enhanced using 1-bit, 2-bit, or a combined 1-bit-and-2-bit LBDS, at the cost of an insignificant increase in computational complexity.

From [16], the one-bit descent search can be carried out by evaluating $\mathbf{z} \odot \boldsymbol{\xi}$ (here $\odot$ denotes component-wise multiplication), where $\boldsymbol{\xi} = \tilde{\mathbf{Q}}\mathbf{z} + \mathbf{q}/2$ and $\tilde{\mathbf{Q}}$ is generated from $\mathbf{Q}$ with its diagonal components set to zero. Index k^* is then identified as the case where the corresponding component ξ_{k^*} has maximum value, and the sign of z_{k^*} is switched to obtain an improved solution. Similarly, a 2-bit LBDS is performed by computing $\mathbf{G} = \boldsymbol{\xi}e^T + \mathbf{e}^T\boldsymbol{\xi} - 2\mathbf{Q} \odot (\mathbf{z}\mathbf{z}^T)$, where $\mathbf{e}$ is the all-one vector. The index (k^*, m^*) is identified as the case where the component P_{k^*,m^*} reaches its maximum value, and an improved solution is then obtained by switching the signs of the k^*-th and m^*-th components of $\mathbf{z}^*$.

IV. Extension to higher-order QAM OFDM systems

With minor modifications, the proposed algorithms can be readily extended to higher-order M-QAM OFDM systems. For example, for a 16-QAM OFDM system, the complex-valued ICI reduction problem (9) can be reformulated as a real-valued problem, i.e.,

$$\text{minimize } \hat{\mathbf{X}}^T\hat{\mathbf{Q}}\hat{\mathbf{X}} + \hat{\mathbf{q}}^T\hat{\mathbf{X}} \tag{17.a}$$
$$\text{subject to } \hat{X}_k = \{\pm1, \pm3\} \quad \text{for} \quad k = 0, \ldots, 2N-1, \tag{17.b}$$

where $\hat{\mathbf{Q}} = \hat{\mathbf{A}}^T\hat{\mathbf{A}}$, $\hat{\mathbf{q}} = -2\hat{\mathbf{A}}^T\hat{\mathbf{Y}}$, and $\hat{\mathbf{A}}$, $\hat{\mathbf{Y}}$ are as given in (10). The variable set in (17) can be characterized as

$$\hat{\mathbf{X}} = 2\boldsymbol{\alpha} + \boldsymbol{\beta}, \tag{18}$$

where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $2N$-dimensional vectors with components α_k and $\beta_k \in \{-1, 1\}$ for $k = 0, \ldots, 2N-1$. Problem (17) then assumes the form

$$\text{minimize } \mathbf{z}^T\mathbf{Q}\mathbf{z} + \mathbf{q}^T\mathbf{z} \tag{19.a}$$
$$\text{subject to } z_k = \{-1, 1\} \quad \text{for} \quad k = 0, \ldots, 4N-1, \tag{19.b}$$

where

$$\mathbf{z} = \begin{bmatrix}\boldsymbol{\alpha}\\ \boldsymbol{\beta}\end{bmatrix}, \quad \mathbf{Q} = \begin{bmatrix}4\hat{\mathbf{Q}} & 2\hat{\mathbf{Q}}\\ 2\hat{\mathbf{Q}} & \hat{\mathbf{Q}}\end{bmatrix}, \quad \mathbf{q} = \begin{bmatrix}2\hat{\mathbf{q}}\\ \hat{\mathbf{q}}\end{bmatrix}.$$

Similarly, for the case of 64-QAM OFDM systems, the ICI reduction problem can be formulated as a real-valued optimization problem:

$$\text{minimize } \hat{\mathbf{X}}^T\hat{\mathbf{Q}}\hat{\mathbf{X}} + \hat{\mathbf{q}}^T\hat{\mathbf{X}} \tag{20.a}$$
$$\text{subject to } \hat{X}_k = \{\pm1, \pm3, \pm5, \pm7\} \quad \text{for} \quad k = 0, \ldots, 2N-1, \tag{20.b}$$

where $\hat{\mathbf{Q}} = \hat{\mathbf{A}}^T\hat{\mathbf{A}}$ and $\hat{\mathbf{q}} = -2\hat{\mathbf{A}}^T\hat{\mathbf{Y}}$. The variable set in (20) can be characterized as

$$\hat{\mathbf{X}} = 4\boldsymbol{\alpha} + 2\boldsymbol{\beta} + \boldsymbol{\gamma}, \tag{21}$$

where $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, and $\boldsymbol{\gamma}$ are $2N$-dimensional vectors with components α_k, β_k, and $\gamma_k \in \{-1, 1\}$ for $k = 0, \ldots, 2N-1$. Problem (20) then assumes the form

$$\text{minimize } \mathbf{z}^T\mathbf{Q}\mathbf{z} + \mathbf{q}^T\mathbf{z} \tag{22.a}$$
$$\text{subject to } z_k = \{-1, 1\} \quad \text{for} \quad k = 0, \ldots, 6N-1, \tag{22.b}$$

where

$$\mathbf{z} = \begin{bmatrix}\boldsymbol{\alpha}\\ \boldsymbol{\beta}\\ \boldsymbol{\gamma}\end{bmatrix}, \quad \mathbf{Q} = \begin{bmatrix}16\hat{\mathbf{Q}} & 8\hat{\mathbf{Q}} & 4\hat{\mathbf{Q}}\\ 8\hat{\mathbf{Q}} & 4\hat{\mathbf{Q}} & 2\hat{\mathbf{Q}}\\ 4\hat{\mathbf{Q}} & 2\hat{\mathbf{Q}} & \hat{\mathbf{Q}}\end{bmatrix}, \quad \mathbf{q} = \begin{bmatrix}4\hat{\mathbf{q}}\\ 2\hat{\mathbf{q}}\\ \hat{\mathbf{q}}\end{bmatrix}.$$

Problems (19) and (22) can be solved in a recursive manner to improve system performance, in which case only some binary components of $\mathbf{z}$ in (19) or (22) are determined in each iteration by solving a corresponding combinatorial problem of type (11). Algorithmic details of a given, say the i-th, iteration are described as follows. Suppose that prior to the i-th iteration, several binary components of vector $\mathbf{z}$ have already been determined. Let $\mathbf{z}_i$ be the reduced-size vector that collects all undecided components of $\mathbf{z}$, let Ω_i be the index set corresponding to $\mathbf{z}_i$, and let N_i be the size of $\mathbf{z}_i$. By substituting the known binary components of $\mathbf{z}$ into (19) or (22), a reduced-size problem similar to (19) or (22) is obtained as

$$\text{minimize } \mathbf{z}_i^T\mathbf{Q}_i\mathbf{z}_i + \mathbf{q}_i^T\mathbf{z}_i \tag{23.a}$$
$$\text{subject to } z_k^{(i)} = \{-1, 1\} \quad \text{for} \quad k = 0, \ldots, N_i, \tag{23.b}$$

where $z_k^{(i)}$ denotes the k-th component in the reduced-size vector $\mathbf{z}_i$. The relaxation and solution techniques described in Sections III.A to III.C can then be applied with straightforward modifications.

Next, the magnitudes of the components of $\mathbf{z}_i^*$ are examined. If $|z_k^{*(i)}|$ exceeds a given threshold ρ, the corresponding variable is detected as $\text{sign}(z_k^{*(i)})$; otherwise component $z_k^{*(i)}$ remains undetermined and will be considered as a design variable in the next iteration. The components just detected are then used in (23) to produce a similar QP problem of reduced size, where the vector $\mathbf{z}_i$ contains only the undecided variables. This iterative process continues until all the variables have been identified to produce an estimate of the transmitted data.

Note that in the first iteration, $\mathbf{Q}_i$ is the entire matrix $\mathbf{Q}$, and $\mathbf{Q}$ is merely positive semidefinite (see (19) and (22)). Thus problem (23) cannot be solved directly using a two-dimensional search method. This difficulty can be readily fixed by adding $\epsilon\mathbf{I}$ with a small ϵ greater than 0 to $\mathbf{Q}$ so that the slightly modified $\mathbf{Q} + \epsilon\mathbf{I}$ becomes positive definite and thus nonsingular. This modification does not affect the solution because the modification amounts to changing the objective function in (23.a) to $\mathbf{z}_i^T(\mathbf{Q}_i + \epsilon\mathbf{I})\mathbf{z}_i + \mathbf{q}_i^T\mathbf{z}_i$, which in conjunction with the constraint in (23.b) equals $\mathbf{z}_i^T\mathbf{Q}_i\mathbf{z}_i + \mathbf{q}_i^T\mathbf{z}_i + N_i\epsilon$, and adding a constant to the objective function does not alter the solution. As the iterations continue, matrix $\mathbf{Q}_i$ may or may not be singular; the technique outlined above can be used in case $\mathbf{Q}_i$ is singular.

V. Performance evaluation

The proposed algorithms were applied to an OFDM system with $N = 64$ subcarriers and a system bandwidth of 200 kHz. The length of the cyclic prefix was chosen to be $N_p = N/8$. A two-ray WSSUS fading channel was employed, where each path is an independent complex Gaussian random process with Jakes' Doppler spectrum [13]. The delay of the first path was set to zero, and the delay of the second path was randomly generated with a uniform distribution from $\{T_c, \ldots, N_pT_c\}$. The normalized Doppler frequency of the channel

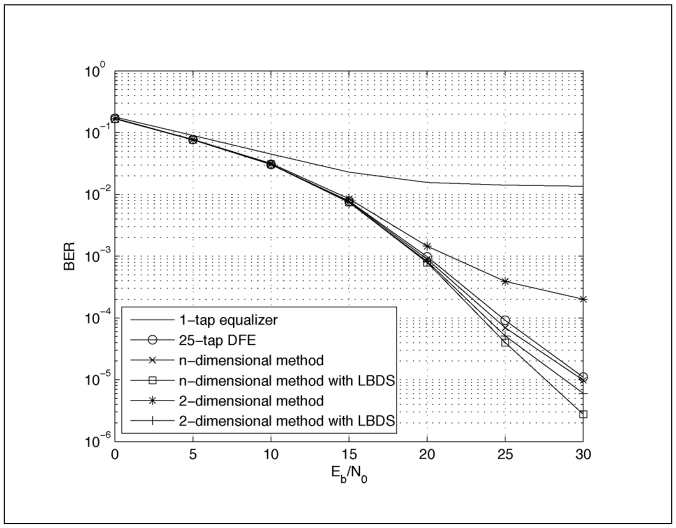

Figure 4: *BER performance of the bounded constraint relaxation method with $f_dT_s = 0.1$ in a 4-QAM OFDM system.*

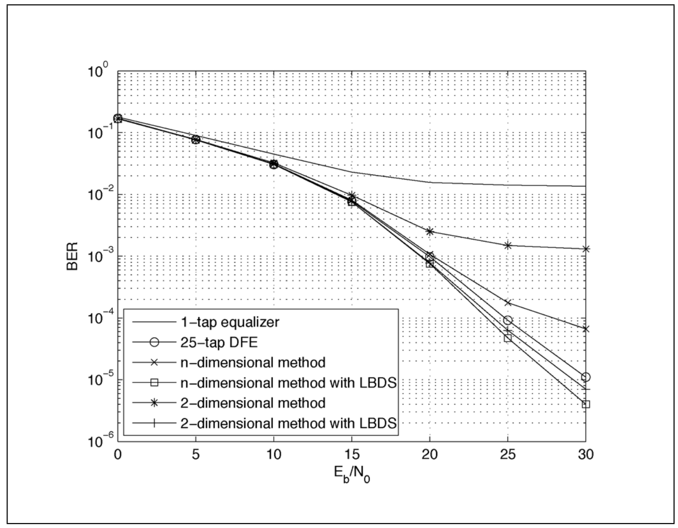

Figure 5: *BER performance of the quadratic constraint relaxation method with $f_dT_s = 0.1$ in a 4-QAM OFDM system.*

is f_dT_s. Simulations were carried out to evaluate the performance in terms of bit error rate (BER) and computational complexity. The BER performances of a conventional one-tap equalizer and a 25-tap DFE [8] are provided for comparison purposes. Perfect channel information is assumed, and combined 1-bit-and-2-bit LBDS was adopted to improve system performance of the proposed algorithms. The algorithms were implemented using the MATLAB SeDuMi toolbox [17].

A. Proposed algorithms in a 4-QAM OFDM system

First, we examine the performance of the proposed integer QP relaxation-based ICI reduction algorithms in a 4-QAM–modulation OFDM system.

The BER performance of an OFDM system with $f_dT_s = 0.1$ and bounded constraint relaxation is shown in Fig. 4. The performance results with a one-tap equalizer and a 25-tap DFE [8] are also given for comparison. It can be observed that the one-tap equalizer provides unsatisfactory performance in time-varying channels, but the bounded constraint relaxation methods considerably mitigate the intercarrier interference. The performance can be further improved by employing the LBDS method. Both the n-dimensional and two-dimensional algorithms offer superior performance to that with the DFE, but with higher computational complexity. Because the solution of (15) is an approximation to that of (12), the n-dimensional algorithm outperforms the two-dimensional algorithm. However, it is more complex. For example, at an E_b/N_0 of 25 dB, the DFE has a BER of 9×10^{-5}, while the two-dimensional bounded constraint relaxation algorithm with LBDS has a BER of 5×10^{-5} (with a 20% increase in CPU running time). The n-dimensional algorithm has a BER of 2.5×10^{-5} with LBDS (with a 40% increase in CPU running time).

The BER performance of the quadratic convex relaxation algorithms is given in Fig. 5. It can be observed that the performance results are better than those for the one-tap equalizer and DFE. However, the quadratic convex relaxation algorithms performed slightly worse than the bounded constraint algorithms. This is because the optimization problem in (13) can be obtained by relaxing (12), so one would expect the bounded constraint relaxation algorithm to offer superior performance, but with a slightly higher computational complexity. For example, for $E_b/N_0 = 25$ dB, the two-dimensional quadratic convex relaxation algorithm with LBDS has a BER of 7×10^{-5} (with an 18% increase in CPU running time compared to the DFE case), whereas the n-dimensional algorithm with LBDS offers a BER of 4.5×10^{-5} (with a 35% increase in CPU running time).

B. Proposed algorithms in higher-order QAM OFDM systems

The proposed algorithms with iterative detection were also employed in a 16-QAM OFDM system to evaluate the performance. For the sake of simplicity, only bounded constraint relaxation performance is presented, as quadratic convex relaxation provides similar performance, as in the 4-QAM OFDM case. Fig. 6 shows that the bounded constraint relaxation method outperforms the 25-tap DEF and provides significant improvement over the performance of the one-tap equalizer. The n-dimensional method offers better performance than the two-dimensional method at the price of higher computational complexity.

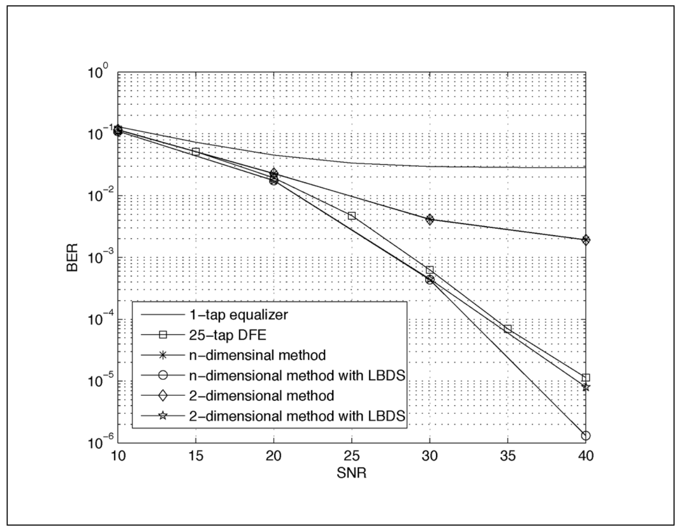

Figure 6: *BER performance of the bounded constraint relaxation method with $\rho = 0.5$ and $f_dT_s = 0.1$ in a 16-QAM OFDM system.*

As iterative detection is utilized, a larger threshold ρ will provide better performance at the cost of increased complexity, as shown in Fig. 7. The proposed two-dimensional bounded constraint relaxation method exhibits an error floor at high SNR, but this can be effectively suppressed by performing LBDS with a slightly increased computational complexity. For the 16-QAM OFDM system with Doppler spread $f_dT_s = 0.1$, the algorithm with $\rho = 0.8$ requires 35% more CPU time than that with $\rho = 0.5$ at 35 dB to achieve a BER of 9×10^{-4} before LBDS is employed. This result can be improved to 4×10^{-5} by performing LBDS with 26% more CPU time than is required with $\rho = 0.5$.

A small constant was added to matrix $\mathbf{Q}$ to make it nonsingular. The system performance for constants $\epsilon = 0.1$ and $\epsilon = 10^{-6}$ is shown in Fig. 8. Although the performance is quite close for both constants

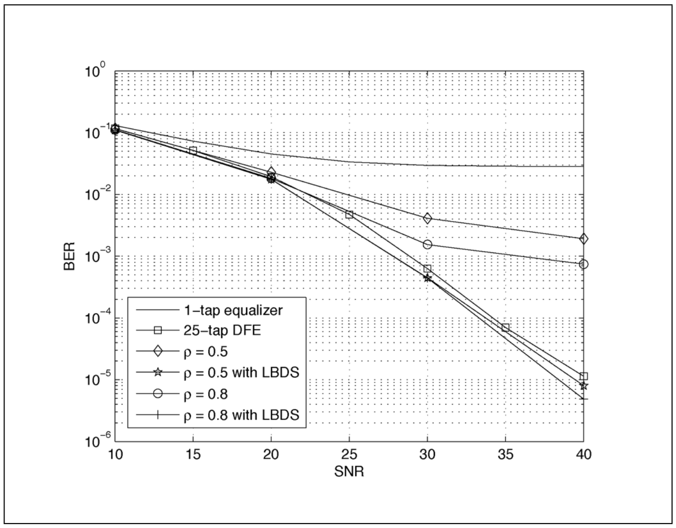

Figure 7: *BER performance of the two-dimensional bounded constraint relaxation method with various thresholds and $f_dT_s = 0.1$ in a 16-QAM OFDM system.*

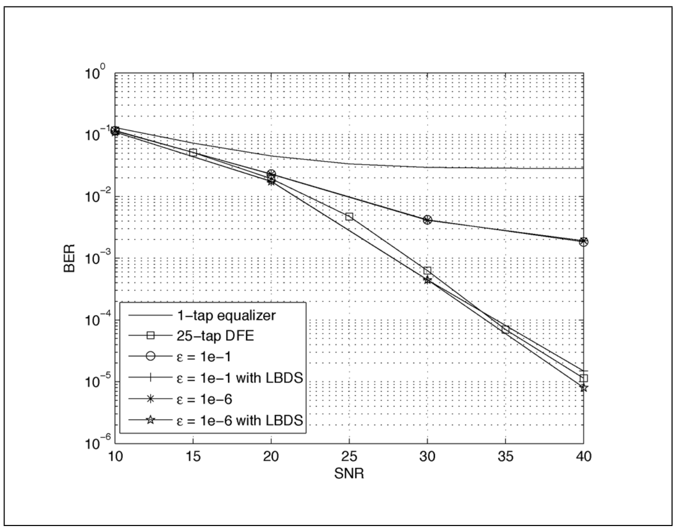

Figure 8: *BER performance of the bounded constraint relaxation method with $f_dT_s = 0.1$ for various values of ϵ in a 16-QAM OFDM system.*

before LBDS is employed, the smaller constant provides better performance at high SNR with LBDS. For example, in a 16-QAM OFDM system with $f_dT_s = 0.1$, the two-dimensional bounded constraint relaxation method with $\epsilon = 0.1$ achieves a BER of 8×10^{-5} at $35\,\text{dB}$, which can be improved to 5×10^{-5} with a smaller constant $\epsilon = 10^{-6}$.

C. The effect of Doppler spread

Simulations were also carried out to determine the impact of normalized Doppler spread f_dT_s on performance. The BER of the two-dimensional bounded constraint relaxation algorithm for $f_dT_s = 0.05$, 0.1, and 0.3 in OFDM systems with different modulation schemes is plotted in Figs. 9–11. It can be observed that the performance of the two-dimensional bounded constraint relaxation algorithm degrades as the Doppler spread increases, while time diversity can be achieved by adding the LBDS method. For example, in a 4-QAM OFDM system, an E_b/N_0 of $25\,\text{dB}$ is required to achieve a BER of 10^{-4} for $f_dT_s = 0.05$ with LBDS, whereas with $f_dT_s = 0.1$, an E_b/N_0 of $24\,\text{dB}$ is required to achieve the same BER. The E_b/N_0 required to obtain the same BER drops to $22.5\,\text{dB}$ for $f_dT_s = 0.3$. This improvement with increasing f_dT_s can be attributed to the increased temporal diversity introduced by the larger Doppler spread [18]. Similar diversity gain can also be realized by employing the quadratic convex relaxation algorithm.

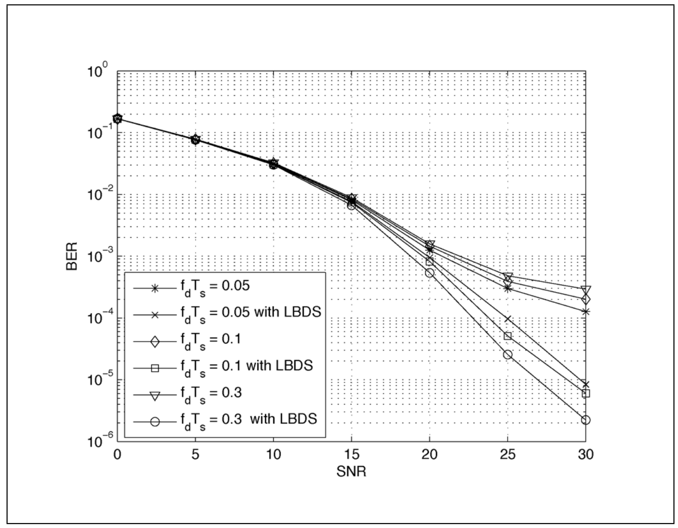

Figure 9: *BER performance of the two-dimensional bounded constraint relaxation method with various Doppler spreads in a 4-QAM OFDM system.*

VI. Conclusions

In this paper, the OFDM ICI reduction problem was first formulated as a combinatorial optimization problem with integer constraints. Two relaxation methods were then utilized to convert the discrete ML detection problem into convex QP problems. To further reduce the computational complexity, the QP problems were solved by limiting the search to a two-dimensional subspace. An LBDS method was employed to improve the system performance with slightly increased computational complexity. The proposed algorithms could also be employed in higher-order OFDM systems with minor modifications, and iterative detection can be used to improve performance at the cost of higher complexity. Simulations were carried out to examine the performance of the proposed ICI reduction algorithms. The results demonstrated that the integer QP relaxation-based algorithms provide excellent performance with reasonable computational complexity and that temporal diversity can be achieved with increased Doppler spread.

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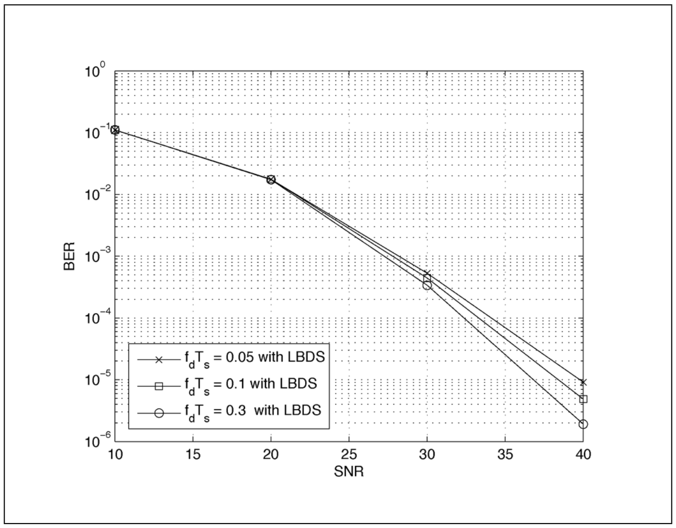

Figure 10: *BER performance of the two-dimensional bounded constraint relaxation method with various Doppler spreads in a 16-QAM OFDM system.*

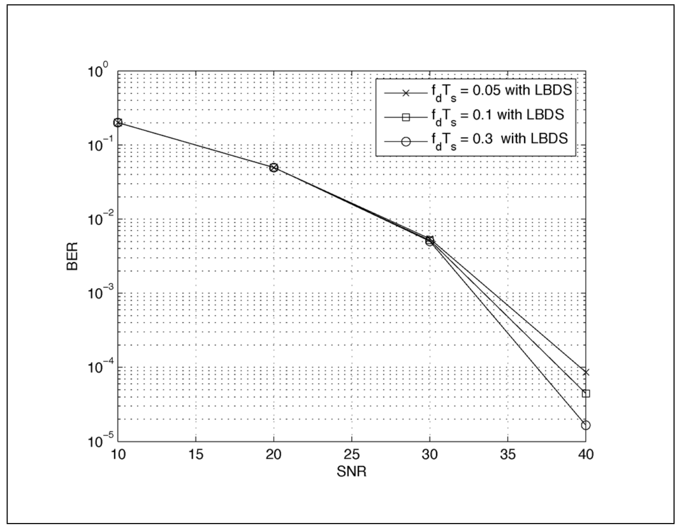

Figure 11: *BER performance of the two-dimensional bounded constraint relaxation method with various Doppler spreads in a 64-QAM OFDM system.*

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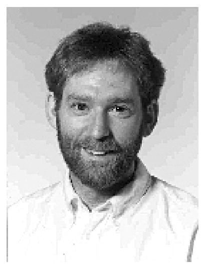

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