# **Constrained Minimum-BER Multiuser Detection**

Xiaofeng Wang, Wu-Sheng Lu, Fellow, IEEE, and Andreas Antoniou, Fellow, IEEE

Abstract—A new linear multiuser detector that directly minimizes the bit-error rate (BER) subject to a set of reasonable constraints is proposed. It is shown that the constrained BER cost function has a unique global minimum. This allows us to develop an efficient barrier Newton method for finding the coefficients of the proposed detector using information about timing, amplitudes, channels, and the signature signals of all users. Although the new detector cannot be shown to be optimal among linear multiuser detectors without the constraints imposed, extensive simulations demonstrate that it achieves the lowest BER. Furthermore, in some cases, the BER of the proposed detector can be significantly lower than that of the decorrelating and MMSE detectors.

*Index Terms*—Bit-error rate minimization, interior-point numerical optimization, multiuser detection.

#### I. INTRODUCTION

THE CAPACITY of direct-sequence code-division multiple-access (DS-CDMA) systems is limited primarily by the near-far problem. This has motivated considerable effort to develop near-far resistant multiuser detectors. Linear multiuser detectors such as the decorrelating detector [1], [2] and the minimum mean-squared error (MMSE) detector [3] are among the most popular due to a number of advantages. As indicated by their names, the decorrelating detector and the MMSE detector minimize the multiple-access interference (MAI) and the mean-squared error, respectively. These detectors achieve optimal near-far resistance, and hence, both are worst-case optimal linear multiuser detectors [2]. It has been shown that in many cases, the output error of the MMSE detector can be assumed to be a Gaussian random process [4]. However, there are situations in practice where this is not a valid assumption. For example, this will be the case when the number of simultaneous users in a microcell is small, or the crosscorrelation properties among signature signals are poor, and the energy of the interferers is small relative to that of the desired signal. In general, the decorrelating and MMSE detectors do not provide the lowest bit-error rate (BER) even among linear detectors. Hence, it is of significant interest to develop a new linear multiuser detector that minimizes BER directly.

In [5], an *approximate* minimum BER criterion was proposed for combating intersymbol interference (ISI) in single-user communication systems and was shown to yield significant performance gain over the conventional zero-forcing and MMSE criteria [5]. In this paper, we study the minimum BER criterion as applied to linear multiuser detection for binary signaling and its

The authors are with the Department of Electrical and Computer Engineering, University of Victoria, Victoria, BC, Canada V8W 3P6.

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biorthogonal extensions to DS-CDMA communication systems. Note that the BER function is highly nonlinear and several local minima may exist. In order to avoid suboptimal local minima, we propose a constrained minimum-BER (CMBER) multiuser detector that minimizes the BER cost function directly subject to a set of convex constraints. It is shown that if the decorrelating detector exists, then there exist an infinite number of detectors satisfying the constraints and that the constrained optimization problem at hand has a unique minimizer. We focus our attention on base stations where information about the signature signals, timing, channels, and received amplitudes of all active users is available or can be accurately estimated. Hence, a deterministic approach can be taken and the linear multiuser detector that minimizes the BER directly can be designed prior to its application. Adaptive methods that yield linear detectors in which approximately minimum BER is achieved without knowledge of the parameters of the interferers have been proposed recently in [6] and [7].

To obtain the proposed detector, we convert the constrained optimization problem to an equivalent convex programming problem and then develop a Newton barrier method that requires considerably less computation than that required by the method of sequential quadratic programming [10]. Even though the proposed detector cannot be shown to be optimal without the constraints, our simulations demonstrate that its BER performance is the best.

The paper is organized as follows. In Section II, the system model considered is described, and the decorrelating and MMSE detectors are briefly reviewed. In Section III, the CMBER detector is proposed, and issues concerning convergence are studied. In Section IV, the Newton barrier method for designing the CMBER detector is presented. Numerical examples are given in Section V, and conclusions are drawn in Section VI.

#### **II. PRELIMINARIES**

We consider binary phase-shift-keying (BPSK) transmission for a channel with additive white Gaussian noise (AWGN) in a DS-CDMA system. For synchronous systems, user detection can be performed symbol by symbol. For asynchronous systems, a window approach is usually adopted that results in a symbol-by-symbol detection [8], [9]. As an alternative, each symbol within the observation window, which usually spans an odd number of symbol intervals [8], can be deemed to originate from a different synchronous user. By doing this, an asynchronous system can be interpreted as an equivalent synchronous system [2].

Assume that there are K synchronous users, and denote the information bit of the *i*th user and its amplitude as  $b_i$  and  $A_i$ , respectively. Within the observation window, the critically

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sampled version of the received baseband signal r can be expressed as

where

$$\mathbf{r} = \mathbf{S}\mathbf{b} + \mathbf{n} \tag{1}$$

$$\mathbf{r} = [r_1 \ r_2 \cdots r_N]^T$$
  
$$\mathbf{b} = [b_1 \ b_2 \cdots b_K]^T$$
  
$$\mathbf{S} = [A_1 \mathbf{s}_1 \ A_2 \mathbf{s}_2 \cdots A_K \mathbf{s}_K] \in \mathcal{R}^{N \times K}.$$

In (1), **n** is an AWGN signal with zero mean and variance  $\sigma^2$ , and  $\mathbf{s}_k \in \mathcal{R}^{N \times 1}$  is the signature signal of the *k*th synchronous user. It is important to note that **S** contains user amplitudes that need not equal each other. For asynchronous systems, if the original signature signal of the real user, who transmitted the *i*th information bit, is  $\hat{\mathbf{s}}_k$ , then  $\mathbf{s}_i$  will be of the form  $[0 \cdots 0 \ \hat{\mathbf{s}}_k^T \ 0 \cdots 0]^T$ .

Note that if the channel coefficients are known and the ISI due to the channel's time spread is negligible compared with the multiuser interference, then the above model applies to frequency-selective channels [10], [11]. For a frequency-selective channel with L resolvable paths,  $\mathbf{s}_k$  in (1) becomes

$$\mathbf{s}_k = \sum_{l=1}^L c_k(l) \hat{\mathbf{s}}_k^l$$

where  $c_k(l)$  is the *l*th tap weight of the tapped-delay line model of the channel [12],  $\hat{\mathbf{s}}_k^l = [\mathbf{0}_{(l-1)\times 1} \ \hat{s}_k(1) \cdots \hat{s}_k(N-l+1)]^T$ , and  $\hat{s}_k(i)$  is the *i*th chip of the original signature signal of user k.

A linear multiuser detector can be viewed as a linear filter followed by a sampler that samples the output of the filter at t = nT, where T is the duration of the symbol interval. The decorrelating detector attempts to completely eliminate MAI, regardless of the presence of background noise. This so-called zero-forcing (ZF) solution can be achieved by employing a receiving filter with the coefficient vector

$$\mathbf{c}_d = \mathbf{S}(\mathbf{S}^T \mathbf{S})^{-1} \mathbf{e}_k \tag{2}$$

where

 $\mathbf{S}^T \mathbf{S}$  crosscorrelation matrix among signature signals;

 $\mathbf{e}_k$  kth coordinate vector;

k index of the desired user.

Note that we have assumed that  $\mathbf{S}^T \mathbf{S}$  is positive definite, as is usually the case in practice [1]; otherwise, a nearly ZF solution can be achieved by replacing  $\mathbf{S}(\mathbf{S}^T\mathbf{S})^{-1}$  in (2) by the Moore-Penrose pseudoinverse of  $\mathbf{S}^T$ .

In contrast to the decorrelating detector, the MMSE detector attempts to minimize the mean-squared error or, equivalently, maximize the signal-to-interference-plus-noise ratio. The coefficient vector of the MMSE detector is given by

$$\mathbf{c}_m = \mathbf{S}(\mathbf{S}^T \mathbf{S} + \sigma^2 \mathbf{I})^{-1} \mathbf{e}_k \tag{3}$$

where **I** is the identity matrix.

Both of the above two linear detectors achieve the optimal near-far resistance and both provide significant performance gain compared with the conventional matched-filter receiver [1]–[3]. However, since their decision criteria are not related to the BER directly, the possibility to develop an improved linear detector that directly minimizes the BER exists, as will be shown in the rest of the paper.

### III. CONSTRAINED MINIMUM-BER MULTIUSER DETECTION

Consider the BER performance of a linear multiuser receiver with coefficient vector **c** for a multiuser channel, and assume that the two binary values of the signal, i.e.,  $\pm 1$ , are equally likely. The BER of the *k*th user can be readily found to be

$$P(\mathbf{c}) = \frac{1}{2^{K-1}} \sum_{i=1}^{2^{K-1}} Q\left(\frac{\mathbf{c}^T \mathbf{v}_i}{||\mathbf{c}||\sigma}\right)$$
(4)

where  $\mathbf{v}_i = \mathbf{S}\hat{\mathbf{b}}_i$ , and  $\hat{\mathbf{b}}_i$  for  $1 \leq i \leq 2^{K-1}$  is a possible information vector with its kth entry  $b_k = 1$ , and

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-v^2/2} dv.$$

Since the BER cost function with respect to  $\mathbf{c}$  given in (4) depends only on the direction of  $\mathbf{c}$ , the existence of a global minimum of  $P(\mathbf{c})$  is obvious. The detector, whose coefficient vector  $\mathbf{c}^*$  minimizes (4), is optimal among linear detectors and will be referred to as the *optimal linear detector*. However, there is no closed-form expression for  $\mathbf{c}^*$  as in the case of  $\mathbf{c}_d$  and  $\mathbf{c}_m$ . Furthermore, since the BER function is highly nonlinear and there may exist more than one local minimum, convergence to  $\mathbf{c}^*$  cannot generally be guaranteed for most optimization algorithms. A detailed interpretation of the minima of the BER cost function can be found in [4].

The following proposition will be useful in the subsequent analysis.

*Proposition 1:* Any local minimizer of the BER cost function in (4) subject to

$$\mathbf{c}^T \mathbf{v}_i \ge 0 \quad \text{for } 1 \le i \le 2^{K-1} \tag{5}$$

is a global minimizer. Furthermore, with the constraint  $||\mathbf{c}|| = 1$ , the global minimizer is unique.

*Proof:* Since the BER cost function is independent of the length of  $\mathbf{c}$ , it is sufficient to consider minimizing  $P(\mathbf{c})$  on the set

$$I = \{ \mathbf{c} \colon ||\mathbf{c}|| = 1 \text{ and } \mathbf{c} \text{ satisfies } (5) \}.$$
(6)

Let the global minimizer of the above constrained minimization problem be  $\mathbf{c}_1 \in I$ , and assume that there exists another local minimizer  $\mathbf{c}_2 \in I$  such that

$$P(\mathbf{c}_1) < P(\mathbf{c}_2). \tag{7}$$

Let  $\alpha < 1$  be a positive constant, and assume that

$$\mathbf{c} = \frac{\alpha \mathbf{c}_1 + (1 - \alpha) \mathbf{c}_2}{\|\alpha \mathbf{c}_1 + (1 - \alpha) \mathbf{c}_2\|}.$$
(8)

Since  $\|\mathbf{c}\| = 1$  and  $\mathbf{c}^T \mathbf{v}_i \ge 0$  for  $1 \le i \le 2^{K-1}$ , we conclude that  $\mathbf{c} \in I$ . Furthermore, because  $\|\alpha \mathbf{c}_1 + (1 - \alpha)\mathbf{c}_2\| \le 1$ , we have

$$\mathbf{c}^T \mathbf{v}_i \ge \alpha \mathbf{c}_1^T \mathbf{v}_i + (1 - \alpha) \mathbf{c}_2^T \mathbf{v}_i \tag{9}$$

Hence

$$Q\left(\frac{\mathbf{c}^{T}\mathbf{v}_{i}}{\sigma}\right) \leq Q\left[\frac{\alpha \mathbf{c}_{1}^{T}\mathbf{v}_{i} + (1-\alpha)\mathbf{c}_{2}^{T}\mathbf{v}_{i}}{\sigma}\right]$$
$$\leq \alpha Q\left(\frac{\mathbf{c}_{1}^{T}\mathbf{v}_{i}}{\sigma}\right) + (1-\alpha)Q\left(\frac{\mathbf{c}_{2}^{T}\mathbf{v}_{i}}{\sigma}\right) \quad (10)$$

where the second inequality follows from the fact that Q(x) for  $x \ge 0$  is a convex function. From (4) and (10), we have

$$P(\mathbf{c}) = \frac{1}{2^{K-1}} \sum_{i=1}^{2^{K-1}} Q\left(\frac{\mathbf{c}^{T}\mathbf{v}_{i}}{\sigma}\right)$$
$$\leq \frac{\alpha}{2^{K-1}} \sum_{i=1}^{2^{K-1}} Q\left(\frac{\mathbf{c}_{1}^{T}\mathbf{v}_{i}}{\sigma}\right)$$
$$+ \frac{1-\alpha}{2^{K-1}} \sum_{i=1}^{2^{K-1}} Q\left(\frac{\mathbf{c}_{2}^{T}\mathbf{v}_{i}}{\sigma}\right)$$
$$\leq \alpha P(\mathbf{c}_{1}) + (1-\alpha)P(\mathbf{c}_{2})$$
$$< P(\mathbf{c}_{2}) \quad \text{for all } \alpha \in (0,1). \tag{11}$$

Since  $\mathbf{c} \to \mathbf{c}_2$  as  $\alpha \to 0$ , the inequality in (11) implies that in any arbitrarily small neighborhood centered at  $\mathbf{c}_2$ , there always exists a vector  $\mathbf{c}$  such that  $P(\mathbf{c}) < P(\mathbf{c}_2)$ . This contradicts the fact that  $\mathbf{c}_2$  is a local minimizer, and hence, we have  $P(\mathbf{c}_1) \ge P(\mathbf{c}_2)$ . Since  $\mathbf{c}_1$  is the global minimizer, we also have  $P(\mathbf{c}_1) \le P(\mathbf{c}_2)$ . Therefore,  $P(\mathbf{c}_1) = P(\mathbf{c}_2)$ , and  $\mathbf{c}_2$  is a global minimizer.

To show the uniqueness of the minimizer in set I, we note that any point in the set

$$I_0 = \{ \mathbf{c} : \|\mathbf{c}\| = 1 \text{ and } \mathbf{c}^T \mathbf{v}_i = 0 \text{ for } 1 \le i \le 2^{K-1} \}$$
 (12)

is a global maximizer. Hence, it is sufficient to consider only those points in the convex set  $I_1 = I - I_0$ .

Since Q(x) is strictly convex for x > 0,  $P(\mathbf{c})$  in (4) is strictly convex on set  $I_1$ . Now, assume that there are two distinct global minimizers  $\mathbf{c}_1$  and  $\mathbf{c}_2$  with  $\mathbf{c}_1 \neq \mathbf{c}_2$ . In such a case, any point

$$\mathbf{c} = \frac{\alpha \mathbf{c}_1 + (1 - \alpha)\mathbf{c}_2}{\|\alpha \mathbf{c}_1 + (1 - \alpha)\mathbf{c}_2\|}$$

with  $0 < \alpha < 1$  would satisfy the inequality

$$P(\mathbf{c}) \ge P(\mathbf{c}_1). \tag{13}$$

Since  $P(\mathbf{c})$  is strictly convex on  $I_1$ , we have

$$P(\mathbf{c}) < \alpha P(\mathbf{c}_1) + (1-\alpha)P(\mathbf{c}_2) = P(\mathbf{c}_1)$$

which contradicts (13). Therefore, the global minimizer is unique.

Note that in the above proposition, we have assumed that the set I defined by (6) is not empty. This assumption is true for most practical systems, as stated in the following proposition.

Proposition 2: If the signature signals  $\mathbf{s}_i$ ,  $1 \le i \le K$  are linearly independent of each other, then there always exist an infinite number of elements in I.

*Proof:* It is easy to show that if  $\mathbf{s}_i$ ,  $1 \le i \le K$  are linearly independent of each other, then  $\mathbf{S}^T \mathbf{S}$  is positive definite, and the ZF solution can be achieved. Consequently, from (2), we have

$$\frac{\mathbf{c}_{d}^{T}\mathbf{v}_{i}}{\|\mathbf{c}_{d}\|} = \frac{\mathbf{e}_{k}^{T}(\mathbf{S}^{T}\mathbf{S})^{-1}\mathbf{S}^{T}\mathbf{S}\hat{\mathbf{b}}_{i}}{\|\mathbf{c}_{d}\|} = \frac{\mathbf{e}_{k}^{T}\hat{\mathbf{b}}_{i}}{\|\mathbf{c}_{d}\|} = \frac{1}{\|\mathbf{c}_{d}\|} > 0 \quad \text{for } 1 \le i \le 2^{K-1}.$$
(14)

This means that  $\mathbf{c}_d / \|\mathbf{c}_d\|$  is in set *I*. Now, consider vector

$$\mathbf{c}_p = \frac{\mathbf{c}_d + \mathbf{p}}{\|\mathbf{c}_d + \mathbf{p}\|} \tag{15}$$

where **p** is a perturbation vector to be determined later. We have  $||\mathbf{c}_p|| = 1$  and

$$\mathbf{c}_{p}^{T}\mathbf{v}_{i} = \frac{\mathbf{c}_{d}^{T} + \mathbf{p}^{T}\mathbf{v}_{i}}{\|\mathbf{c}_{d} + \mathbf{p}\|} = \frac{1 + \mathbf{p}^{T}\mathbf{v}_{i}}{\|\mathbf{c}_{d} + \mathbf{p}\|}$$
$$\geq \frac{1 - \|\mathbf{p}\|\|\mathbf{v}_{i}\|}{\|\mathbf{c}_{d} + \mathbf{p}\|} \geq \frac{1 - \|\mathbf{p}\|(\max_{i} \|\mathbf{v}_{i}\|)}{\|\mathbf{c}_{d} + \mathbf{p}\|}.$$
 (16)

It follows that if

$$\|\mathbf{p}\| \le \frac{1}{\max_{1 \le i \le 2^{K-1}} \|\mathbf{v}_i\|} \tag{17}$$

then (16) implies that  $\mathbf{c}_p^T \mathbf{v}_i \ge 0$  for  $1 \le i \le 2^{K-1}$ . In other words, any vector  $\mathbf{c}_p$  given by (15) with  $\mathbf{p}$  satisfying (17) belongs to set I.

The CMBER multiuser detector is defined as the detector whose coefficient vector is the global minimizer of  $P(\mathbf{c})$  in (4) subject to the constraints in (5). From Proposition 2, once the ZF solution can be achieved, the CMBER detector exists and outperforms the decorrelating detector. Several remarks are now in order.

- a) In our simulations, we found out that for systems where the signal-to-noise ratio (SNR) at the output of the decorrelating detector is not very low (e.g., the SNR is greater than 0 dB), the CMBER multiuser detector is identical with the linear minimum-BER detector whose coefficient vector minimizes the BER cost function (4) without any constraints. An intuitive explanation is as follows. Since a detector with a coefficient vector c that does not satisfy (5) would usually yield a poorer BER than that of the decorrelating detector, the global minimizer most likely satisfies (5). If so, from Proposition 1, this global minimizer is the same as the coefficient vector of the CMBER detector.
- b) The above results can be readily extended to the case of channel equalization for single-user communication systems. Even though an exact ZF solution is not always achievable with a finite-length linear equalizer, the channel is still *equalizable* for most cases. In other words, set *I* is most likely not empty, and a constrained minimum-BER equalizer exists.
- c) As will be shown in the next section, the problem of minimizing  $P(\mathbf{c})$  in (4) subject to the constraints in (5) can be converted to a standard convex programming problem, and an efficient constrained Newton method can be developed to obtain the CMBER solution quickly.

#### IV. NEWTON BARRIER METHOD FOR CMBER PROBLEM

The problem of minimizing the BER in (4) subject to the constraints in (5) is equivalent to

minimize 
$$\hat{P}(\mathbf{c})$$
 (18a)

subject to 
$$\mathbf{c}^T \hat{\mathbf{v}}_i \ge 0$$
 for  $1 \le i \le 2^{K-1}$  (18b)

$$\|\mathbf{c}\| = 1 \tag{18c}$$

where

$$\hat{P}(\mathbf{c}) = \frac{1}{2^{K-1}} \sum_{i=1}^{2^{K-1}} Q(\mathbf{c}^T \hat{\mathbf{v}}_i)$$

and

$$\hat{\mathbf{v}}_i = \frac{\mathbf{v}_i}{\sigma} \quad \text{for } 1 \le i \le 2^{K-1}.$$

Note that the problem in (18) is *not* a convex programming problem because the feasible region characterized by (18b) and (18c) is not convex. However, it can be readily verified that the solution of (18) coincides with the solution of the constrained optimization problem

minimize 
$$\hat{P}(\mathbf{c})$$
 (19a)

subject to  $\mathbf{c}^T \hat{\mathbf{v}}_i \ge 0$  for  $1 \le i \le 2^{K-1}$  (19b)

$$\|\mathbf{c}\| \le 1. \tag{19c}$$

This is because for any **c** with  $||\mathbf{c}|| < 1$ , one always has  $\hat{P}(\hat{\mathbf{c}}) \leq \hat{P}(\mathbf{c})$ , where  $\hat{\mathbf{c}} = \mathbf{c}/||\mathbf{c}||$ . In other words, the minimizer  $\mathbf{c}^*$  of problem (19) always satisfies  $||\mathbf{c}^*|| = 1$ . A key distinction between the problems in (18) and (19) is that the latter one is a convex programming problem for which a number of efficient algorithms are available. The optimization algorithm described below fits into the class of barrier function methods [13], [14], but it has several additional features that are uniquely associated with the present problem. These include a closed-form formula for evaluating the Newton direction and an efficient line search.

By using a barrier function approach, we can further drop the nonlinear constraint in (19c) and convert the problem in (19) into

minimize 
$$F_{\mu}(\mathbf{c}) = \hat{P}(\mathbf{c}) - \mu \log(1 - \mathbf{c}^T \mathbf{c})$$
 (20a)

subject to 
$$\mathbf{c}^T \hat{\mathbf{v}}_i \ge 0$$
 for  $1 \le i \le 2^{K-1}$  (20b)

where  $\mu > 0$  is the barrier parameter. With a strictly feasible initial point  $c_0$ , which strictly satisfies the constraints in (19b) and (19c), the logarithmic term in (20a) is well defined. It is also evident that regardless of the value of  $\mu$ , the minimum of (20) is the global minimum of the problem of minimizing (4) subject to the constraints in (5). The gradient and Hessian matrixes of  $F_{\mu}(\mathbf{c})$  are given by

$$\nabla F_{\mu}(\mathbf{c}) = -\sum_{i=1}^{M} \frac{1}{M} e^{-\beta_{i}^{2}/2} \hat{\mathbf{v}}_{i} + \frac{2\mu \mathbf{c}}{1 - ||\mathbf{c}||^{2}}$$
(21)

$$\nabla^{2} F_{\mu}(\mathbf{c}) = \sum_{i=1}^{M} \frac{1}{M} e^{-\beta_{i}^{2}/2} \beta_{i} \hat{\mathbf{v}}_{i} \hat{\mathbf{v}}_{i}^{T} + \frac{2\mu}{1 - ||\mathbf{c}||^{2}} \mathbf{I} + \frac{4\mu}{(1 - ||\mathbf{c}||)^{2}} \mathbf{c} \mathbf{c}^{T}$$
(22)

where  $M = 2^{K-1}$ , and  $\beta_i = \mathbf{c}^T \hat{\mathbf{v}}_i$  for  $1 \le i \le M$ . Note that the Hessian matrix in the interior of the feasible region, i.e.,  $\mathbf{c}$ with  $\beta_i = \mathbf{c}^T \hat{\mathbf{v}}_i > 0$  and  $||\mathbf{c}|| < 1$ , is positive definite. This suggests that at the (k+1)th iteration,  $\mathbf{c}_{k+1}$  can be obtained as

$$\mathbf{c}_{k+1} = \mathbf{c}_k + \alpha_k \mathbf{d}_k \tag{23}$$

where the search direction  $d_k$  is computed using

$$\mathbf{d}_{k} = -[\nabla^{2} F_{\mu}(\mathbf{c}_{k})]^{-1} \nabla F_{\mu}(\mathbf{c}_{k}).$$
(24)

The positive scalar  $\alpha_k$  in (23) is determined by a linear search step, as follows. First, note that the one-variable function  $F_{\mu}(\mathbf{c}_k + \alpha \mathbf{d}_k)$  is strictly convex on the interval  $[0, \overline{\alpha}]$ , where  $\overline{\alpha}$  is the largest positive scalar such that  $\mathbf{c}_k + \alpha \mathbf{d}_k$  remains feasible for  $0 \le \alpha \le \overline{\alpha}$ . Once  $\overline{\alpha}$  is determined,  $F_{\mu}(\mathbf{c}_k + \alpha \mathbf{d}_k)$  is a unimodal function on  $[0, \overline{\alpha}]$ , and the search for the minimizer of the function can be carried out using one of the well-known methods such as quadratic or cubic interpolation, the Golden-section method, or some direct search method [13], [15], [16]. To find  $\overline{\alpha}$ , we note that a point  $\mathbf{c}_k + \alpha \mathbf{d}_k$  satisfies the constraints in (19b) if

$$(\mathbf{c}_k + \alpha \mathbf{d}_k)^T \hat{\mathbf{v}}_i \ge 0 \quad \text{for } 1 \le i \le M.$$
 (25)

Since  $\mathbf{c}_k$  is feasible, we have  $\mathbf{c}_k^T \hat{\mathbf{v}}_i \ge 0$  for  $1 \le i \le M$ . Hence, for those indices i such that  $\mathbf{d}_k^T \hat{\mathbf{v}}_i \ge 0$ , any non-negative  $\alpha$  will satisfy (25). In other words, only those constraints in (19b) whose indices are in the set

$$\mathcal{I}_k = \{i: \mathbf{d}_k^T \hat{\mathbf{v}}_i < 0\}$$
(26)

will affect the largest value of  $\alpha$  that satisfies (25), and the largest value of  $\alpha$  can be computed as

$$\overline{\alpha}_1 = \min_{i \in \mathcal{I}_k} \left( \frac{\mathbf{c}_k^T \mathbf{v}_i}{-\mathbf{d}_k^T \mathbf{v}_i} \right).$$
(27)

In order to satisfy the constraint in (19c), we solve  $\|\mathbf{c}_k + \alpha \mathbf{d}_k\|^2 = 1$  for  $\alpha$ , and the solution is given by  $\alpha = \overline{\alpha}_2$  with

$$\overline{\alpha}_{2} = \frac{[(\mathbf{c}_{k}^{T}\mathbf{d}_{k})^{2} - ||\mathbf{d}_{k}||^{2}(||\mathbf{c}_{k}||^{2} - 1)]^{1/2} - \mathbf{c}_{k}^{T}\mathbf{d}_{k}}{||\mathbf{d}_{k}||^{2}}.$$
 (28)

The value of  $\overline{\alpha}$  can now be taken as  $\min(\overline{\alpha}_1, \overline{\alpha}_2)$ . In practice, one must keep the next iterate strictly inside the feasible region to ensure that the barrier function in (21) is well defined. To this end, we use

$$\overline{\alpha} = 0.99 \min(\overline{\alpha}_1, \overline{\alpha}_2). \tag{29}$$

This iterative optimization procedure continues until the difference between two successive solutions is less than a prescribed tolerance. Even though, with a strictly feasible initial point the barrier Newton method described above always converges to the global minimizer for an arbitrary positive  $\mu$ , the value of  $\mu$  does affect the behavior of the algorithm. A small  $\mu$  would lead to an ill-conditioned Hessian matrix, whereas a large  $\mu$  would lead to slow convergence. Hence, a  $\mu$  in the interval [0.001, 0.1], which would guarantee a well-conditioned Hessian matrix and allow a fast convergence, is desirable.

## V. NUMERICAL EXAMPLES

#### A. Example 1

As a first example, we consider a two-user system. The main purpose of the example is to illustrate how and when the CMBER detector outperforms the MMSE detector. Assume that the signature signals multiplied by the corresponding amplitudes are  $\hat{s}_0$  and  $\hat{s}_1$ , as depicted in Fig. 1, and the desired user is user 0. From the figure, the normalized signature crosscorrelation  $\rho$  can be found to be 0.894. The corresponding coefficient vector  $\mathbf{c}_d$  of the decorrelating detector must be orthogonal to  $\hat{s}_1$ , as illustrated in Fig. 1. Clearly, in this case, the CMBER detector is equivalent to the optimal linear detector since the energy of the desired signal is stronger than the energy of the interferer. If a linear detector is used and the



Fig. 1. Signature signals and multiuser detectors for a two-user system.

TABLE I BERS FOR A TWO-USER SYSTEM (SNR = 15 dB)

Detectors	Decorrelating	MMSE	MFR	Minimum-BER
BER	$1.91  imes 10^{-4}$	$2.09  imes 10^{-7}$	$6.40  imes 10^{-10}$	$3.86\times10^{-10}$

angle between its coefficient vector and  $\hat{s}_0$  is  $\theta$ , the BER for this example can be obtained as

$$P(\theta) = \frac{1}{2} \left[ Q \left( \frac{1.25 \cos \theta + 0.125 \sin \theta}{\sigma} \right) + Q \left( \frac{0.75 \cos \theta - 0.125 \sin \theta}{\sigma} \right) \right].$$
(30)

Hence, the BER of the decorrelating detector is

$$P(\mathbf{c}_d) = Q\left(\frac{1}{\sqrt{5}\sigma}\right) \tag{31}$$

and the BER of the matched-filter receiver (MFR) with coefficient vector  $\mathbf{s}_0$  is

$$P(\mathbf{s}_0) = \frac{1}{2} \left[ Q\left(\frac{5}{4\sigma}\right) + Q\left(\frac{3}{4\sigma}\right) \right]. \tag{32}$$

Comparing (31) with (32), we can see that the MFR, in this case, outperforms the decorrelating detector regardless of the SNR because the crosscorrelation of the two signature signals is large, whereas the MAI is sufficiently small relative to the desired signal energy. Under these circumstances, the loss of signal energy inherent in the decorrelating detection is always larger than the loss of signal energy due to signal cancellation when the MFR is employed. Even though the MMSE detector can to some extent balance the effects of MAI and AWGN, its performance is still close to that of the decorrelating detector since it treats the residual MAI and AWGN as equally harmful. On the other hand, the optimal linear detector can better balance the effects of MAI and AWGN, and as a result, its performance is always better than that of the MFR. For a given  $\sigma = 0.126$ , the coefficient vector  $\mathbf{c}_m$  of the MMSE detector and the coefficient vector  $\mathbf{c}_c$  of the optimal linear detector are illustrated in Fig. 1. The BER's of the decorrelating detector, MMSE, MFR, and optimal linear detectors are given in Table I. As can be observed from the table, for user 0, the optimal linear detector and even the MFR significantly outperform the decorrelating and MMSE detectors. However, this occurs only when the power the user of interest is sufficiently high compared with the power of the interferers.



Fig. 2. Performance comparison of linear multiuser detectors: Ten equal-power users.

#### B. Example 2

As a second example, we compare the performance of the CMBER with that of the optimal linear detector. Only synchronous systems were considered. The BER curves of different detectors for a system with ten equal-power users are shown in Fig. 2. The optimal linear detector was taken as the best solution of 40 runs of a quasi-Newton optimization algorithm. In order to test a case in which the CMBER detector differs from the optimal linear multiuser detector, the user signature signals were randomly selected, and the SNR was chosen to be unrealistically small. As can be seen in Fig. 2, the BER curve of the CMBER detector and that of the optimal linear detector are indistinguishable. In fact, these two detectors have approximately the same coefficient vector when the SNR is greater than -3 dB. From the figure, it is also evident that the MMSE detector achieves very similar performance as that of the optimal linear detector, whereas the decorrelating detector yields a considerably poorer performance.

We now consider more practical situations where the SNR is greater than 0 dB and the near-far effect needs to be taken into account. Figs. 3 and 4 show the BER curves for ten-user single-path and multipath systems, respectively. In both types of systems, the powers of interferers were chosen to be in the range 0 to 10 dB below the power of the user of interest, and the interferers' powers were approximately uniformly distributed. The number of resolvable paths was assumed to be six for Fig. 4. For the multipath system, the tap weights were assumed to be known. Hence, the single-path model in (1) applies, as described in Section II. Only conditional BER curves were evaluated, and the SNR for a particular user is defined as the ratio of the total received signal energy of the user to the energy of the Gaussian noise. As can be observed, in both cases, the CMBER detector outperforms the decorrelating and MMSE detectors. For the single-path system, the crosscorrelation among signature signals is small, and the three BER curves are close to each other. On the other hand, for the multipath system, where the crosscorrelation properties are relatively poorer, the performance gain of the CMBER detector over those of the decorrelating and the



Fig. 3. Performance comparison of linear multiuser detectors: 31-chip Gold codes and single path.



Fig. 4. Performance comparison of linear multiuser detectors: 31-chip Gold codes and multipaths.

MMSE detectors is significant. Note that the coefficient vector of the CMBER detector was equal to that of the optimal linear detector, but its performance curve is not shown in the figures.

## C. Example 3

As a last example, we applied the proposed CMBER detector, the ZF equalizer, and the MMSE equalizer to a single-user system over a dispersive channel with impulse response  $h = \{0.7317, 0.6707, -0.1219\}$ . The detector and equalizers were assumed to be of three taps, and the detection delay was assumed to be 2. The BER curves of the ZF equalizer, the MMSE equalizer, and the CMBER detector are illustrated in Fig. 5. As can be seen in the figure, the performance of the ZF equalizer is similar to that of the MMSE equalizer, whereas the CMBER detector offers a performance gain as much as 5 dB over the MMSE equalizer.



Fig. 5. Performance comparison of linear equalizers for a dispersive channel.

## D. Use of MATLAB

We have also compared the proposed barrier Newton method with the constrained optimization method provided by MATLAB routine *constr* in terms of computational complexity. We found out that for a system with ten users and signature signals of length 31 chips, the MATLAB routine required 40 to 100 times more flops than the barrier Newton method that converged after about 10 to 20 iterations. As the number of users and the length of signature signals increase, the barrier Newton method becomes increasingly more efficient in terms of computation than the MATLAB routine. It was noted that most of the performance gain is usually achieved in the first couple of iterations. This implies that only a few iterations would be needed in practice.

#### E. Adaptive Implementation

In principle, one needs to perform the optimization again when the channel changes. Since the proposed optimizationbased algorithm converges as long as the initial point satisfies the constraints in (19b) and (19c), it is possible to develop an adaptive version of the algorithm to accommodate channel variations. For a slowly varying channel, the optimization will converge quickly by using the previous coefficient vector as the initial point.

## VI. CONCLUSIONS

We have studied the minimum BER criterion as applied to multiuser detection as well as channel equalization for single-user systems and proposed a constrained minimum-BER multiuser detector. The proposed detector minimizes the BER cost function directly subject to the constraint that the corresponding eye pattern is open independently of the information bits transmitted by the interferers. We have shown that under this constraint, the BER cost function has a unique minimizer. Consequently, we were able to use a Newton barrier method to find the coefficients of the proposed detector, which requires only a very small amount of computation relative to that required by the popular method of sequential quadratic programming. The analysis and numerical examples presented demonstate that, in many practical situations, the proposed detector offers a significant performance advantage over the decorrelating and MMSE detectors.

The analysis and results presented in this paper, especially the identification of the convergence region defined by the set of convex constraints, are also useful for adaptive implementations of the proposed algorithm. Even though identifying the proposed convergence region exactly would need information about the interferers, it is possible to identify a closely related convergence region without such information. For example, a constraint on the angle between the coefficient vector to be optimized and the coefficient vector of the decorrelator can be used to define such a region.

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Xiaofeng Wang received the B.S. degree in electronics from Wuhan University, Wuhan, China, in 1991 and the M.E. degree in electronic engineering from Beijing University of Posts and Telecommunications, Beijing, China, in 1994. Currently, he is pursuing the Ph.D. degree with the Department of Electrical and Computer Engineering, University of Victoria, Victoria, BC, Canada.

Since February 2000, he has been working for PMC-Sierra, Inc. His research interests include digital signal processing for communications and

broadband network architectures.



**Wu-Sheng Lu** (F'99) received the B.S. degree in mathematics from Fudan University, Fudan, China, in 1964, and the M.S. degree in electrical engineering and the Ph.D. degree in control science from the University of Minnesota, Minneapolis, in 1983 and 1984, respectively.

He was a post-doctoral fellow at the University of Victoria, Victoria, BC, Canada, in 1985 and a Visiting Assistant Professor with the University of Minnesota in 1986. Since 1987, he has been with the University of Victoria, where he is currently a Professor. His

teaching and research interests are in the areas of digital signal processing and numerical optimization. He is the co-author, with A. Antoniou, of *Two-Dimensional Digital Filters* (New York: Marcel Dekker, 1992).

Dr. Lu was an Associate Editor of the *Canadian Journal of Electrical and Computer Engineering* in 1989 and the Editor of the same journal from 1990 to 1992. He was an Associate Editor of IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS II from 1993 to 1995. Presently, he serves as an Associate Editor for the IEEE TRANS. ON CIRCUITS AND SYSTEMS I and for the *International Journal of Multidimensional Systems and Signals*. He is a Fellow of the Engineering Institute of Canada.



Andreas Antoniou (F'82) received the B.Sc. (Eng.) and Ph.D. degrees in electrical engineering from London University, London, U.K., in 1963 and 1966, respectively.

From 1966 to 1969, he was Senior Scientific Officer at the Post Office Research Department, London, and from 1969 to 1970, he was a Member of the Scientific Staff at the R&D Laboratories, Northern Electric Company Ltd., Ottawa, ON, Canada. From 1970 to 1983, he served in the Department of Electrical and Computer Engineering,

Concordia University, Montreal, PQ, Canada, as Professor from June 1973 and as Chairman from December 1977. He served as founding Chairman of the Department of Electrical and Computer Engineering, University of Victoria, Victoria, BC, Canada, from July 1, 1983 to June 30, 1990 and is now Professor in the same department. His teaching and research interests are in the areas of electronics, network synthesis, digital system design, active and digital filters, and digital signal processing. He published extensively in these areas. He is the author of *Digital Filters: Analysis, Design, and Applications* (New York: McGraw-Hill) and the co-author with W.-S. Lu of *Two-Dimensional Digital Filters* (New York: Marcel Dekker).

Dr. Antoniou is a member of the Association of Professional Engineers and Geoscientists of British Columbia and Fellow of the Institution of Electrical Engineers. He served as Associate Editor of IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS from June 1983 to May 1985 and as Editor from June 1985 to May 1987. He was a member of the Board of Governors of the Circuits and Systems Society from 1995 to 1997. He was recently awarded a CAS Golden Jubilee Medal by the IEEE Circuits and Systems Society in recognition of outstanding achievements in the area of circuits and systems and the B.C. Science Council Chairman's Award for Career Achievement for 2000. One of his papers on gyrator circuits was awarded the Ambrose Fleming Premium by the Institution of Electrical Engineers of the UK.