A New Method for the Design of FIR Quadrature Mirror-Image Filter Banks

Wu-Sheng Lu, Hua Xu, and Andreas Antoniou

***Abstract*—A new algebraic method for the design of two-channel finite-duration impulse response quadrature mirror-image filter (FIR QMF) banks is proposed. The method uses a self-convolution technique to reformulate a fourth-order objective function whose minimization leads to the design of QMF banks. It is shown that the reformulated optimization problem can be solved by an iterative technique in which the major part of each iteration is carried out in terms of a closed-form formula. This leads to improved computational efficiency relative to that in several existing design methods. The method is then extended to the design of QMF banks with low reconstruction delay. Two examples are included which show that the proposed design method leads to filter banks with improved performance.**

I. Introduction

Finite-duration impulse response quadrature mirror-image filter (FIR QMF) banks are widely used and many methods have been developed for their design [1]–[12]. Some of the design methods lead to near-perfect reconstruction QMF banks [1]–[5], [12] while others lead to perfect-reconstruction QMF banks [6]–[11].

A time-domain iterative algorithm for the design of QMF banks was proposed in [2]. The method involves calculating the eigenvalues and eigenvectors of a matrix in each iteration. Another iterative algorithm which uses a linearization of the error function in the frequency domain was proposed in [5]. This algorithm needs less computation than other QMF design methods [1]–[3] and produces improved filter banks. Furthermore, improvements in the computational efficiency over that of the method in [5] have been achieved by deriving explicit and precise expressions for the objective function [12].

In this paper, a new algebraic method for the design of QMF banks with near-perfect reconstruction is proposed. The method uses a self-convolution technique to reformulate a fourth-order objective function as a quadratic function whose minimization leads to the design of QMF banks. It is shown that the reformulated optimization problem can be solved by an iterative technique in which the major part of each iteration is carried out in terms of a closed-form formula. This leads to improved computational efficiency relative to that in several existing design methods. The method is then extended to the design of QMF banks with low reconstruction delay. Two examples are included which show that the proposed design method leads to filter banks with improved performance.

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II. Design Method

A. Generalized FIR QMF Banks

Consider the two-channel filter bank shown in Fig. 1. The output and input of the system are related in terms of

$$\hat{X}(z) = \frac{1}{2}[H_0(z)G_0(z) + H_1(z)G_1(z)]X(z) + \frac{1}{2}[H_0(-z)G_0(z) + H_1(-z)G_1(z)]X(-z) \quad (1)$$

where the second term in the right-hand side represents aliasing.

By assuming that $H_1(z) = H_0(-z), G_1(z) = -H_0(-z)$ and $G_0(z) = H_0(z)$, the aliasing term is canceled and (1) becomes

$$\hat{X}(z) = \frac{1}{2}\left[H_0^2(z) - H_0^2(-z)\right]X(z).$$

To reconstruct the input signal at the output, it is required that

$$H_0^2(z) - H_0^2(-z) = z^{-k_d} \quad (2)$$

where k_d is the system delay. Note that if filter H_0 has a linear phase response and $k_d = N-1$ where $N-1$ is the order of $H_0(z)$, then the filter bank is a conventional QMF bank; if $k_d < N-1$ and one does not assume that the filter H_0 has a linear phase response, then the filter bank has a low reconstruction delay. In effect, (2) can represent both conventional and low-delay QMF banks depending on the numerical value of k_d and whether or not H_0 has a linear phase response.

B. A Closed-Form Formula for the Gradient of a Least-Squares Objective Function

Let

$$H_0(z) = h_0 + h_1 z^{-1} + \cdots + h_{N-1} z^{-(N-1)}$$

with N an even number, and

$$\begin{aligned}\mathbf{h} &= [h_0 \quad h_1 \quad \cdots \quad h_{N-1}]^T \\ \mathbf{z}_{2N} &= [1 \quad z^{-1} \quad \cdots \quad z^{-2(N-1)}]^T \\ \hat{\mathbf{z}}_{2N} &= [1 \quad (-z)^{-1} \quad \cdots \quad (-z)^{-2(N-1)}]^T.\end{aligned}$$

We can write

$$\begin{aligned}H_0^2(z) &= \mathbf{g}^T \mathbf{z}_{2N} \\ H_0^2(-z) &= \mathbf{g}^T \hat{\mathbf{z}}_{2N}\end{aligned}$$

where

$$\mathbf{g} = \mathbf{h} * \mathbf{h} \quad (3)$$

with $*$ denoting the convolution operation. Therefore,

$$H_0^2(z) - H_0^2(-z) = \mathbf{g}^T(\mathbf{z}_{2N} - \hat{\mathbf{z}}_{2N}) \equiv \mathbf{g}^T \tilde{\mathbf{z}}_{2N}$$

where $\tilde{\mathbf{z}}_{2N} = 2[0 \quad z^{-1} \quad 0 \quad z^{-3} \quad 0 \quad \cdots \quad z^{-2N+3} \quad 0]^T$. If we let $z = e^{j\omega}$, we have

$$H_0^2(e^{j\omega}) - H_0^2(-e^{j\omega}) = 2\mathbf{g}^T \mathbf{c}_0(\omega) e^{-jk_d\omega}$$

where $\mathbf{c}_0(\omega) = [0 \quad e^{j(k_d-1)\omega} \quad \cdots \quad 0 \quad 1 \quad 0 \quad \cdots \quad e^{j(k_d-2N+3)\omega} \quad 0]^T$ and (2) becomes

$$2\mathbf{g}^T \mathbf{c}_0(\omega) = 1. \quad (4)$$

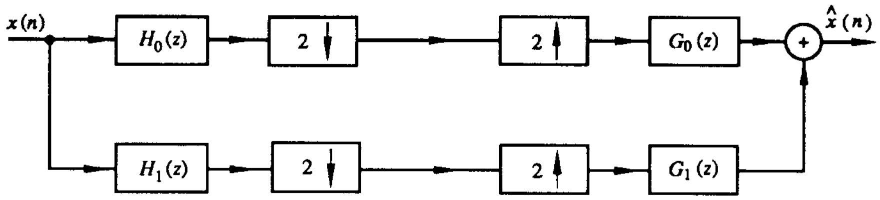

Fig. 1. A two-band filter bank.

Now if a QMF bank with near-perfect reconstruction is designed by minimizing the objective function

$$E = \int_0^{\pi} \left|H_0^2(e^{j\omega}) - H_0^2(-e^{j\omega}) - e^{-j\omega k_d}\right|^2 d\omega + \alpha \int_{\omega_s}^{\pi} |H_0(e^{j\omega})|^2 \, d\omega$$

where ω_s is the stopband edge of the lowpass filter H_0 and α is a weight, then the linearization of (2), namely (4), implies that

$$E = \int_0^{\pi} \left|2\mathbf{g}^T \mathbf{c}_0(\omega) - 1\right|^2 d\omega + \alpha \int_{\omega_s}^{\pi} |H_0(e^{j\omega})|^2 \, d\omega. \tag{5}$$

It follows that E can be expressed as a *quadratic function* of $\mathbf{h}$ and $\mathbf{g}$, namely,

$$E = 4\mathbf{g}^T \mathbf{Q}_r \mathbf{g} - 4\mathbf{g}^T \mathbf{b} + \alpha \mathbf{h}^T \mathbf{Q}_a \mathbf{h} + \pi \tag{6}$$

where $\mathbf{Q}_r = \pi \cdot \mathrm{diag}\{0, 1, 0, \ldots, 1, 0\}$,

$$\mathbf{Q}_a = \begin{bmatrix} \pi - \omega_s & \phi_1 & \phi_2 & \cdots & \phi_{N-1} \\ \phi_1 & \pi - \omega_s & \phi_1 & \cdots & \phi_{N-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \phi_{N-1} & \phi_{N-2} & \phi_{N-3} & \cdots & \pi - \omega_s \end{bmatrix}$$

with $\phi_k = -\frac{1}{k} \sin k\omega_s$, and $\mathbf{b} = [0 \ \cdots \ 0 \ \pi \ 0 \ \cdots \ 0]^T$ where only the $(k_d + 1)$th entry is nonzero.

The gradient vector of E with respect to $\mathbf{h}$ is given by

$$\nabla_{\mathbf{h}} E = 2[(4\mathbf{J}\mathbf{Q}_r\mathbf{H} + \alpha \mathbf{Q}_a)\mathbf{h} - 2\mathbf{J}\mathbf{b}] \tag{7}$$

where $\mathbf{J}$ is the $N \times (2N-1)$ Jacobian of $\mathbf{g}$ with respect to $\mathbf{h}$, i.e.,

$$\mathbf{J} = \frac{\partial \mathbf{g}}{\partial \mathbf{h}} = \frac{\partial(\mathbf{h} * \mathbf{h})}{\partial \mathbf{h}} = \frac{\partial(\mathbf{H}\mathbf{h})}{\partial \mathbf{h}}.$$

Matrix $\mathbf{H}$ is the $(2N-1) \times N$ Toeplitz matrix whose first row and first column are $[h_0 \ 0 \ \cdots \ 0]$ and $[h_0 \ h_1 \ \cdots \ h_{N-1} \ 0 \ \cdots \ 0]^T$. It can be shown that $\mathbf{J}$ is the $N \times (2N-1)$ Toeplitz matrix whose first row and first column are $2[h_0 \ h_1 \ \cdots \ h_{N/2-1} \ h_{N/2} \ \cdots \ h_{N-1} \ 0 \ \cdots \ 0]$ and $[2h_0 \ 0 \ \cdots \ 0]^T$.

C. Design of Linear-Phase QMF Banks

If $k_d = N - 1$ and $\mathbf{h}$ is symmetrical with respect to its midpoint, then the filter H_0 has a linear phase response. In this case the number of design variables is reduced to $N/2$ and (7) is replaced by

$$\nabla_{\mathbf{h}_s} E = 2[(4\mathbf{J}_s\mathbf{Q}_r\mathbf{H}_s + \alpha \mathbf{Q}_{as})\mathbf{h}_s - 2\mathbf{J}_s\mathbf{b}]. \tag{8}$$

In (8), $\mathbf{h}_s = [h_0 \ h_1 \ \cdots \ h_{N/2-1}]^T$; $\mathbf{J}_s$ is the $N/2 \times (2N-1)$ Jacobian matrix given by

$$\mathbf{J}_s = \frac{\partial \mathbf{g}}{\partial \mathbf{h}_s} = [\mathbf{J}_{s1} \, \mathrm{fliplr}(\mathbf{J}_{s1}(:, 1:N-1))]$$

with

$$\mathbf{J}_{s1} = \mathbf{J}_t + \mathbf{J}_h$$

where $\mathbf{J}_t$ is the $N/2 \times N$ Toeplitz matrix whose first column and first row are $[2h_0 \ 0 \ \cdots \ 0]^T$ and $2 \cdot [h_0 \ h_1 \ \cdots \ h_{N/2-1} \ h_{N/2-1} \ \cdots \ h_1 \ h_0]$, $\mathbf{J}_h$ is the $N/2 \times N$ Hankel matrix whose first column is a zero vector and last row is $[0 \ \cdots \ 0 \ 2h_0 \ 2h_1 \ \cdots \ 2h_{N/2-1}]$, and fliplr $(\mathbf{J}_{s1}(:, 1:N-1))$ is the $N/2 \times (N-1)$ matrix generated by flipping the first $N-1$ columns of matrix $\mathbf{J}_{s1}$ from left to right[1]; $\mathbf{H}_s$ is given by

$$\mathbf{H}_s = \begin{bmatrix} \mathbf{H}_{s1} \\ \text{flipud } [\mathbf{H}_{s1}(1:N-1, :)] \end{bmatrix}$$

where $\mathbf{H}_{s1}$ is the $N \times N/2$ Toeplitz matrix whose first column and first row are

$$[h_0 \ h_1 \ \cdots \ h_{N/2-1} \ 2h_{N/2-1} \ \cdots \ 2h_1 \ 2h_0]^T$$

and $[h_0 \ 0 \ \cdots \ 0]$, and flipud $[\mathbf{H}_{s1}(1:N-1, :)]$ denotes the matrix generated by flipping the first $N-1$ rows of $\mathbf{H}_{s1}$ upside down; and $\mathbf{Q}_{as}$ assumes the form

$$\mathbf{Q}_{as} = \mathbf{K}^T \mathbf{Q}_a \mathbf{K}$$
$$\mathbf{K} = \begin{bmatrix} \mathbf{I}_{N/2} \\ \hat{\mathbf{I}}_{N/2} \end{bmatrix}$$

where $\mathbf{I}_{N/2}$ is the $N/2 \times N/2$ identity matrix, and $\hat{\mathbf{I}}_{N/2}$ = flipud $(\mathbf{I}_{N/2})$.

From the above analysis, we see that the linearization through the self-convolution technique described in conjunction with the simple expression for the Jacobian $\mathbf{J}$ has led to analytic expressions for the gradient for the linear-phase case given by (8) as well as for the general case given by (7). These equations are of critical importance for design efficiency and system performance. By letting $\nabla_{\mathbf{h}_s} E = 0$, we obtain

$$\mathbf{h}_s = 2(4\mathbf{J}_s\mathbf{Q}_r\mathbf{H}_s + \alpha \mathbf{Q}_{as})^{-1}\mathbf{J}_s\mathbf{b}. \tag{9}$$

Equation (9) suggests an iterative algorithm for the design problem. One starts by designing a half-band, linear-phase, lowpass FIR filter of length N and uses its first $N/2$ coefficients to construct an initial vector $\mathbf{h}_0$. This $\mathbf{h}_0$ is then used to evaluate matrices $\mathbf{J}_s$ and $\mathbf{H}_s$. A new coefficient vector, denoted as $\mathbf{h}_s$, is calculated by using (9), and $\mathbf{h}_0$ is updated as a linear combination of the previous $\mathbf{h}_0$ and $\mathbf{h}_s$. The iteration continues until $\|\mathbf{h}_0 - \mathbf{h}_s\|$ is less than a prescribed tolerance. A step-by-step description of the algorithm is as follows.

Algorithm 1

Step 1: Design a half-band linear-phase lowpass FIR filter of even length N, and use the first $N/2$ of its coefficients to form a vector denoted as $\mathbf{h}_0$.

Step 2: Calculate $\mathbf{J}_s$ and $\mathbf{H}_s$.

Step 3: Evaluate $\mathbf{h}_s$ using (9).

Step 4: Update $\mathbf{h}_0$ as

$$\mathbf{h}_0 = \beta \mathbf{h}_s + (1 - \beta)\mathbf{h}_0$$

where the weight $\beta \in (0, 1)$ usually assumes a value between 0.3 and 0.8.

[1] fliplr and flipud are MATLAB commands for the matrix operations described here.

Step 5: If $\|\mathbf{h}_0 - \mathbf{h}_s\|_2 < \varepsilon$ where ε is a prescribed tolerance, use $\mathbf{h}_s$ as the first $N/2$ of the coefficients of filter H_0 and construct $G_0(z) = H_0(z)$, $H_1(z) = H_0(-z)$, and $G_1(z) = -H_0(-z)$. Otherwise repeat from Step 2.

The proposed algorithm differs from the existing iterative algorithms proposed in [5] and [12] in several respects. The approaches in [5] and [12] *transform* the fourth-order objective function into a quadratic function by replacing the integrand $|H_0^2(e^{j\omega}) - H_0^2(e^{j(\omega+\pi)}) - e^{-jk_d\omega}|^2$ by $|H_0(e^{j\omega})F_0(e^{j\omega}) - H_0(e^{j(\omega+\pi)})F_0(e^{j(\omega+\pi)}) - e^{-jk_d\omega}|^2$. The proposed approach does *not* modify the objective function E. Instead it utilizes a self-convolution technique to express E as a *formal* quadratic function as in (6). Here the term "formal" is used to stress that vector $\mathbf{g}$ in (6) depends on $\mathbf{h}$ nonlinearly as can been seen in (3). This leads to different formulas that can be used to compute the intermediate impulse response of H_0 in each iteration. For example, in [12] the impulse response is evaluated using [12, eqs. (10), (5), and (7)], while the proposed method computes $\mathbf{h}_s$ using (9). Note that $\mathbf{h}_s$ in (9) can be evaluated more efficiently than its counterpart in [12] since the matrices used in (9) are simple Hankel matrices while matrices $\mathbf{U}$ and $\mathbf{U}_s$ used in [12] involve more computations and many logical decisions. In Section III, design examples will be presented to demonstrate the efficiency of the proposed method as compared to the methods of [5] and [12].

D. Design of Low-Delay QMF Banks

In a linear-phase FIR QMF bank, the reconstruction delay is $N-1$, i.e., the order of the analysis and synthesis filters. In some applications, QMF banks with reconstruction delays less than $N-1$ are desired. In these applications, Algorithm 1 needs to be modified accordingly as shown below.

Algorithm 2

Step 1: Design a half-band, lowpass FIR filter of length N with group delay $k_d/2$. The coefficients of the filter designed are then used as an initial $\mathbf{h}_0$.

Step 2: Calculate $\mathbf{J}$ and $\mathbf{H}$.

Step 3: Evaluate $\mathbf{h}$ using

$$\mathbf{h} = 2(4\mathbf{J}\mathbf{Q}_r\mathbf{H} + \alpha\mathbf{Q}_a)^{-1}\mathbf{J}\mathbf{b}. \tag{10}$$

Step 4: Update $\mathbf{h}_0$ using

$$\mathbf{h}_0 = \beta\mathbf{h} + (1-\beta)\mathbf{h}_0. \tag{11}$$

Step 5: If $\|\mathbf{h} - \mathbf{h}_0\|_2 < \varepsilon$, use $\mathbf{h}$ as the impulse response of H_0, and construct $G_0(z) = H_0(z)$, $H_1(z) = H_0(-z)$, and $G_1(z) = -H_0(-z)$. Otherwise, repeat from Step 2.

It has been observed from experiments that undesirable artifacts may occur in the transition region of filter H_0 when the reconstruction delay k_d is significantly smaller than $N-1$. A possible approach to reduce the amplitudes of these artifacts is to modify the objective function in (6) by including an additional term

$$\alpha_1 \int_{\omega_{t1}}^{\omega_{t2}} |H_0(e^{j\omega}) - e^{-j\omega k_d/2}|^2 d\omega$$

where $[\omega_{t1}, \omega_{t2}]$ is an interval in the transition region where the artifacts occur. It can be shown that

$$E = 4\mathbf{g}^T\mathbf{Q}_r\mathbf{g} - 4\mathbf{g}^T\mathbf{b} + \mathbf{h}^T(\alpha\mathbf{Q}_a + \alpha_1\mathbf{Q}_t)\mathbf{h} - 2\alpha_1\mathbf{d}^T\mathbf{h} + \pi + \alpha_1(\omega_{t2} - \omega_{t1})$$

where $\mathbf{Q}_t = \{\mathbf{q}_{ij}\}$ is a symmetric matrix with

$$q_{ij} = \begin{cases} \omega_{t2} - \omega_{t1} & i = j \\ \frac{1}{|i-j|}[\sin(|i-j|\omega_{t2}) - \sin(|i-j|\omega_{t1})] & i \neq j \end{cases}$$

for $i, j = 1, \ldots, N$, and

$$\mathbf{d} = \begin{bmatrix} d_0 \\ \vdots \\ d_{N-1} \end{bmatrix}$$

with $d_i = [\sin(\rho_i\omega_{t2}) - \sin(\rho_i\omega_{t1})]/\rho_i$ and $\rho_i = -i + k_d/2$. By setting $\nabla_{\mathbf{h}}E = 0$, we obtain

$$\mathbf{h} = (4\mathbf{J}\mathbf{Q}_r\mathbf{H} + \alpha\mathbf{Q}_a + \alpha_1\mathbf{Q}_t)^{-1}(2\mathbf{J}\mathbf{b} + \alpha_1\mathbf{d}). \tag{12}$$

Equation (12) can now replace (10) to provide effective control on the amplitudes of artifacts in the design of low-delay QMF banks.

TABLE I
COMPARISONS OF THE PROPOSED METHOD WITH THE METHOD OF [5]

	Proposed	Method of [5]	Method of [12]
MFLOPS	0.20	1.18	0.45
A_a (dB)	34.70	35.27	34.77
A_p (dB)	0.0114	0.0131	0.0124
PRE (dB)	0.0140	0.0152	0.0147
SNR_s (dB)	81.9	83.4	82.2
SNR_r (dB)	70.6	67.8	69.5
CPU time (s)	0.39	2.42	2.13

III. DESIGN EXAMPLES

In this section, two design examples are given to illustrate the proposed method. The performance of the designs is evaluated in terms of:

- floating-point operations in millions (MFLOPS) used;
- minimum stopband attenuation

$$A_a = \min_{\omega_s \le \omega \le \pi} [-20\log_{10}|H_0(e^{j\omega})|]$$

- peak-to-peak passband ripple

$$A_p = \max_{0 \le \omega \le \omega_p} [20\log_{10}|H_0(e^{j\omega})|] - \min_{0 \le \omega \le \omega_p} [20\log_{10}|H_0(e^{j\omega})|]$$

  where ω_p is the passband edge;
- peak reconstruction error

$$\text{PRE} = \max_{\omega} \left|20\log_{10}\left[\left|H_0^2(\omega) - H_0^2(\omega+\pi)\right|\right]\right|$$

- signal-to-noise ratio (SNR)

$$\text{SNR} = 10\log_{10}\left(\frac{\text{energy of the signal}}{\text{energy of the reconstruction noise}}\right) = 10\log_{10}\left\{\frac{\Sigma x^2(n)}{\Sigma[x(n) - \hat{x}(n+k_d)]^2}\right\}.$$

As Example 1, a linear-phase QMF bank was designed on the basis of the following specifications: $N = 32$, $\alpha = 1$, $\omega_p = 0.2$, $\omega_s = 0.3$, $\beta = 0.6$, and $\varepsilon = 5 \times 10^{-4}$. The initial $\mathbf{h}$ was obtained by using the window method. For comparison purposes, the method of Chen and Lee [5] and the method in [12] were applied to design QMF banks with the same design parameters. These methods were programmed using MATLAB and run on a Pentium PC/100. The number of frequency sampling points was set to $8N$ when implementing the method of [5], where N is the filter length. The results are summarized in Table I where SNR_s and SNR_r denote the SNR with a step input and a random input, respectively. The amplitude responses of the analysis filters and the reconstruction error of the QMF bank designed are depicted in Figs. 2 and 3, respectively. The amplitude responses of the filters designed by the method in [5] are also shown in Fig. 2. As can be observed from the comparisons,

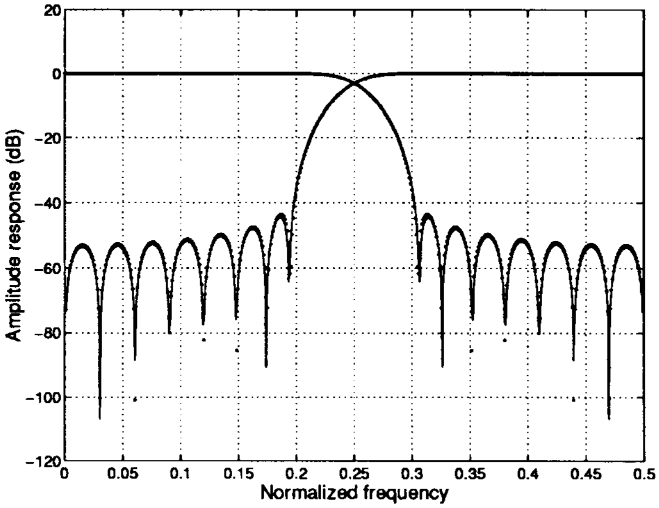

Fig. 2. Amplitude responses of the analysis filters of Example 1: solid line: the proposed method; dotted line: the method of [5].

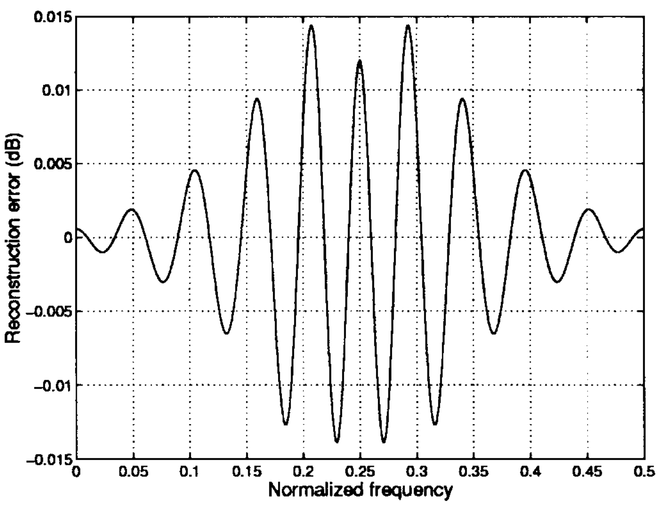

Fig. 3. Reconstruction error for Example 1.

TABLE II
COEFFICIENTS OF THE LOWPASS ANALYSIS FILTER IN EXAMPLE 1

i	h_i	i	h_i
0	0.00129077162668	8	−0.00575709539395
1	−0.00229138388989	9	−0.03301353302975
2	−0.00164465165080	10	0.01672138265228
3	0.00579583179341	11	0.05390461015107
4	0.00131634631555	12	−0.04168419440744
5	−0.01150641455856	13	−0.09966338208708
6	0.000673347241838	14	0.13056029831012
7	0.02013693728515	15	0.46517801436741

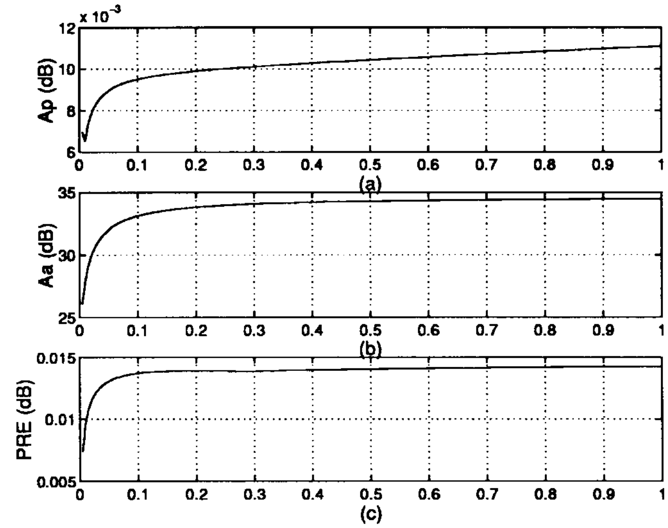

Fig. 4. (a) A_p versus α, (b) A_a versus α, and (c) PRE versus α for $N = 32$, $\omega_p = 0.2$, and $\omega_s = 0.3$.

TABLE III
COMPARISONS OF THE PROPOSED METHOD WITH THE METHOD OF [4] AND [12]

	Proposed	Method of [4]	Method of [12]
MFLOPS	1.44	–	1.81
A_a (dB)	53.15	36.96	52.66
A_p (dB)	0.0036	0.0237	0.0030
PRE (dB)	1.2×10^{-3}	1.2×10^{-3}	1.2×10^{-3}
SNR_s (dB)	82.6	75.8	82.9
SNR_r (dB)	81.7	77.4	80.2
CPU time (s)	0.44	–	2.01

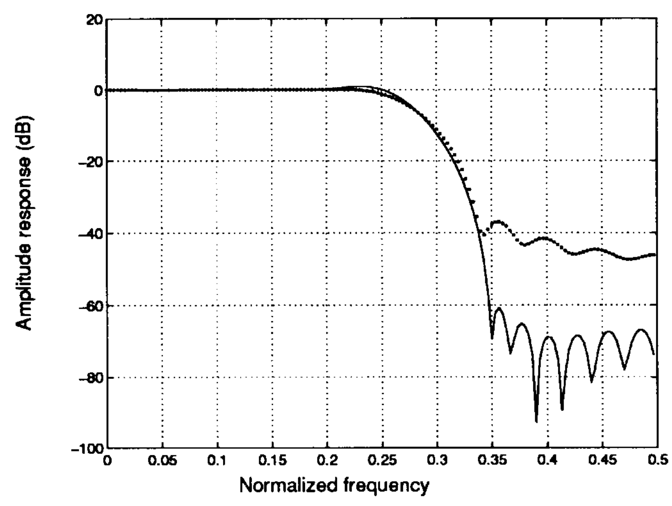

Fig. 5. Amplitude responses of the analysis filters for Example 2: solid line: our design; dotted line: from [4].

the proposed method can achieve almost the same design as the methods of [5] and [12] but with much less computation than that of [5] and less CPU time than that of [5] and [12]. The coefficients of the lowpass analysis filter obtained by the proposed method are listed in Table II.

It follows from (5) that the weight α provides a useful tradeoff among the parameters A_a, PRE, and A_p. In general, with fixed N, ω_p, and ω_s, a large α leads to improved stopband attenuation but degraded A_p, while a small α improves PRE and A_p but tends to reduce A_a. The variation of A_a, A_p, and PRE with α for $0.005 \leq \alpha \leq 1$, $N = 32$, $\omega_p = 0.2$, and $\omega_s = 0.3$ is illustrated in Fig. 4. Note that a very small α, say less than 0.005, should be avoided to prevent matrix $4\mathbf{J}_s\mathbf{Q}_r\mathbf{H}_s + \alpha\mathbf{Q}_{as}$ in (9) from becoming ill-conditioned.

As Example 2, an FIR QMF bank with low reconstruction delay was designed. The design parameters were $N = 32$, $k_d = 15$, $\alpha = 0.3$, $\alpha_1 = 4\times10^{-5}$, $\omega_p = 0.175$, $\omega_s = 0.345$, $\omega_{t1} = 0.175$, $\omega_{t2} = 0.225$, $\beta = 0.5$, and $\varepsilon = 10^{-4}$. The initial $\mathbf{h}_0$ was obtained

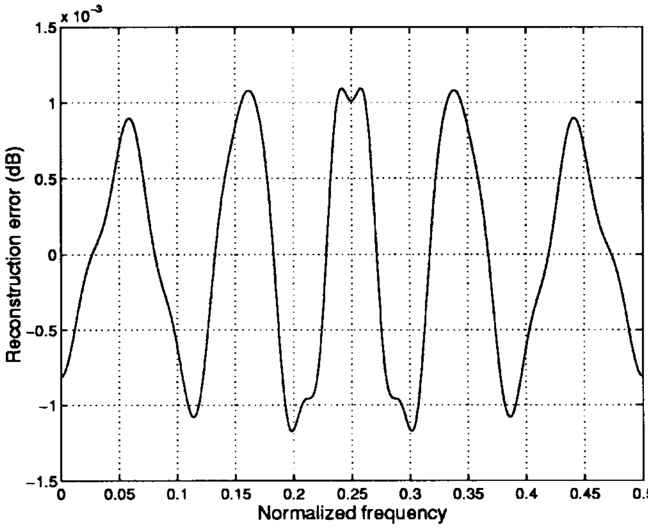

Fig. 6. Reconstruction error for Example 2.

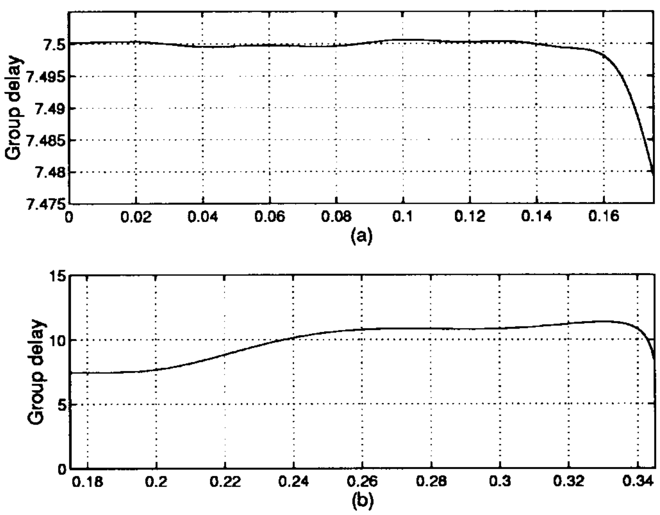

Fig. 7. Group delay of filter H_0 for Example 2: (a) passband and (b) transition band.

by first designing a linear-phase filter using the window method and then truncating its impulse response to the length $N/2+$ floor $(k_d/2)+1$ and padding it with $N/2-$ floor $(k_d/2)-1$ zeros. To identify the interval $[\omega_{t1}, \omega_{t2}]$, we first designed a filter bank with Algorithm 2 in which $\mathbf{h}$ is computed by using (10). The amplitude response of H_0 showed a region in the transition band with undesirable artifacts. Based on this, $[\omega_{t1}, \omega_{t2}]$ was identified as [0.175, 0.225]. For comparison purposes, we refer to [4, Example 6.6.1] which was designed with a time-domain approach. The method proposed in [12] was also applied to design the same filter bank. The results are summarized in Table III. The amplitude responses of the lowpass analysis filters designed with the proposed method and the method of [4] are depicted in Fig. 5. The reconstruction error of the filter bank designed is shown in Fig. 6. The group delay characteristic of the designed filter H_0 is shown in Fig. 7(a) and (b). In Fig. 7(a) we note that the filter designed has approximate linear phase in the passband. Fig. 7(b) shows that the group delay varies somewhat in the transition band but since the signal is significantly attenuated at these frequencies, the delay distortion introduced is less important. As can be observed from Table III and Fig. 5, the proposed method improves the stopband attenuation by more than 14 dB relative to that in the example from [4]. In addition, the quadrature mirror-image structure of the proposed design is amenable to efficient polyphase-type implementation which needs only half of the computation required by the low-delay filter banks of [4].

MATLAB codes for the design of conventional and low delay QMF banks using the proposed method are available from the authors upon request.

IV. Conclusion

A new algebraic method for the design of two-channel QMF banks has been proposed. The new method is efficient and can be used to design both linear-phase and low-delay QMF banks. From the design examples, it is observed that the proposed method leads to filter banks with improved performance in terms of increased minimum stopband attenuation.

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References

[1] J. D. Johnston, "A filter family designed for use in quadrature mirror filter banks," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Processing*, Mar. 1980, pp. 291–294.

[2] V. K. Jain and R. E. Crochiere, "Quadrature mirror filter design in the time domain," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-32, pp. 353–361, Apr. 1984.

[3] G. Pirani and V. Ziegarelli, "An analytical formula for the design of quadrature mirror filters," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-32, pp. 645–648, June 1984.

[4] K. Nayebi, T. P. Barnwell, III, and M. J. T. Smith, "Time-domain filter bank analysis: A new design theory," *IEEE Trans. Signal Processing*, vol. 40, pp. 1412–1429, June 1992.

[5] C.-K. Chen and J.-H. Lee, "Design of quadrature mirror filters with linear phase in the frequency domain," *IEEE Trans. Circuits Syst.*, vol. 39, pp. 593–605, Sept. 1992.

[6] M. J. T. Smith and T. P. Barnwell, III, "Exact reconstruction techniques for tree structured subband coders," *IEEE Trans. Acoust. Speech Signal Processing*, vol. ASSP-34, pp. 434–441, June 1986.

[7] P. P. Vaidyanathan, "Theory and design of M-channel maximally decimated quadrature mirror filters with arbitrary M, having perfect reconstruction property," *IEEE Trans. Acoust. Speech Signal Processing*, vol. ASSP-35, pp. 476–492, Apr. 1987.

[8] P. P. Vaidyanathan and P. Q. Hoang, "Lattice structures for optimal design and robust implementation of two-channel perfect reconstruction QMF banks," *IEEE Trans. Acoust. Speech Signal Processing*, vol. 36, pp. 81–94, Jan. 1988.

[9] T. Q. Nguyen and P. P. Vaidyanathan, "Two-channel perfect-reconstruction FIR QMF structures which yield linear-phase analysis and synthesis filters," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 37, pp. 676–690, May 1989.

[10] B.-R. Horng and A. N. Willson, Jr., "Lagrange multiplier approaches to the design of two-channel perfect-reconstruction linear-phase FIR filter banks," *IEEE Trans. Signal Processing*, vol. 40, pp. 364–374, Feb. 1992.

[11] H. Xu, W.-S. Lu, and A. Antoniou, "Design of perfect reconstruction QMF banks by a null-space-projection method," in *Proc. IEEE Int. Symp. Circuits and Syst.*, May 1995, pp. 965–968.

[12] ——, "Improved iterative methods for the design of quadrature-mirror filter banks," *IEEE Trans. Circuits Syst. II*, vol. 43, pp. 363–371, May 1996.