Five design examples are provided to illustrate our proposed approach. The specifications and the peak stopband attenuation achieved for each examples are tabulated in Table I. The lattice or ladder coefficients and the extension block coefficients for each of the examples are tabulated in Table II. The frequency responses of the analysis filters are shown in Fig. 4. All the design examples are verified to be perfect reconstruction.

IV. Conclusions

We have presented a novel approach for the design of two channel PR filter banks employing linear phase FIR filters. The problem of factorizing the pair of polyphase components $[E_{j0}(z), E_{j1}(z)]$ for each filter is first examined. This differs from existing work [9]–[14], where both $H_0(z)$ and $H_1(z)$ of a PR filter bank were considered in the factorization of the polyphase matrix $\boldsymbol{E}(z) = [E_{jl}(z)]$, $0 \le j, l \le 1$. We showed that under PR condition, the low-pass and high-pass filters must share a common set of generic blocks (either lattice or ladder blocks). Based on this result, a broad class of PR linear phase FIR banks (including all previously reported cases) can be easily obtained by reordering those cascaded structures synthesizing the polyphase components of each filter. Our procedure determines the coefficients of the filter banks in a reverse order in comparison with existing works [9]–[15], without the need for additional transformation [13], [16]. As the PR condition is “structurally” ensured, unconstrained optimization techniques can be used to obtain the coefficients.

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A Direct Method for the Design of 2-D Nonseparable Filter Banks

Wu-Sheng Lu, Andreas Antoniou, and Hua Xu

***Abstract*— A direct approach for the design of 2-D nonseparable diamond-shaped filter banks is proposed. The design goal is achieved by formulating the problem as an iterative quadratic programming problem in which the perfect reconstruction condition is incorporated in a weighted quadratic objective function. Satisfactory stopband attenuation is assured by imposing a set of constraints on the amplitude response of the low-pass analysis filter. Formulas for the exact and efficient evaluation of the objective function are derived, which facilitate precise and efficient function evaluations. The brief concludes with two design examples that illustrate the proposed design method.**

***Index Terms*— 2-D diamond-shaped digital filters, filter banks, optimization.**

I. Introduction

One of the important classes of two-dimensional (2-D) filter banks is the class of nonseparable diamond-shaped filter banks [1], [2]. A low-pass 2-D filter with a diamond-shaped passband retains significant horizontal as well as vertical frequency components while rejecting less important diagonal frequency components. Filter banks of this type have been applied to image and video compression and HDTV with satisfactory results [3]–[5]. Many of the existing methods for the design of 2-D nonseparable filter banks are based on the application of transformations to one-dimensional (1-D) prototype filters [3], [5]–[9], and are referred to as indirect methods here. A direct method for the design of these filter banks by using linear programming is described in [10].

The purpose of this brief is to propose a new direct approach for the design of 2-D nonseparable diamond-shaped filter banks. To achieve reconstruction error and intra-band error minimization, we employ an objective function which is a weighted sum of two error components.

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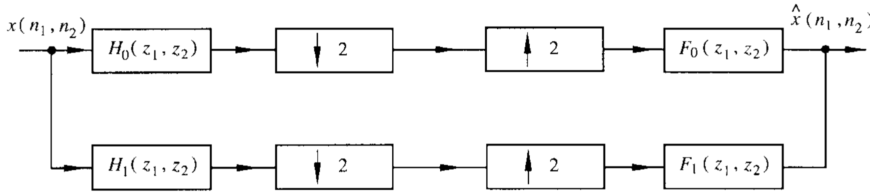


Fig. 1. 2-D diamond-shaped filter bank.

This objective function turns out to be a fourth-order function of the filter coefficients. Constraints on the maximum stopband error in the low-pass analysis filter are imposed to achieve improved stopband attenuation. To solve this nonlinear constrained optimization problem efficiently, a technique similar to the one used in the design of 1-D filter banks [11]–[13] is used to modify the objective function into a quadratic function with positive definite Hessian matrix. Furthermore, analytic formulas are derived to evaluate the terms involved in the modified objective function. These formulas facilitate precise and efficient function evaluations as compared to summing up a large number of discretized terms, which would be time consuming and inevitably introduce additional errors into the design. As a result, the design problem at hand is reduced to an iterative quadratic programming problem which can be solved by using, for example, the active set method [14], [15]. Two examples are given to illustrate the design method proposed and to compare it to the unconstrained least-squares design. These design examples demonstrate that the availability of the weight in the objective function and the constraints on the stopband attenuation of the low-pass analysis filter render the approach more flexible, and enable the designer to choose between low peak reconstruction error for the filter bank and high minimum stopband attenuation for the individual filters.

II. Proposed Design Method

A. Formulation of the Objective Function

A 2-D nonseparable diamond-shaped filter bank can be represented by the system shown in Fig. 1. The ideal frequency response of the analysis low-pass filter H_0 is illustrated in Fig. 2(a). After filtering by the analysis filters,the signals are quincunx downsampled as shown in Fig. 2(b), where half of the samples are discarded. At the reconstruction end, the signals are upsampled by inserting zero values between adjacent samples. Analytically the input and output relationship of the filter bank can be expressed as

$$\begin{aligned}\hat{X}&(z_1, z_2)\\ &= \frac{1}{2}\Big[H_0(z_1, z_2)F_0(z_1, z_2)\\ &\quad + H_1(z_1, z_2)F_1(z_1, z_2)\Big]X(z_1, z_2)\\ &\quad + \frac{1}{2}\Big[H_0(-z_1, -z_2)F_0(z_1, z_2) + H_1(-z_1, -z_2)F_1(z_1, z_2)\Big]\\ &\quad \cdot X(-z_1, -z_2) \qquad (1)\end{aligned}$$

where the first term represents the input signal component and the second term represents the aliasing component. If

$$H_1(z_1, z_2) = z_1^{-1}H_0(-z_1, -z_2) \qquad (2a)$$

$$F_0(z_1, z_2) = H_0(z_1, z_2) \qquad (2b)$$

$$F_1(z_1, z_2) = z_1 H_0(-z_1, -z_2) \qquad (2c)$$

the aliasing term in (1) is cancelled and (1) becomes

$$\hat{X}(z_1, z_2) = [H_0^2(z_1, z_2) + H_0^2(-z_1, -z_2)]X(z_1, z_2). \qquad (3)$$

So if the condition

$$H_0^2(\omega_1, \omega_2) + H_0^2(\omega_1 + \pi, \omega_2 + \pi) = 1 \qquad (4)$$

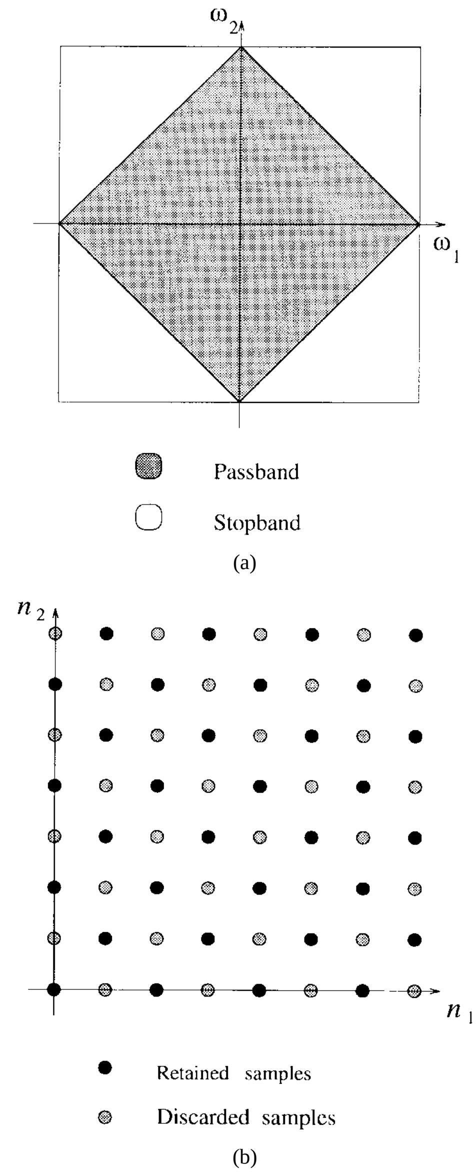


Fig. 2. Ideal frequency response and quincunx downsampling.

is satisfied for $-\pi \leq \omega_1 \leq \pi, -\pi \leq \omega_2 \leq \pi$, then the output signal will be a replica of the input signal.

Assuming that the region of support of filter H_0 is a $(2N_1 - 1) \times (2N_2 - 1)$ rectangle centered at the origin of the (n_1, n_2) plane and its impulse response $h(n_1, n_2)$ for $n_1 = -(N_1 - 1), \cdots, N_1 - 1$ and $n_2 = -(N_2 - 1), \cdots, N_2 - 1$ is of quadrantal symmetry, then the frequency response of filter H_0 can be expressed as [16]

$$\begin{aligned}H_0(\omega_1, \omega_2) &= \sum_{n_1=0}^{N_1-1}\sum_{n_2=0}^{N_2-1} a(n_1, n_2)\cos(n_1\omega_1)\cos(n_2\omega_2)\\ &= \boldsymbol{h}^T \mathbf{c}(\omega_1, \omega_2) \qquad (5)\end{aligned}$$

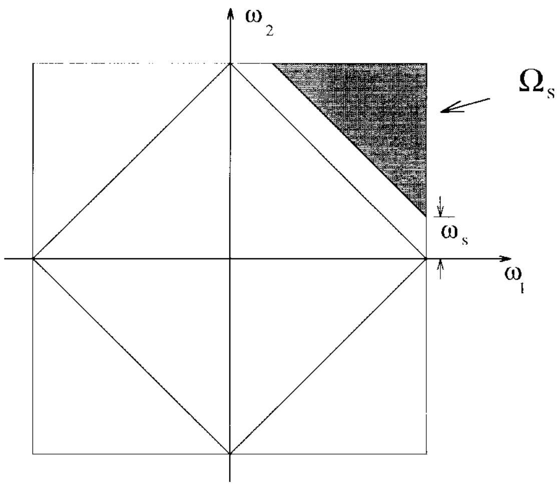


Fig. 3. Stopband region of filter H_0.

where

$$\begin{aligned} a(0,\,0) &= h(0,\,0) \\ a(n_1,\,0) &= 2h(n_1,\,0), \qquad \text{for } 1 \le n_1 \le N_1 - 1 \\ a(0,\,n_2) &= 2h(0,\,n_2), \qquad \text{for } 1 \le n_2 \le N_2 - 1 \\ a(n_1,\,n_2) &= 4h(n_1,\,n_2), \qquad \text{for } 1 \le n_i \le (N_i - 1) \\ & \qquad\qquad\qquad\qquad i = 1,\,2 \end{aligned}$$

and

$$\begin{aligned} \boldsymbol{h} = [&a(0,\,0) \quad a(1,\,0) \,\cdots\, a(N_1-1,\,0) \quad a(0,\,1)\cdots \\ &a(N_1-1,\,1)\cdots a(N_1-1,\,N_2-1)]^T. \end{aligned}$$

Vector $\boldsymbol{c}(\omega_1,\,\omega_2)$ is a column vector whose ith entry can be expressed as $c(i) = \cos p(i)\omega_2 \cdot \cos q(i)\omega_1$ with $p(i) = \text{int}[(i-1)/N_1]$ and $q(i) = i - 1 - \text{int}[(i-1)/N_1] \cdot N_1$ for $1 \le i \le N_1 \times N_2$.

As for the 1-D case, there are two requirements for a 2-D analysis/synthesis system, namely, perfect reconstruction error and intra-band aliasing minimization. To satisfy these requirements, an objective function can be formed as

$$E = E_1 + \alpha E_2 \tag{6}$$

where $\alpha > 0$ is a weight and the first error component E_1, defined as

$$E_1 = \int_0^\pi \int_0^\pi \left[H_0^2(\omega_1,\,\omega_2) + H_0^2(\omega_1+\pi,\,\omega_2+\pi) - 1\right]^2 d\omega_1\, d\omega_2$$

is used to approximate perfect reconstruction and the second error component E_2, defined as

$$E_2 = \int\int_{\boldsymbol{\Omega}_s} H_0^2(\omega_1,\,\omega_2)\, d\omega_1\, d\omega_2$$

is used to reduce intra-band aliasing. The region of integration Ω_s is the shaded area in Fig. 3, which corresponds to the stopband of filter H_0.

B. Iterative Design Technique

Here we use an iterative method in which the error function E in (6) is modified as

$$E' = E_1' + \alpha E_2' \tag{7}$$

where

$$\begin{aligned} E_1' = \int_0^\pi \int_0^\pi &[H_0(\omega_1,\,\omega_2)G_0(\omega_1,\,\omega_2) + H_0(\omega_1+\pi,\,\omega_2+\pi) \\ &\cdot G_0(\omega_1+\pi,\,\omega_2+\pi) - 1]^2 d\omega_1\, d\omega_2 \end{aligned}$$

and

$$E_2' = \int\int_{\Omega_s} G_0^2(\omega) d\omega_1\, d\omega_2.$$

Like filter H_0, filter G_0 is a low-pass FIR filter of the same order with quadrantally symmetrical impulse response and a frequency response $G_0(\omega_1,\,\omega_2) = \boldsymbol{g}^T\boldsymbol{c}(\omega_1,\,\omega_2)$ where $\boldsymbol{g}$ is a column vector whose elements are the values of the impulse response of filter G_0. In order to assure a satisfactory stopband attenuation in filter G_0, we impose the constraint

$$|G_0(e^{j\omega_1},\,e^{j\omega_2})| \le \delta_s \quad \text{for} \quad (\omega_1,\,\omega_2) \in \hat{\Omega}_s \tag{8}$$

where $\hat{\Omega}_s$ denotes a discrete set of sampling points in region Ω_s and δ_s is the maximum error allowed in the stopband of filter G_0. By using (5), (8) can be written as

$$\boldsymbol{c}^T(\omega_1,\,\omega_2)\boldsymbol{g} \le \delta_s \quad \text{and} \quad -\boldsymbol{c}^T(\omega_1,\,\omega_2)\boldsymbol{g} \le \delta_s \tag{9}$$

for $(\omega_1,\,\omega_2) \in \hat{\Omega}_s$. If $\hat{\Omega}_s$ contains K points $(\omega_{1i},\,\omega_{2i})$, $1 \le i \le K$, then the $2K$ constraints in (9) can be expressed in matrix notation as

$$\boldsymbol{C}\boldsymbol{g} \le \delta_s \boldsymbol{e} \tag{10}$$

where

$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{c}^T(\omega_{11},\,\omega_{21}) \\ -\mathbf{c}^T(\omega_{11},\,\omega_{21}) \\ \vdots \\ \boldsymbol{c}^T(\omega_{1K},\,\omega_{2K}) \\ -\mathbf{c}^T(\omega_{1K},\,\omega_{2K}) \end{bmatrix}_{2K\times N_1N_2} \quad \text{and} \quad \boldsymbol{e} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ 1 \end{bmatrix}_{2K\times 1}.$$

To start the iteration, we design a 2-D diamond-shaped low-pass filter with a conventional 2-D FIR filter design method [16], and use its coefficients to form the initial $\boldsymbol{h}$. After extensive manipulation, it can be shown that E' in (7) can be written in terms of a quadratic form of $\boldsymbol{g}$ as

$$E' = \boldsymbol{g}^T(\boldsymbol{U} + \alpha\boldsymbol{U}_s)\boldsymbol{g} - 2\mathbf{h}^T\boldsymbol{B}\boldsymbol{g} + \pi^2 \tag{11}$$

where $\boldsymbol{B} = \{b_{ij},\,1 \le i,\,j \le N_1N_2\}$ with

$$\begin{aligned} b_{ij} = \frac{\pi^2}{2}&\{\delta[p(i)+p(j)] + \delta[p(i)-p(j)]\} \\ &\cdot \{\delta[q(i)+q(j)] + \delta[q(i)-q(j)]\}. \end{aligned} \tag{12a}$$

In the above equations, $\delta(k)$ is the unit impulse and $\boldsymbol{U}$ is given by $\boldsymbol{U} = \{u_{ij},\,1 \le i,\,j \le N_1N_2\}$ with

$$\begin{aligned} u_{ij} = 2 &\sum_{n_1=0}^{N_1-1}\sum_{n_2=0}^{N_2-1}\sum_{m_1=0}^{N_1-1}\sum_{m_2=0}^{N_2-1} h(n_1,\,n_2)h(m_1,\,m_2) \\ &\cdot [1 + (-1)^{m_1+m_2+p(i)+q(j)}] \cdot I[n_1,\,m_1,\,q(i),\,q(j)] \\ &\cdot I[n_2,\,m_2,\,p(i),\,p(j)] \end{aligned} \tag{12b}$$

where

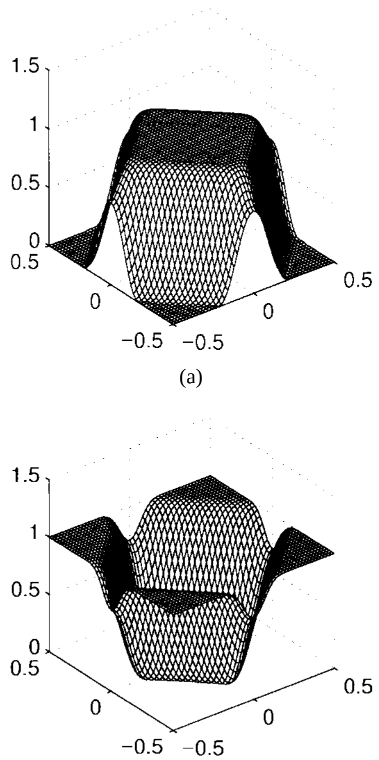
$$I(n_1,\,n_2,\,n_3,\,n_4) = \int_0^\pi \cos n_1\omega \cos n_2\omega \cos n_3\omega \cos n_4\omega\, d\omega. \tag{12c}$$

To evaluate $\boldsymbol{U}_s$, we note from Fig. 3 that

$$E_2' = \boldsymbol{g}^T\left[\int_{\omega_s}^\pi \int_{\pi-(\omega_2-\omega_s)}^\pi \mathbf{c}(\omega_1,\,\omega_2)\mathbf{c}^T(\omega_1,\,\omega_2)\, d\omega_1\, d\omega_2\right]\boldsymbol{g}$$

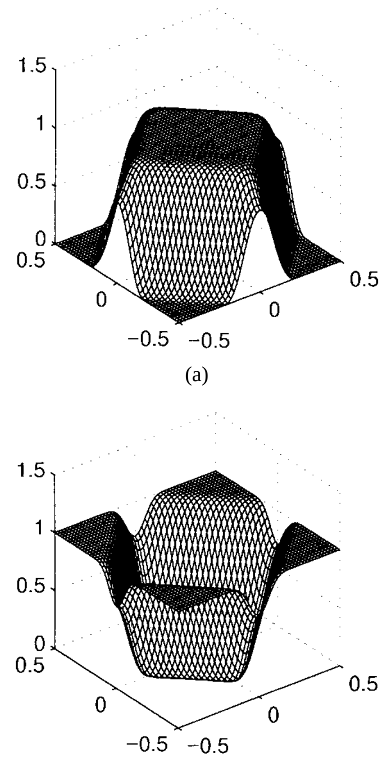
where ω_s is the stopband edge and hence $\boldsymbol{U}_s = \{u_{ij}^{(s)},\,1 \le i,\,j \le N_1N_2\}$ with $u_{ij}^{(s)} = \sigma_{ij}/4$, and

$$\begin{aligned} \sigma_{ij} = &\int_{\omega_s}^\pi \Big\{\cos\,[p(i)+p(j)]\omega_2 + \cos\,[p(i)-p(j)]\omega_2\Big\}\, d\omega_2 \\ &\cdot \int_{\pi-(\omega_2-\omega_s)}^\pi \Big\{\cos\,[q(i)+q(j)]\omega_1 + \cos\,[q(i)-q(j)]\omega_1\Big\} \\ &\cdot d\omega_1. \end{aligned} \tag{12d}$$



(a) (b)

Fig. 4. (a) Amplitude response of filter H_0 for Example 1 and (b) amplitude response of filter H_1 for Example 1.



(a) (b)

Fig. 5. (a) Amplitude response of filter H_0 for Example 2. (b) amplitude response of filter H_1 for Example 2.

Since $\boldsymbol{U}^T\boldsymbol{U} + \alpha\boldsymbol{U}_s^T\boldsymbol{U}_s$ is positive definite, E' is a strictly globally convex quadratic function, and minimizing E' subject to constraints (9) is a typical *quadratic programming* (QP) problem which can be solved efficiently by using the active set method [14], [15]. Having obtained $\boldsymbol{g}$, linear interpolation can be used to update $\boldsymbol{h}$ as

$$\boldsymbol{h} := (1-\tau)\boldsymbol{h} + \tau\boldsymbol{g} \tag{13}$$

where τ is a smoothing parameter in the range $0 < \tau < 1$. The above process is repeated until $\|\boldsymbol{h} - \boldsymbol{g}\|$ is less than a prescribed tolerance. The design procedure can now be summarized in terms of the following algorithm.

Algorithm:

Step 1: Use a conventional method (e.g., the singular-value decomposition method) to design a 2-D, $(2N_1-1)\times(2N_2-1)$ diamond-shaped FIR filter and use its impulse response as the initial $\boldsymbol{h}$.

Step 2: Calculate $\boldsymbol{B}$ and $\boldsymbol{U}_s$ using (12a) and (12d), respectively.

Step 3: Use (12b) to form $\boldsymbol{U}$ and solve the QP problem

$$\text{minimize} \quad E' = \boldsymbol{g}^T(\boldsymbol{U} + \alpha\boldsymbol{U}_s)\boldsymbol{g} - 2\boldsymbol{h}^T\boldsymbol{B}\boldsymbol{g} + \pi^2 \tag{14a}$$

$$\text{subject to} \quad \boldsymbol{C}\boldsymbol{g} \le \delta_s\boldsymbol{e}. \tag{14b}$$

Step 4: If $\|\boldsymbol{h} - \boldsymbol{g}\| < \epsilon$, where ϵ is a prescribed tolerance, output $\boldsymbol{g}$ as the design result and stop. Otherwise, update $\boldsymbol{h}$ using (13) and repeat from Step 3.

Several comments on the above design algorithm are now in order. Matrices $\boldsymbol{B}$ and $\boldsymbol{U}_s$ can be pre-calculated as soon as N_1, N_2, and ω_s are determined; this would make the design algorithm computationally more efficient. In addition, the integrals involved in determining the elements of $\boldsymbol{U}$ can be pre-calculated and stored in an array which can be loaded for use when the iteration starts. Note that $\boldsymbol{B}$, $\boldsymbol{U}_s$, and $\boldsymbol{U}$ are all symmetrical matrices and, therefore, only about half of their entries need to be computed. Our numerical experiments indicate that a value of τ around 0.5 leads the fastest convergence.

TABLE I
PERFORMANCE EVALUATIONS OF FILTER BANKS

		Example 1	Example 2
with constraints (14b)	Iterations	11	19
	PRE (dB)	3.9210e−4	2.4852e−4
	MSA (dB)	40.6129	44.2448
without constraints (14b)	Iterations	13	10
	PRE (dB)	3.6362 e−4	1.7251e−4
	MSA (dB)	31.1483	34.4826

III. DESIGN EXAMPLES

We now present two design examples to illustrate the proposed algorithm. The design parameters were $N_1 = N_2 = 8$, $\alpha = 0.001$, $\tau = 0.5$, $\omega_s = 1.2$, $\delta_s = 0.016$, $\epsilon = 10^{-3}$ for Example 1 and $N_1 = N_2 = 9$, $\alpha = 0.001$, $\tau = 0.5$, $\omega_s = 1.2$, $\delta_s = 0.01$, $\epsilon = 10^{-3}$ for Example 2. The perfect reconstruction performance of the filter banks obtained was evaluated in terms of the peak reconstruction error PRE $= \max_{\omega_1,\omega_2} |20\log_{10}[H_0^2(\omega_1, \omega_2) + H_0^2(\omega_1+\pi, \omega_2+\pi)]|$. The stopband attenuation of the individual filters was evaluated in terms of the minimum stopband attenuation MSA $= \min_{\omega_1,\omega_2\in\Omega_s}[-20\log_{10}|H_0(\omega_1, \omega_2)|]$.

To compare the results obtained with constraints (14b) to those obtained without these constraints, the designs were first carried out by applying the algorithm described in Section II-B. The constraints in (14b) were formulated on a set $\hat{\Omega}_s$ consisting of 15 points that are evenly placed in region Ω_s (see Fig. 3). The results obtained are listed in the top half of Table I. Next, Step 3 of the algorithm was modified by dropping the constraints in (14b). In this case, the vector $\boldsymbol{g}$ that minimizes (14a) is given by $\boldsymbol{g} = (\boldsymbol{U} + \alpha\boldsymbol{U}_s)^{-1}\boldsymbol{B}\boldsymbol{h}$. The modified algorithm was applied to redesign the two filter banks and the results obtained are listed in the bottom half of Table I. It is observed that the QP problem with constraints (14b) improves the stopband attenuation significantly compared to that obtained with the weighted least-squares design but at the expense of a slight increase in PRE. As

TABLE II
COEFFICIENTS OF $H_0(z_1, z_2)$ FOR EXAMPLE 1

Columns 1 through 3		
−1.2972901e−5	1.2316237e−4	−7.5212207e−5
1.2355540e−4	−1.2780882e−4	−1.9663685e−5
−7.5319075e−5	−1.9350810e−5	−7.1947086e−4
6.3400249e−5	−2.4947178e−4	4.4905711e−4
9.6404973e−5	−4.6116254e−5	1.2371871e−3
−7.6955555e−5	−6.5618379e−4	5.7752252e−4
2.9802986e−4	−2.1365364e−5	1.9701350e−3
−1.8556637e−4	−7.1279432e−4	3.3839195e−4
Columns 4 through 6		
6.3686104e−5	9.4800610e−5	−7.5862919e−5
−4.5956538e−5	−6.5195193e−4	−2.4722942e−4
1.2295412e−3	5.7411214e−4	4.4705422e−4
−4.1053629e−3	−2.8588307e−3	4.3782726e−3
−1.1675575e−2	1.6173911e−2	−4.1197559e−3
1.6211426e−2	2.1310378e−2	−2.8712621e−3
4.5047673e−3	−4.8489158e−2	−3.1809526e−3
1.1662001e−2	−5.9398223e−3	−3.4467920e−3
Columns 7 through 8		
2.9586164e−4	−1.8430337e−4	
−2.1538331e−5	−7.0882131e−4	
1.9623554e−3	3.3596717e−4	
−3.1690509e−3	−3.4371484e−3	
4.4911165e−3	1.1635130e−2	
−4.8422645e−2	−5.9246338e−3	
−2.9406153e−2	1.8993868e−1	
1.9006499e−1	5.7235228e−1	

can be seen in Table I, the algorithm with constraints (14b) converged after 11 and 19 iterations for Examples 1 and 2, respectively. The amplitude responses of filters H_0 and H_1 for Examples 1 and 2 are shown in Figs. 4 and 5, respectively. The coefficient matrix of filter H_0, $\{h(7-i,\ 7-j), 0 \le i, j \le 7\}$, for Example 1 using constrained optimization is given in Table II.

IV. CONCLUSIONS

A direct method for the design of 2-D nonseparable diamond-shaped filter banks has been proposed. The method is based on an iterative quadratic programming technique that leads to an efficient design algorithm. The availability of parameter α and the linear inequality constraints on the stopband attenuation of filter H_0 render the approach more flexible and enable the designer to choose between low PRE in the filter bank and high MSA in the individual filters.

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A Subband Adaptive Filter With the Statistically Optimum Analysis Filter Bank

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***Abstract*— Conventional subband adaptive digital filters using filter banks have shown degradation in their performance because of the nonideal nature of analysis filters. For this problem, we propose two kinds of subband adaptive digital filters. The first one has the statistically optimum analysis filter bank which minimizes error signals. This first method, however, needs some statistics of the signals; therefore, we propose another type of subband adaptive digital filter which requires no *a priori* knowledge of the statistics regarding the signals. Computer simulations are also given in order to show the efficiency of the proposed subband adaptive digital filters.**

***Index Terms*—Filter bank, multirate signal processing, statistical analysis, subband adaptive digital filter.**

I. INTRODUCTION

In recent years, there has been an interest in adaptive signal processing applications that require filters with very long impulse responses; for example, several thousands of FIR (Finite Impulse

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