A Lower Bound for the Half-Power Frequency in *RC* Amplifiers

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***Abstract*—It is shown that in *RC* amplifiers there exists a simple relationship between the half-power frequency ω_h and the so-called first-order time moment; a more precise estimate of ω_h than that usually adopted is, therefore, given.**

INTRODUCTION

In accordance with the classic time moment methods, the half-power frequency in *RC* amplifiers is commonly estimated by means of one of the following relationships:

$$\omega_h \simeq \frac{1}{T_1} \tag{1}$$

$$\times \frac{\sqrt{\ln 2}}{T_2} < \omega_h \leqslant \frac{1}{T_2} \tag{2}$$

where T_1 and T_2 are the time moments of first- and second-order, respectively (see, for example, [1], [2]). T_1 and T_2 can be calculated directly either as a function of the circuit parameters or by means of the time constants τ_i of the transfer function

$$H(s) = \frac{1}{\prod_{i=1}^{n} (1+\tau_i s)} \tag{3}$$

and are given by

$$T_1 = \sum_{i=1}^{n} \tau_i \tag{4}$$

$$T_2 = \left(\sum_{i=1}^{n} \tau_i^2 \right)^{1/2}. \tag{5}$$

In this paper it is shown the following relationship:

$$\omega_h > \frac{1}{T_1} \tag{6}$$

so that ω_h lies within the range

$$\frac{1}{T_1} < \omega_h \leqslant \frac{1}{T_2} \tag{7}$$

from whence ω_h can be evaluated more precisely than in either (1) or in (2) when $T_2/T_1 > \sqrt{\ln^2}$.

Proof: Indicating by σ_k the fundamental symmetric polynomial of order k [3], [4] of the n time constants $\tau_1, \tau_2, \cdots, \tau_n$, i.e., by putting

$$\sigma_k = \sum_{i_1, i_2, \cdots, i_k} \tau_{i_1} \cdot \tau_{i_2} \cdots \tau_{i_k}, \qquad k = 1, 2, \cdots, n \tag{8}$$

where the summation is extended to all the k-ples $(i_1, i_2, \cdots, i_k)$ with

$$1 \leqslant i_1 < i_2 < \cdots < i_k \leqslant i_n \tag{9}$$

from (3) it can be derived that ω_h must be a solution of the equation

$$\sum_{k=1}^{n} (a_k - b_k) \omega^{2k} - 1 = 0 \tag{10}$$

where a_k and b_k are given by

$$a_k = \sigma_k^2 + 2 \sum_{r \text{odd}} \sigma_{k-r} \sigma_{k+r} \tag{11}$$

$$b_k = 2 \sum_{r \text{even}} \sigma_{k-r} \sigma_{k+r}, \qquad r, k = 1, 2, \cdots, n \tag{12}$$

by assuming that $\sigma_0 = 1$ and $\sigma_k = 0$ for $k < 0$ and for $k > n$, respectively, (10) can be rewritten as

$$(\sigma_1^2 \omega^2 - 1) + \sum_{k=1}^{n-1} (a_{k+1} \omega^2 - b_k) \omega^{2k} = 0 \tag{13}$$

by substituting the index $k+1$ for k and by observing that $a_1 = \sigma_1^2$ and $b_n = 0$. Since it is always verified that

$$\sigma_{k+2} < \sigma_1^2 \sigma_k \tag{14}$$

it follows that

$$a_{k+1} \left(\frac{1}{\sigma_1} \right)^2 - b_k < 0$$

and the first term of (13) is negative when $\omega = 1/\sigma_1$; moreover, since this term is an increasing function for $\omega > 0$, it is evident that

$$\omega_h > \frac{1}{\sigma_1} \tag{15}$$

and thus (6) is proved.

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A Note on the Roundoff Noise in 2-D State-Space Digital Filters

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***Abstract*—An example is provided which illustrates that a proof provided recently by Mertzios in connection with the roundoff noise in 2-D state-space digital filters includes a step that is not valid in general.**

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In a recent contribution, Mertzios [1] relates the roundoff noise variance and the scaling of the internal registers in 2-D state-space digital filters with two impulse response sequences. In addition, he establishes two Lyapunov equations which correspond to similar equations used for the noise analysis and optimal synthesis of 1-D digital filters.

The 2-D filter considered is represented by Roesser's local state-space model

$$\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u(i,j)$$

$$\equiv Ax + bu \tag{1a}$$

$$y(i,j) = [c_1 \quad c_2] \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + du(i,j)$$

$$\equiv cx + du. \tag{1b}$$

Let us define

$$A^{0,0} = I, \quad A^{1,0} = \begin{bmatrix} A_1 & A_2 \\ 0 & 0 \end{bmatrix} \quad A^{0,1} = \begin{bmatrix} 0 & 0 \\ A_3 & A_4 \end{bmatrix}$$

$$A^{i,j} = A^{1,0}A^{i-1,j} + A^{0,1}A^{i,j-1}$$

$$= A^{i-1,j}A^{1,0} + A^{i,j-1}A^{0,1}, \qquad \text{for } (i,j) > (0,0)$$

$$A^{-i,j} = A^{i,-j} = 0, \qquad \text{for } i \geqslant 1, \; j \geqslant 1$$

$$b^{1,0} = \begin{bmatrix} b_1 \\ 0 \end{bmatrix} \qquad b^{0,1} = \begin{bmatrix} 0 \\ b_2 \end{bmatrix}$$

$$K = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \left(A^{i-1,j}b^{1,0} + A^{i,j-1}b^{0,1} \right) \cdot \left(A^{i-1,j}b^{1,0} + A^{i,j-1}b^{0,1} \right)^T \tag{2}$$

and

$$W = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \left(A^{i,j} \right)^T c^T c A^{i,j}. \tag{3}$$

Mertzios states that matrices K and W defined in (2) and (3) satisfy, respectively, the following two Lyapunov equations:

$$K - AKA^T = \begin{bmatrix} b_1 b_1^T & 0 \\ 0 & b_2 b_2^T \end{bmatrix} \tag{4}$$

$$W - A^T W A = c^T c. \tag{5}$$

However, the proof of the above statement includes a step which is not valid in general. Specifically, once the way of ordering components $f(\cdot,\cdot)$ in double-indexed matrix $l(i,j)$ is determined as in (A3) of [1], equalities

$$\lim_{\substack{i \to \infty \\ j \to \infty}} l(i,j) l^T(i,j) = \lim_{\substack{i \to \infty \\ j \to \infty}} l(i-1,j) l^T(i,j-1)$$

$$= \lim_{\substack{i \to \infty \\ j \to \infty}} l(i,j-1) l^T(i-1,j)$$

do not hold in general and, consequently, matrices K and W defined in (2) and (3), respectively, do not satisfy Lyapunov equations (4) and (5) in general. This is demonstrated through the following example.

Example: Consider a first-order 2-D filter represented by state-space model (1) where

$$A = \begin{bmatrix} 0.6 & 0.3 \\ 0.3 & 0.6 \end{bmatrix} \quad b = \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix} \quad \text{and} \quad c = [1.0 \quad 0.5]. \tag{6}$$

Since the filter is asymptotically stable, both double summations (2) and (3) are convergent. One can, therefore, compute numerically within an accuracy of six decimal places matrices K and W directly from (2) and (3) to obtain

$$K = \begin{bmatrix} 2.183894 & 0.558728 \\ 0.558728 & 2.183894 \end{bmatrix} \tag{7}$$

and

$$W = \begin{bmatrix} 1.851003 & 1.119576 \\ 1.119576 & 0.823606 \end{bmatrix} \tag{8}$$

which give

$$K - AKA^T = \begin{bmatrix} 1.0 & -0.478901 \\ -0.478901 & 1.0 \end{bmatrix}$$

$$\neq \begin{bmatrix} b_1 b_1^T & 0 \\ 0 & b_2 b_2^T \end{bmatrix} = \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix}$$

and

$$W - A^T W A = \begin{bmatrix} 0.707470 & 0.134337 \\ 0.134337 & -0.042529 \end{bmatrix} \neq c^T c = \begin{bmatrix} 1.0 & 0.5 \\ 0.5 & 0.25 \end{bmatrix}$$

In fact, both Lyapunov equations (4) and (5) have unique, positive-definite solutions as

$$K = \begin{bmatrix} 3.181029 & 2.082128 \\ 2.082128 & 3.181029 \end{bmatrix}$$

and

$$W = \begin{bmatrix} 3.542906 & 2.891845 \\ 2.891845 & 2.515509 \end{bmatrix}$$

that are obviously different from the matrices given in (7) and (8), respectively.

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On the Aggregation of Interconnected Systems

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Abstract —**The paper presents a theorem showing that in the component connection model framework a large scale interconnected system is aggregable if and only if one of its components is aggregable.**

I. Introduction

The inherent structure of large scale interconnected systems provides the basis for modeling these systems through their individual components and the interconnecting equations. Decomposition techniques have been applied in many different

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