Transactions Briefs

Two-Dimensional FIR Notch Filter Design Using Singular Value Decomposition

S.-C. Pei, W.-S. Lu, and C.-C. Tseng

***Abstract*— This paper is concerned with the two-dimensional (2-D) linear-phase FIR notch filter-design problem. First, the singular value decomposition (SVD) is used to reduce the 2-D notch filter-design problem to two pairs of one-dimensional (1-D) filter design problems. Then, we derive an analytic least-squares solution for the design of two pairs of 1-D linear-phase FIR filters. Again, the coefficients of the filter designed are given by closed-form formulas and the filter gain at the notch frequencies is exactly zero. One example is included to illustrate the proposed design methods. To demonstrate the usefulness of the 2-D notch filters designed, the 2-D notch filter is applied to eliminate the sinusoidal interference superimposed on an image.**

***Index Terms*—Interference removal, singular value decomposition, 2-D FIR notch filter.**

I. Introduction

Notch filters have been an effective means for eliminating narrow-band or sinusoidal interferences in certain signal-processing applications ranging from power line-interference cancellation for electrocardiograms to multiple sinusoidal interference removal for corrupted images. For the one-dimensional (1-D) case, several methods for the design and performance analysis of IIR and FIR notch filters have been developed [1]–[5]. For the two-dimensional (2-D) case, [6] and [7] proposed a method that reduces the design of a stable IIR 2-D notch filter to the design of a 2-D parallel-line filter and a 2-D straight-line filter. However, the IIR notch filter in [6] is nonlinear phase, so it is meaningful to design a linear-phase notch filter for the image processing applications. Thus, an approach for 2-D linear phase FIR notch filter design will be addressed in this paper.

In the 2-D filter-design literature, the singular value decomposition (SVD) has been widely used to reduce the 2-D filter-design problem to a set of 1-D subfilter-design problems. The design examples of the low-pass, bandpass, and fan digital filters have been investigated in [8] and [9]. In this paper, we will use the SVD method to design a 2-D FIR notch filter. Due to the simplicity of the specification of the notch filter, this SVD-based design has the following two unique features which do not exist for general filter-design cases in [9]. First, the SVD of the sampled magnitude response matrix of the desired 2-D linear phase notch filter reveals that the design problem can be solved by only finding two pairs of 1-D transfer functions. Second, the approximation of each 1-D transfer function involved has the closed-form formula without requiring any iterative optimization procedure. Due to these two features, it is interesting to investigate this design problem in this paper.

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S.-C. Pei and C.-C. Tseng are with the Department of Electrical Engineering, National Taiwan University, Taipei, Taiwan, R.O.C.

W.-S. Lu is with the Department of Electrical and Computer Engineering, University of Victoria, Victoria, B.C., Canada V8W 3P6.

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II. The Design of 2-D Linear-Phase FIR Notch Filters

In this section, we will concentrate on the design of a 2-D linear-phase FIR notch filter. The desired magnitude response is given by

$$\left|H_d(e^{j\omega_1},\, e^{j\omega_2})\right| = \begin{cases} 0, & (\omega_1,\, \omega_2) = (\omega_1^*,\, \omega_2^*) \text{ and } (-\omega_1^*,\, -\omega_2^*) \\ 1, & \text{otherwise.} \end{cases} \tag{1}$$

We denote the sampled magnitude response matrix by $\mathbf{D} \in R^{M_1 \times M_2}$. If the notch frequencies $(\omega_1^*,\, \omega_2^*)$ and $(-\omega_1^*,\, -\omega_2^*)$ are among the sampling grids, then the entries of matrix $\mathbf{D}$ are given by

$$\mathbf{D}(n_1,\, n_2) = \begin{cases} 0, & (n_1,\, n_2) = (p_1,\, p_2) \ \text{ and } \ (q_1,\, q_2) \\ 1, & \text{otherwise} \ \ 1 \le n_1 \le M_1,\ 1 \le n_2 \le M_2 \end{cases} \tag{2}$$

where the $(p_1,\, p_2)$th and $(q_1,\, q_2)$th entries with $q_1 = M_1 - p_1 + 1$ and $q_2 = M_2 - p_2 + 1$ correspond to the notch frequencies $(\omega_1^*,\, \omega_2^*)$ and $(-\omega_1^*,\, -\omega_2^*)$, respectively. Evidently, the rank of matrix $\mathbf{D}$ is three as it only contains three linearly independent columns. By using the SVD, the matrix $\mathbf{D}$ can be written as

$$\mathbf{D} = \mathbf{w}_1\mathbf{w}_2^t - \mathbf{u}_1\mathbf{u}_2^t - \mathbf{v}_1\mathbf{v}_2^t \tag{3}$$

where the elements of the column vectors involved are given by

$$\mathbf{w}_k(n_k) = 1$$

$$\mathbf{u}_k(n_k) = \begin{cases} \dfrac{1}{\sqrt{2}}, & n_k = p_k \ \text{ and } \ q_k \\ 0, & \text{otherwise} \end{cases}$$

$$1 \le n_k \le M_k \quad k = 1,\, 2$$

$$\mathbf{v}_k(n_k) = \begin{cases} \dfrac{1}{\sqrt{2}}, & n_k = p_k \\ \dfrac{-1}{\sqrt{2}}, & n_k = q_k \\ 0, & \text{otherwise.} \end{cases}$$

This decomposition suggests that the magnitude response $\mathbf{D}$ can be approximated by that of 1-D FIR filters whose magnitude responses approximate $\mathbf{w}_k$, $\mathbf{u}_k$, and $\mathbf{v}_k$ $(k = 1,\, 2)$. In this section, we shall use this decomposition to design 2-D linear-phase FIR notch filter. It follows from (3) that the transfer function of a 2-D FIR notch filter should be approximated by the following form:

$$H(z_1,\, z_2) = z_1^{-[(N_1-1)/2]}\, z_2^{-[(N_2-1)/2]} - f_1(z_1)f_2(z_2) + g_1(z_1)g_2(z_2) \tag{4}$$

where the 1-D FIR filters are denoted by

$$f_k(z_k) = \sum_{i=0}^{N_k-1} a_i^{(k)} z_k^{-i}$$

$$g_k(z_k) = \sum_{i=0}^{N_k-1} b_i^{(k)} z_k^{-i}, \qquad k = 1,\, 2 \tag{5}$$

with N_k odd $(k = 1,\, 2)$. The linear phase constraints on $f_k(z_k)$ and $g_k(z_k)$ $(k = 1,\, 2)$ imply that

$$a_{N_k-i-1}^{(k)} = a_i^{(k)}$$

$$b_{N_k-i-1}^{(k)} = -\, b_i^{(k)}, \qquad 0 \le i \le \frac{N_k - 1}{2} \quad k = 1,\, 2 \tag{6}$$

and the frequency response of 1-D filters $f_k(z_k)$ and $g_k(z_k)$ are given by

$$\begin{aligned} f_k(e^{j\omega_k}) &= \left[a^{(k)}_{(N_k-1)/2} + \sum_{i=0}^{(N_k-3)/2} 2a_i^{(k)} \cos\left(\frac{N_k-1}{2} - i\right)\omega_k\right] \\ &\quad \cdot e^{-j\omega_k[(N_k-1)/2]} \\ &\equiv M_{f_k}(\omega_k) e^{-j\omega_k[(N_k-1)/2]} \qquad (7) \\ g_k(e^{j\omega_k}) &= j\left[\sum_{i=0}^{(N_k-3)/2} 2b_i^{(k)} \sin\left(\frac{N_k-1}{2} - i\right)\omega_k\right] \\ &\quad \cdot e^{-j\omega_k[(N_k-1)/2]} \\ &\equiv jM_{g_k}(\omega_k) e^{-j\omega_k[(N_k-1)/2]}, \qquad k = 1, 2. \end{aligned}$$

Substituting (7) into (4), the frequency response of the proposed 2-D FIR filter can be written as

$$H(e^{j\omega_1}, e^{j\omega_2}) = M(\omega_1, \omega_2) e^{-j\omega_1[(N_1-1)/2]} e^{-j\omega_2[(N_2-1)/2]} \qquad (8)$$

where the magnitude response is given by

$$M(\omega_1, \omega_2) = 1 - M_{f_1}(\omega_1) M_{f_2}(\omega_2) - M_{g_1}(\omega_1) M_{g_2}(\omega_2). \qquad (9)$$

Now, we will design 1-D filters $f_k(z_k)$ and $g_k(z_k)$ such that magnitude response $M(\omega_1, \omega_2)$ approximates $|H_d(\omega_1, \omega_2)|$ defined by (1) as close as possible. From (3), we have the following fact.

Fact: Let the two desired magnitude functions be

$$M_{df_k}(w_k) = \begin{cases} \dfrac{1}{\sqrt{2}}, & \omega_k = \omega_k^* \text{ and } -\omega_k^* \\ 0, & \text{otherwise} \end{cases} \qquad (10)$$

$$M_{dg_k}(\omega_k) = \begin{cases} \dfrac{1}{\sqrt{2}}, & \omega_k = \omega_k^* \\ -\dfrac{1}{\sqrt{2}}, & \omega_k = -\omega_k^* \\ 0, & \text{otherwise} \end{cases} \qquad (11)$$

then

$$\begin{aligned} &|H_d(e^{j\omega_1}, e^{j\omega_2})| \\ &\quad = 1 - M_{df_1}(\omega_1) M_{df_2}(\omega_2) - M_{dg_1}(\omega_1) M_{dg_2}(\omega_2). \end{aligned} \qquad (12)$$

Based on this fact, the task of designing a linear-phase 2-D FIR notch filter reduces to the following two approximation problems: 1) find filter coefficients such that $M_{f_k}(\omega_k)$ approximates $M_{df_k}(\omega_k)$ for $k = 1, 2$, and 2) find filter coefficients such that $M_{g_k}(\omega_k)$ approximates $M_{dg_k}(\omega_k)$ for $k = 1, 2$. In order to satisfy $|H_d(\omega_1^*, \omega_2^*)| = 0$ exactly, the following constraints need to be imposed:

$$\begin{aligned} M_{f_k}(\omega_k^*) &= M_{df_k}(\omega_k^*) = \frac{1}{\sqrt{2}} \\ M_{g_k}(\omega_k^*) &= M_{dg_k}(\omega_k^*) = \frac{1}{\sqrt{2}}, \qquad k = 1, 2. \end{aligned} \qquad (13)$$

We are now in a position to derive closed-form least squares solutions to these approximation problems.

A. Approximation of $M_{f_k}(\omega_k)$

The least-squares approximation of $M_{f_k}(\omega_k)$ to $M_{df_k}(\omega_k)$ for $k = 1, 2$ can be unified as one approximation problem. That is, we shall seek to find a parameter vector $\mathbf{a} = [a_0 \cdots a_N]^t$ such that

$$J(\mathbf{a}) = \int_0^\pi w(\omega)[f(\omega) - f_d(\omega)]^2 \, d\omega \qquad (14)$$

is minimized, where

$$f(\omega) = \sum_{i=0}^{N} a_i \cos(N-i)\omega$$

$$f_d(\omega) = \begin{cases} \dfrac{1}{\sqrt{2}}, & \omega \in \left[\omega^* - \dfrac{\delta}{2}, \omega^* + \dfrac{\delta}{2}\right] \\ 0, & \text{otherwise} \end{cases} \qquad (15)$$

and

$$w(\omega) = \begin{cases} \alpha, & \omega \in \left[\omega^* - \dfrac{\delta}{2}, \omega^* + \dfrac{\delta}{2}\right]. \\ 1-\alpha, & \text{otherwise} \end{cases} \qquad (16)$$

The width δ in (15) is small and positive, and is assumed to be given as a design parameter. The weight α is fairly close to 1 in order to ensure a narrow notch. A formula that determines the right value of α will be derived later in this section. Using (14)–(16), we can write

$$\begin{aligned} J(\mathbf{a}) = (1-\alpha) \int_0^\pi f^2(\omega)\, d\omega + \int_{\omega^*-\delta/2}^{\omega^*+\delta/2} \\ \cdot \left\{\alpha\left[f(\omega) - \frac{1}{\sqrt{2}}\right]^2 - (1-\alpha) f^2(\omega)\right\} d\omega. \end{aligned} \qquad (17)$$

Since δ is very small, the last integral in (17) can be approximated by

$$\alpha\delta\left[f(\omega^*) - \frac{1}{\sqrt{2}}\right]^2 - (1-\alpha)\delta f^2(\omega^*). \qquad (18)$$

Hence $J(\mathbf{a})$ can be written in a matrix–vector form as

$$J(\mathbf{a}) \approx \mathbf{a}^t \mathbf{Q} \mathbf{a} - 2\mathbf{a}^t \mathbf{q} + \text{const} \qquad (19)$$

where

$$\begin{aligned} \mathbf{Q} &= \frac{(1-\alpha)\pi}{2} \mathbf{Q}_0 + \delta(2\alpha - 1)\mathbf{Q}_1(\omega^*) \\ \mathbf{q} &= \frac{\alpha\delta}{\sqrt{2}} \mathbf{c}(\omega^*) \\ \mathbf{Q}_0 &= \begin{bmatrix} 1 & & & \\ & \ddots & & \\ & & 1 & \\ & & & 2 \end{bmatrix} \\ \mathbf{Q}_1(\omega^*) &= \mathbf{c}(\omega^*)\mathbf{c}^t(\omega^*) \\ \mathbf{c}(\omega^*) &= [\cos(N\omega^*), \cdots, \cos(\omega^*) \quad 1]^t \end{aligned} \qquad (20)$$

and const is a constant independent of $\mathbf{a}$. Since the weight α is close to one but less than one, matrix $\mathbf{Q}$ is positive definite, and the global minimum of $J(\mathbf{a})$ is achieved by

$$\mathbf{a}^* = \mathbf{Q}^{-1}\mathbf{q}. \qquad (21)$$

Using the matrix inversion formula for vectors $\mathbf{r}$ and $\mathbf{s}$, i.e.,

$$(\mathbf{I} + \mathbf{r}\mathbf{s}^t)^{-1} = \mathbf{I} - \frac{\mathbf{r}\mathbf{s}^t}{1 + \mathbf{r}^t\mathbf{s}} \qquad (22)$$

we obtain

$$\mathbf{a}^* = \eta_1 \left[\mathbf{Q}_2 - \frac{\tilde{\mathbf{c}}(\omega^*)\tilde{\mathbf{c}}^t(\omega^*)}{\eta_2 + \|\hat{\mathbf{c}}(\omega^*)\|^2}\right] \mathbf{c}(\omega^*) \qquad (23)$$

where

$$\eta_1 = \frac{\sqrt{2}\alpha\delta}{(1-\alpha)\pi} \quad \eta_2 = \frac{(1-\alpha)\pi}{2(2\alpha-1)\delta} \qquad (24)$$

$$\mathbf{Q}_2 = \begin{bmatrix} 1 & & & \\ & \ddots & & \\ & & 1 & \\ & & & \frac{1}{2} \end{bmatrix} \qquad (25)$$

$$\tilde{\mathbf{c}}(\omega^*) = \mathbf{Q}_2 \mathbf{c}(\omega^*) \quad \hat{\mathbf{c}}(\omega^*) = \mathbf{Q}_2^{1/2} \mathbf{c}(\omega^*). \qquad (26)$$

There are two problems with (23) that need to be addressed. First, as a common problem with a least-squares design, the filter with coefficients $\mathbf{a}^*$ determined by (23) often exhibits undesirable Gibbs oscillations in the vicinity of the notch frequency ω^*. An effective way to eliminate these oscillations is to use a certain window function to modify the coefficient vector, i.e., to modify the ith coefficient of the filter to be

$$a(i) = 2w_h(i)a^*(i), \qquad 0 \le i \le N \tag{27}$$

where

$$w_h(i) = 0.54 - 0.46\cos\left(\frac{\pi i}{N}\right), \qquad 0 \le i \le N \tag{28}$$

is from the $(2N+1)$-point Hamming window. The factor 2 introduced in (27) turns out to be necessary to satisfy a constraint at the notch frequency, as will be seen next. The second problem that needs to be dealt with is that the constraint

$$f(\omega^*) = \frac{1}{\sqrt{2}} \tag{29}$$

must be satisfied. In what follows we show that this constraint can be met by properly choosing a weight α.

By writing (29) as

$$\mathbf{c}^t(\omega^*)\mathbf{a} = \frac{1}{\sqrt{2}} \tag{30}$$

and using (23), (27), and (29), we obtain

$$\eta_1\left(r_1 - \frac{r_2}{\eta_2 + \beta}\right) = \frac{1}{\sqrt{2}} \tag{31}$$

where $\beta = \|\hat{\mathbf{c}}(\omega^*)\|^2$, $\mathbf{w}_c = 2[w_h(0)\cos(N\omega^*), \cdots, w_h(N-1)\cos(\omega^*) \;\; w_h(N)]^t$, $r_1 = \tilde{\mathbf{c}}^t(\omega^*)\mathbf{w}_c$, and $r_2 = [\tilde{\mathbf{c}}^t(\omega^*)\mathbf{c}(\omega^*)]\mathbf{c}^t(\omega^*)\mathbf{w}_c$. Using the definitions of η_1 and η_2, straightforward manipulations lead (31) to

$$A\alpha^2 + B\alpha + C = 0 \tag{32}$$

where

$$\begin{aligned} A &= \left(r_1 + \frac{\pi}{2\delta}\right)(4\beta\delta - \pi) - 4r_2\delta \\ B &= r_1(\pi - 2\beta\delta) + 2r_2\delta - \frac{\pi}{\delta}(3\beta\delta - \pi) \\ C &= \frac{\pi}{2\delta}(2\beta\delta - \pi). \end{aligned} \tag{33}$$

Although (32) is a quadratic form, the root

$$\alpha = \frac{-B + \sqrt{B^2 - 4AC}}{2A} \tag{34}$$

falls into the interval $(0, 1)$ and is the right choice of α that satisfies (29). To verify this fact, Fig. 1 shows the relation curves of the parameters α and δ for various lengths N when we choose $\omega^* = 0.5\pi$. From this result, it is clear that α approaches unity when δ approaches zero. In fact, if δ approaches zero, we have

$$\lim_{\delta\to 0} A = \frac{-\pi^2}{2\delta} \quad \lim_{\delta\to 0} B = \frac{\pi^2}{\delta} \quad \lim_{\delta\to 0} C = \frac{-\pi^2}{2\delta} \tag{35}$$

that is, (32) becomes

$$\alpha^2 - 2\alpha + 1 = 0. \tag{36}$$

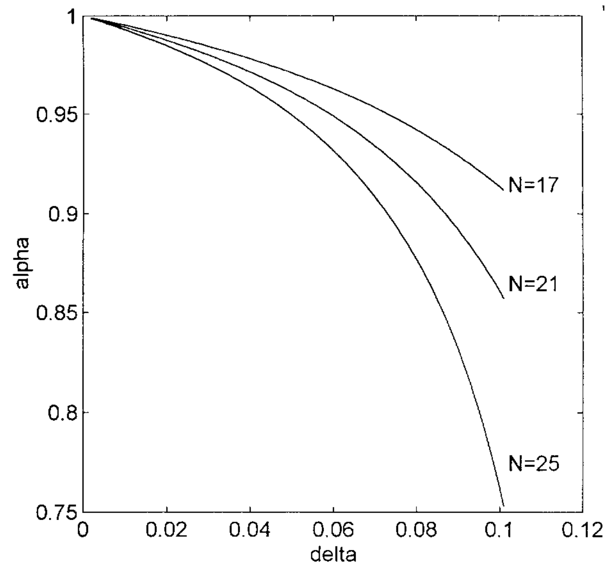


Fig. 1. The relation curves of the parameters α and δ for various lengths N and $\omega^* = 0.5\pi$.

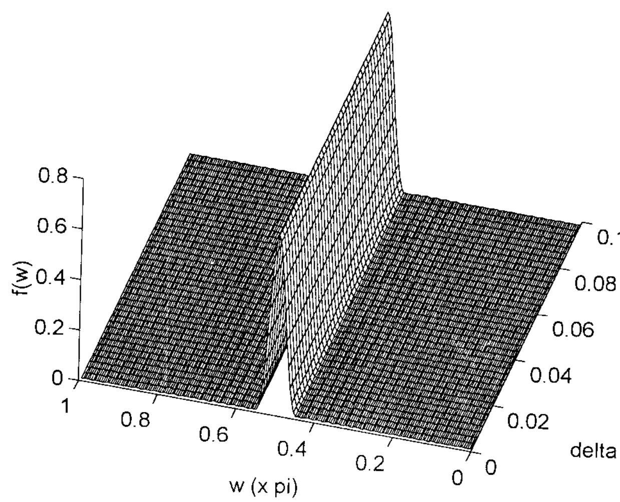


Fig. 2. The responses $f(\omega)$ for $N = 40, \omega^* = 0.5\pi$, and several values δ.

Thus, the fact that α approaches unity when δ approaches zero is obvious. Moreover, Fig. 2 shows the magnitude responses of $f(\omega)$ for $N = 40$, $\omega^* = 0.5\pi$ and various δ. It is clear that the responses $f(\omega)$ are almost the same for several values δ even though δ approaches zero which means α approaches unity. Thus, the choice of α value close to 1 (16) would not deteriorate the filter characteristics in the passband because δ approaches zero under this condition.

In summary, we have developed a closed-form least-squares solution to the approximation problem (14) subject to constraint (29). This $(2N+1)$th-order filter has symmetric coefficients $\mathbf{h} = [\frac{1}{2}a(0), \cdots, \frac{1}{2}a(N-1) \;\; a(N) \;\; \frac{1}{2}a(N-1), \cdots, \frac{1}{2}a(0)]$ where $\{a(i), 0 \le i \le N\}$ are given by (23) and (27) with an α determined by (34).

B. Approximation of $M_{g_k}(\omega_k)$

The least-squares approximation of $M_{g_k}(\omega_k)$ to $M_{dg_k}(\omega_k)$ for $k = 1, 2$ can be unified as one approximation problem. That is, we shall seek to find a parameter vector $\mathbf{b} = [b_0, \cdots, b_{N-1}]^t$ such that

$$J(\mathbf{b}) = \int_0^\pi w(\omega)[g(\omega) - g_d(\omega)]^2\, d\omega \tag{37}$$

is minimized, where

$$g(\omega) = \sum_{i=0}^{N-1} b_i \sin(N-i)\omega$$

$$g_d(\omega) = \begin{cases} \frac{1}{\sqrt{2}}, & \omega \in \left[\omega^* - \frac{\delta}{2},\, \omega^* + \frac{\delta}{2}\right] \\ 0, & \text{otherwise} \end{cases} \tag{38}$$

and $w(\omega)$ is defined by (16). With δ sufficiently small

$$\begin{aligned} J(\mathbf{b}) &= (1-\alpha)\int_0^{\pi} g^2(\omega)\,d\omega + \int_{\omega^*-\delta/2}^{\omega^*+\delta/2} \\ &\quad \cdot \left\{\alpha\left[g(\omega) - \frac{1}{\sqrt{2}}\right]^2 - (1-\alpha)g^2(\omega)\right\} d\omega \\ &\approx (1-\alpha)\int_0^{\pi} g^2(\omega)\,d\omega + \alpha\delta\left[g(\omega^*) - \frac{1}{\sqrt{2}}\right]^2 \\ &\quad - (1-\alpha)\delta g^2(\omega^*) \\ &= \mathbf{b}^t\mathbf{P}\mathbf{b} - 2\mathbf{b}^t\mathbf{p} + \text{const} \end{aligned} \tag{39}$$

where

$$\begin{aligned} \mathbf{P} &= \frac{(1-\alpha)\pi}{2}\mathbf{I} + \delta(2\alpha-1)\mathbf{P}_1(\omega^*) \\ \mathbf{P}_1 &= \mathbf{d}(\omega^*)\mathbf{d}^t(\omega^*) \\ \mathbf{d}(\omega) &= [\sin(N\omega), \cdots, \sin(\omega)]^t \\ \mathbf{p} &= \frac{\alpha\delta}{\sqrt{2}}\mathbf{d}(\omega^*). \end{aligned} \tag{40}$$

With an α close to unity, matrix $\mathbf{P}$ is positive definite and the global minimal of $J(\mathbf{b})$ is achieved by

$$\mathbf{b}^* = \mathbf{P}^{-1}\mathbf{p} \tag{41}$$

which, by using matrix inversion formula (22), can be expressed as

$$\mathbf{b}^* = \xi\mathbf{d}(\omega^*) \tag{42}$$

with

$$\xi = \frac{\sqrt{2}\alpha\delta}{(1-\alpha)\pi + 2(2\alpha-1)\delta\|\mathbf{d}(\omega^*)\|^2}. \tag{43}$$

Like the approximation of $M_{f_k}(\omega_k)$, a Hamming window

$$w_h(i) = 0.54 - 0.46\cos\left(\frac{\pi i}{N}\right), \qquad 0 \le i \le N-1 \tag{44}$$

is used to eliminate the Gibbs oscillations in the frequency response of the filter designed, and the modified filter coefficients become

$$b(i) = 2w_h(i)b^*(i), \qquad 0 \le i \le N-1 \tag{45}$$

where $b^*(i)$ is the ith entry of $\mathbf{b}^*$ in (42). Further, to satisfy the constraint

$$g(\omega^*) = \frac{1}{\sqrt{2}} \tag{46}$$

we write $g(\omega^*) = \mathbf{d}^t(\omega^*)\mathbf{b}$ and use (42), (45), and (46) to obtain an equation for α:

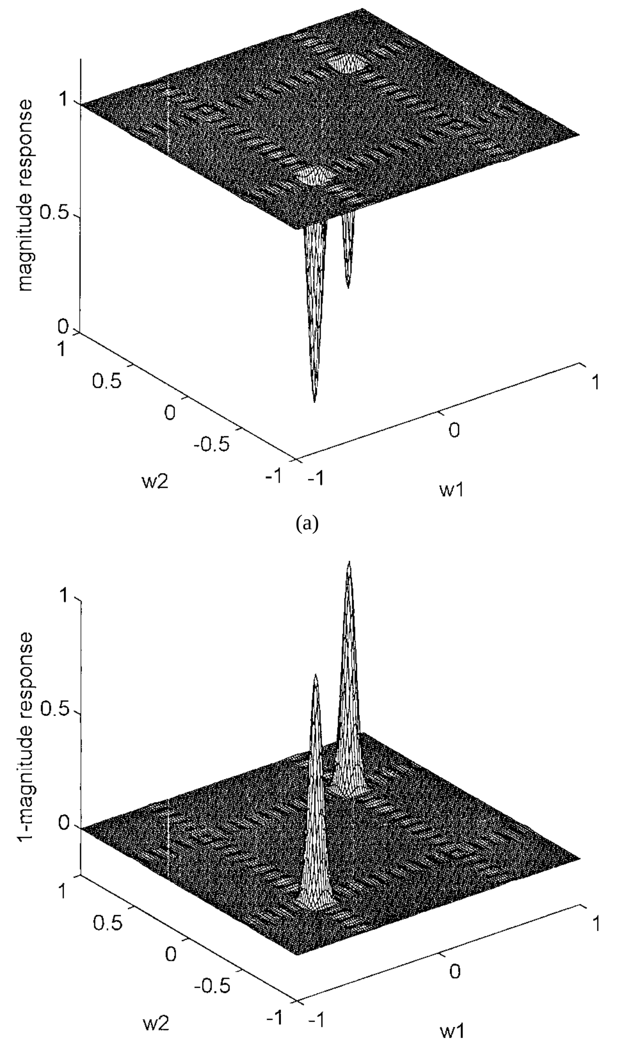
$$\frac{\sqrt{2}\alpha\delta\mathbf{d}^t(\omega^*)\mathbf{w}_d}{(1-\alpha)\pi + 2(2\alpha-1)\delta\|\mathbf{d}(\omega^*)\|^2} = \frac{1}{\sqrt{2}} \tag{47}$$

where

$$\mathbf{w}_d = 2[w_h(0)\sin(N\omega^*), \cdots, w_h(N-1)\sin(\omega^*)]^t. \tag{48}$$

Solving (47) for α gives

$$\alpha = \frac{\pi - 2\delta\|\mathbf{d}(\omega^*)\|^2}{\pi + 2\delta\mathbf{d}^t(\omega^*)\mathbf{w}_d - 4\delta\|\mathbf{d}(\omega^*)\|^2}. \tag{49}$$



(a)

(b)

Fig. 3. The results of the designed notch filter. (a) Magnitude response. (b) 1-magnitude response (loss).

In summary, we have obtained a closed-form least-squares solution to the approximation problem in (37) subject to constraint (46). This $(2N+1)$th-order filter has odd symmetric coefficients $\mathbf{h} = [-\frac{1}{2}b(0), \cdots, \frac{-1}{2}b(N-1) \quad 0 \quad \frac{1}{2}b(N-1), \cdots, \frac{1}{2}b(0)]$ where $\{b(i),\, 0 \le i \le N-1\}$ are given by (42) and (45) with an α determined by (49).

III. Design Example and Application

In this section, one design example of a linear-phase FIR notch filter is first illustrated. Next, we use the 2-D FIR notch filter to remove sinusoidal interferences superimposed on an image.

Example 1: FIR Notch Filter Design

In this example, we use the procedure stated in Section II to design a 2-D linear-phase FIR notch filter with notch frequency $(\omega_1^*, \omega_2^*) = (0.5\pi, 0.5\pi)$. The filter length is 41 × 41 and the design parameter δ is chosen to be 0.001. Fig. 3(a) depicts the resultant magnitude response designed in the least-squares error sense. It is clear that the specification is well satisfied. However, the details of notches are underneath the unity gain plane. In order to show the

(a)

(b)

Fig. 4. Example of single sinusoidal interference removal. (a) corrupted image and (b) image restored by using 2-D FIR notch filter.

performance of the designed filter better, Fig. 3(b) plots the loss (1-gain) instead of the gain. From the result, we observe that the magnitude response has small ripples in the vicinity of the notches.

Example 2: Single Sinusoidal Interference Removal

In the example, we will use the 2-D FIR notch filter to remove a single sinusoidal interference superimposed on an image. The scenario of the experiment is the same as the one described in [6] except that the image size here is 256 × 256. The image shown in Fig. 4(a) is the Lena image corrupted by a sinusoidal pattern of the form

$$30 \sin(0.1\pi m + 0.2\pi n). \tag{50}$$

Now we design a 2-D FIR notch filter with $(\omega_1^*, \omega_2^*) = (0.1\pi, 0.2\pi)$ and $\delta = 0.001$ to remove the interference in spatial domain. The filtered image, shown in Fig. 4(b), is clearly free from interference.

IV. Conclusions

In this paper, the 2-D FIR notch filter-design problem has been investigated. First, the SVD is used to reduce the 2-D notch filter-design problem to two pairs of 1-D filter-design problems. Then, we provide the closed-form solutions for the design of two pairs of 1-D FIR filters. Finally, several design and application examples are presented to demonstrate the performance of the proposed design methods. However, only fixed notch filters are considered here. Thus, it is interesting to develop a 2-D adaptive notch filtering algorithm. This topic will be studied in the future.

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Analysis of the Behavior of a Dynamic Latch Comparator

P. Cusinato, M. Bruccoleri, D. D. Caviglia, and M. Valle

***Abstract*—This brief deals with the behavior of a dynamic latch used as a voltage comparator. A detailed analysis of the fine settling phase is reported, putting in evidence the non-idealities which lead to comparison errors. A technique to minimize such errors is suggested. An experimental chip has been fabricated and measurements are reported and discussed.**

I. Introduction

The dynamic latch (Fig. 1), widely used as a sense amplifier in dynamic RAM's, is a fast CMOS comparator. However, its large input offset voltage, mainly caused by the circuit-parameter deviations and by the charge injection mismatch between switches S_1 and S_2, limits its resolution to about 5 b [1], [2]. A linear amplifier may precede the dynamic latch so that the voltage applied

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P. Cusinato and M. Bruccoleri are with SGS-Thomson Microelectronics, I-20010 Cornaredo, Milan, Italy.

D. D. Caviglia and M. Valle are with DIBE, University of Genoa, I-16145 Genoa, Italy.

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