Transactions Briefs

Order Reduction of 2-D FIR Filters With Applications

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***Abstract*—This brief develops two algorithms for the order reduction of two-dimensional (2-D) FIR digital filters, which lead to two indirect design methods. Specifically, we show how an optimal L_2 design can be derived as a matrix approximation problem in the Frobenius norm, and how a suboptimal L_∞ design can be obtained by solving a matrix approximation problem in the 2-norm using the Davis–Kahan–Weinberger theorem on norm-preserving dilations. A case study is included to demonstrate the usefulness of the proposed methods.**

I. Introduction

Analytically, designing a digital filter amounts to finding a polynomial (FIR) or a stable rational function (IIR) that approximates a given (desired) frequency response. For the design of IIR filters, one of the recent trends is the use of indirect methods that start by designing a high order FIR filter and then approximating it by a stable, low order, IIR transfer function, see for example [1]–[5] for both one-dimensional (1-D) and two-dimensional (2-D) designs. An attractive feature of the indirect methods is that the replacement of the desired frequency response, which is often unrealizable, by a high-order, but realizable transfer function allows the designer to employ systems theory [6], [7] to accomplish the design task. As a matter of fact, the most often utilized approximation methods in the indirect design methods are the balanced approximation [8] and the Hankel-norm optimal approximation [9], and these approximation methods are well known to the control community.

In this brief, we propose two algorithms for the order reduction of 2-D FIR transfer functions, which lead to two indirect design methods. These methods start by designing a high order, linear-phase, FIR 2-D filter and the design problem is then formulated as a matrix approximation problem leading to an optimal solution in L_2 sense and a suboptimal solution in L_∞ sense, respectively. For the L_2 design, we show that an optimal solution can be obtained by simply truncating the central part of the original impulse response, and that the filter obtained has a linear phase response if the original filter does so. For the L_∞ design, the Davis–Kahan–Weinberger (DKW) theorem on norm-preserving dilations [10] and the Parrott theorem on quotient norm [11] are utilized as key tools in the derivation of a suboptimal design. Again, we show that linear phase response is preserved during the design. A case study is included to demonstrate the usefulness of the proposed methods.

II. An Indirect L_2 Design Method

A. Problem Statement

Given a desired frequency response $H_d(e^{j\omega_1},\ e^{j\omega_2})$, we seek to determine the transfer function of a linear-phase FIR filter

$$H(z_1,z_2)=z_1^{-(N_1-1)/2}z_2^{-(N_2-1)/2}\times\sum_{i=-(N_1-1)/2}^{(N_1-1)/2}\ \sum_{j=-(N_2-1)/2}^{(N_2-1)/2} h_{ij}z_1^{-i}z_2^{-j}$$

that approximates $H_d(e^{j\omega_1},\ e^{j\omega_2})$ in a certain sense. The transfer function $H(z_1,z_2)$ can be expressed as

$$H(z_1,z_2)=z_1^{-(N_1-1)/2}z_2^{-(N_2-1)/2}Z_{1s}^T H Z_{2s} \tag{1}$$

where

$$Z_{1s}=\left[z_1^{(N_1-1)/2}\ \cdots\ z_1\ 1\ z_1^{-1}\ \cdots\ z_1^{-(N_1-1)/2}\right]^T$$
$$Z_{2s}=\left[z_2^{(N_2-2)/2}\ \cdots\ z_2\ 1\ z_2^{-1}\ \cdots\ z_2^{-(N_2-1)/2}\right]^T$$

and H is the $N_1\times N_2$ impulse response matrix. If the filter is required to have a linear phase response, then the entries of H must satisfy

$$h_{ij}=h_{-i,-j}. \tag{2}$$

Throughout, a matrix $H=\{h_{ij}\}$ satisfying (2) is said to be *symmetric with respect to center*.

The method starts by designing a high order, linear phase "prototype" filter that well approximates $H_d(e^{j\omega_1},\ e^{j\omega_2})$. This can be accomplished by using an established method such as the window method [13], [12] or the SVD method [3]. Let the transfer function of the prototype filter be given by

$$P(z_1,z_2)=z_1^{-(N_1-1)/2}z_2^{-(N_2-1)/2}\times\sum_{i=-(M_1-1)/2}^{(M_1-1)/2}\ \sum_{j=-(M_2-1)/2}^{(M_2-1)/2} p_{ij}z_1^{-i}z_2^{-j} \tag{3}$$

where

$$p_{ij}=p_{-i,-j}. \tag{4}$$

Note that with (4) the constant group delays of $P(z_1,z_2)$ are N_1-1 in ω_1 and N_2-1 in ω_2, which are identical to that of $H(z_1,z_2)$. Note also that in order to well approximate $H_d(e^{j\omega_1},e^{j\omega_2})$, the order of $P(z_1,z_2)$ is usually high, hence $M_1>N_1$ and $M_2>N_2$ are assumed throughout. These two facts imply that the prototype filter is noncausal and we are now in a scenario that often occurs in system-theoretic research: approximating an "ideal" but noncausal system by a causal system of lower order.

We proceed by writing

$$P(z_1,z_2)=z_1^{-(N_1-1)/2}z_2^{-(N_2-1)/2}Z_1^T P Z_2 \tag{5}$$

with

$$Z_1=\left[z_1^{(M_1-1)/2}\ \cdots\ z_1\ 1\ z_1^{-1}\ \cdots z_1^{-(M_1-1)/2}\right]^T$$
$$Z_2=\left[z_2^{(M_2-1)/2}\ \cdots\ z_2\ 1\ z_2^{-1}\ \cdots z_2^{-(M_2-1)/2}\right]^T$$

and formulating the design problem as to find impulse response matrix H in (1) such that the L_2 approximation error

$$J_2=\|P(z_1,z_2)-H(z_1,z_2)\|_2^2 \equiv \frac{1}{(2\pi j)^2}\oint_{|z_1|=1}\oint_{|z_2|=1}|P(z_1,z_2)-H(z_1,z_2)|^2\frac{dz_1}{z_1}\frac{dz_2}{z_2} \tag{6}$$

is minimized.

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B. Minimization of J_2

Using (1), (5), and bearing in mind that $M_1 > N_1$ and $M_2 > N_2$, we have

$$P(z_1, z_2) - H(z_1, z_2) = z_1^{-(N_1-1)/2} z_2^{-(N_2-1)/2} Z_1^T (P - \tilde{H}) Z_2$$

where

$$\tilde{H} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & H & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{7}$$

Partitioning the impulse response matrix P in (5) conformable to $\tilde{H}$ as

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix} \tag{8}$$

where $P_{22} \in R^{N_1 \times N_2}$, we compute

$$\begin{aligned} J_2 &= \frac{1}{(2\pi j)^2} \oint_{|z_1|=1} \oint_{|z_2|=1} Z_1^T (P - \tilde{H}) Z_2 \\ &\quad \times Z_2^H (P - \tilde{H})^T \bar{Z} \frac{dz_1}{z_1} \frac{dz_2}{z_2} \\ &= \frac{1}{2\pi j} \oint_{|z_1|=1} Z_1^T (P - \tilde{H}) \\ &\quad \times \left(\frac{1}{2\pi j} \oint_{|z_2|=1} Z_2 Z_2^H \frac{dz_2}{z_2} \right) (P - \tilde{H})^T \bar{Z}_1 \frac{dz_1}{z_1} \\ &= \mathrm{tr} \left[(P - \tilde{H})^T \left(\frac{1}{2\pi j} \oint_{|z_1|=1} \bar{Z}_1 Z_1^T \frac{dz_1}{z_1} \right) (P - \tilde{H}) \right] \\ &= \|P_{22} - H\|_F^2 + r_2 \end{aligned} \tag{9}$$

with

$$r_2 = \|P_1\|_F^2 + \|P_2\|_F^2 + \|P_3\|_F^2 \tag{10}$$

where $P_1 = [P_{11} \ P_{12} \ P_{13}]$, $P_2 = [P_{21} \ P_{23}]$, $P_3 = [P_{31} \ P_{32} \ P_{33}]$, and $\|\cdot\|_F$ denotes the Frobenius norm. Since r_2 is independent of H, it follows from (9) that J_2 is minimized if and only if

$$H = P_{22}. \tag{11}$$

Furthermore, from (11) we see that (4) implies (2). In other words, if the prototype filter has a linear phase response, so does the designed filter $H(z_1, z_2)$.

The proposed design is simple and straightforward, and it can be summarized as the following two steps: 1) Design a high order, linear phase, FIR transfer function $P(z_1, z_2)$; 2) Represent the impulse response matrix of $P(z_1, z_2)$ as P in (8), and truncate it to obtain $H = P_{22}$.

A study that compares the performance of an $N_1 \times N_2$ FIR filter obtained by this method with that of FIR filters of same order obtained by some established direct design methods will be presented in Section II-C.

C. A Case Study

As an illustrative example, a circularly symmetric, linear phase, lowpass FIR filter of order $N \times N = 41 \times 41$ with normalized passband edge = 0.23 and stopband edge = 0.27 is designed using the design method described in Section II-B. The prototype filter $P(z_1, z_2)$ of order $M \times M$ is designed by the window method [13], [12] and the SVD method [3], respectively, and the transfer function $H(z_1, z_2)$ of order 41 × 41 is then obtained by truncating the central part of the impulse response matrix of $P(z_1, z_2)$. The prototype filter with order varying from 43 × 43 to 99 × 99 is used to achieve best designs that are evaluated based on the following performance indicies:

- L_2 error in the passband

$$e_{p2} = \left[\iint_{R_p} |H_d(e^{j\omega_1}, e^{j\omega_2}) - H(e^{j\omega_1}, e^{j\omega_2})|^2 d\omega_1 \, d\omega_2 \right]^{1/2}$$

- L_2 error in the stopband

$$e_{s2} = \left[\iint_{R_s} |H_d(e^{j\omega_1}, e^{j\omega_2}) - H(e^{j\omega_1}, e^{j\omega_2})|^2 d\omega_1 \, d\omega_2 \right]^{1/2}$$

- maximum ripple in the passband

$$e_{p\infty} = \max_{(\omega_1, \omega_2) \in R_p} |H_d(e^{j\omega_1}, e^{j\omega_2}) - H(e^{j\omega_1}, e^{j\omega_2})|$$

- maximum ripple in the stopband

$$e_{s\infty} = \max_{(\omega_1, \omega_2) \in R_s} |H_d(e^{j\omega_1}, e^{j\omega_2}) - H(e^{j\omega_1}, e^{j\omega_2})|.$$

TABLE I
COMPARISONS OF THE L_2 DESIGN WITH THE WINDOW AND SVD METHODS

Method	M	e_{p2}	e_{s2}	$e_{p\infty}$	$e_{s\infty}$
The L_2 design with a prototype filter obtained by the window method	61	0.2640	0.2605	0.0216	0.0285
	63	0.2877	0.2692	0.0235	0.0235
The L_2 design with a prototype filter obtained by the SVD method	69	0.2596	0.3488	0.0580	0.0602
	85	0.3753	0.3895	0.0399	0.0344
Direct design by the window method		0.4963	0.7529	0.0977	0.1087
Direct design by the SVD method	–	0.8667	0.9056	0.1404	0.1570

As is shown in Table I, good designs in terms of L_2 error are achieved with $M = 61$ when the prototype filter is designed by the window method, and with $M = 69$ when the prototype filter is designed by the SVD method. In terms of L_∞ error, good designs are obtained with $M = 63$ from the window method and with $M = 85$ from the SVD method. For comparison purposes, FIR filters of order 41 × 41 are also directly designed by using the window method and SVD method, respectively, and are then evaluated using the same criteria. The numerical evaluations of these filters are given in Table I.

Table I shows that with a prototype filter of order 63 × 63 designed by the window method, a linear phase FIR filter of order 41 × 41 can be designed using the proposed method to achieve a less then 2.5% maximum ripple in both passband and stopband which is evidently superior to those designed directly using either the window method or the SVD method.

III. AN INDIRECT L_∞ DESIGN METHOD

A. Problem Formulation

In an L_∞ design, we seek to find the impulse response matrix H in (1) such that

$$J_\infty = \max_{|z_1|=|z_2|=1} |P(z_1, z_2) - H(z_1, z_2)| \tag{12}$$

is minimized, where $P(z_1, z_2)$ is the prototype filter given by (3) and (4). Using Cauchy–Schwarz inequality, we estimate for $z_1 = e^{j\omega_1}$ and $z_2 = e^{j\omega_2}$

$$\begin{aligned} &|P(z_1, z_2) - H(z_1, z_2)| \\ &\quad = \left| Z_1^T (P - \tilde{H}) Z_2 \right| \le \|Z_1\|_2 \, \|Z_2\|_2 \, \|P - \tilde{H}\|_2 \\ &\quad = M \|P - \tilde{H}\|_2 \end{aligned}$$

which implies that

$$J_\infty \le M\|P - \tilde{H}\|_2. \tag{13}$$

By (13), it appears to be reasonable to call an FIR transfer function $H(z_1, z_2)$ L_∞-suboptimal if its impulse response matrix minimizes $\|P - \tilde{H}\|_2$. That is to say, the suboptimal solution to be derived below is the one that only minimizes the upper bound of J_∞.

B. Minimization of $\|P - \tilde{H}\|_2$

By using permutations and noticing that the 2-norm is invariant under permutations, we write

$$\begin{aligned}\|P - \tilde{H}\|_2 &= \left\|\begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} - H & P_{23} \\ P_{31} & P_{32} & P_{33}\end{bmatrix}\right\|_2 \\ &= \left\|\begin{bmatrix} P_{22} - H & P_{21} & P_{23} \\ P_{12} & P_{11} & P_{13} \\ P_{32} & P_{31} & P_{33}\end{bmatrix}\right\|_2 \\ &\equiv \left\|\begin{bmatrix} P_{22} - H & F_1 \\ F_2 & F_3\end{bmatrix}\right\|_2 \end{aligned} \tag{14}$$

where matrices

$$F_1 = [P_{21} \quad P_{23}], \quad F_2 = \begin{bmatrix} P_{12}^T & P_{32}^T \end{bmatrix}^T, \quad F_3 = \begin{bmatrix} P_{11} & P_{13} \\ P_{31} & P_{33}\end{bmatrix}$$

are independent of H. Hence, designing an L_∞-suboptimal FIR filter amounts to finding H that minimizes the 2-norm of

$$\begin{bmatrix} P_{22} - H & F_1 \\ F_2 & F_3\end{bmatrix}.$$

An analytic solution to this problem can be derived using a theorem on quatient norm by Parrott [11] and a theorem on norm-preserving dilations by Davis, Kahn, and Weinberger [10]. Define

$$\gamma_0 = \min_H \left\|\begin{bmatrix} P_{22} - H & F_1 \\ F_2 & F_3\end{bmatrix}\right\|_2$$

then we have Theorem 1.

Theorem 1) [11]:

$$\gamma_0 = \max\left(\left\|\begin{bmatrix} F_1 \\ F_3\end{bmatrix}\right\|_2, \ \|[F_2 \quad F_3]\|_2\right). \tag{15}$$

Theorem 2) [10]: Let $\gamma \ge \gamma_0$. All solutions H such that

$$\|P - \tilde{H}\|_2 \le \gamma \tag{16}$$

are given by

$$H = P_{22} + GF_3^T K - \gamma(I - GG^T)^{1/2} S (I - K^T K)^{1/2} \tag{17}$$

where S is an arbitrary contraction (i.e., $\|S\|_2 < 1$) and G, K solve the linear equations

$$G\left(\gamma^2 I - F_3^T F_3\right)^{1/2} = F_1 \tag{18}$$

$$\left(\gamma^2 I - F_3 F_3^T\right)^{1/2} K = F_2. \tag{19}$$

In a filter design context, the significance of these two theorems is that using (15) the exact minimum error γ_0 can be calculated, and the DKW theorem claims that this error is achievable by a solution class characterized by (17)–(19) with $\gamma = \gamma_0$. For the sake of solution simplicity, one may choose $S = 0$ in (17), which leads to

$$H = P_{22} + G_0 F_3^T K_0 \tag{20a}$$

$$G_0 = F_1\left(\gamma_0^2 I - F_3^T F_3\right)^{-1/2} \tag{20b}$$

$$K_0 = \left(\gamma_0^2 I - F_3 F_3^T\right)^{-1/2} F_2. \tag{20c}$$

An interesting special case is when the prototype filter is circularly symmetric and has a linear phase response. In this case, P is symmetric and consequently $F_2 = F_1^T$, $F_3^T = F_3$, $K_0 = G_0^T$, and (15) implies that

$$\gamma_0 = \|[F_2 \quad F_3]\|_2$$

The impulse response matrix of the filter is then given by

$$H = P_{22} + G_0 F_3 G_0^T$$

where G_0 is given by (20).

In summary, a suboptimal L_∞ design can be obtained using the following three steps: 1) Design a high order, linear phase, FIR filter $P(z_1, z_2)$; 2) Compute γ_0 using (15); and 3) Compute H using (20).

We now conclude this section with two remarks.

1) *Computation Complexity of the Design:* In addition to designing a prototype filter, the most intensive computation in the rest of the design is to evaluate $G_0 F_2^T K_0$. Here, we present an approach to computing $G_0 F_3^T K_0$ using the singular value decomposition (SVD) of F_3. For simplicity, we assume $M_1 = M_2 = M$ and $N_1 = N_2 = N$, and let the SVD of F_3 be given by $F_3 = U\Sigma V^T$ where U, V are orthogonal matrices and $\Sigma = \mathrm{diag}\{\sigma_1, \cdots, \ \sigma_{M-N}\}$. By (20b), $G_0 = F_1 V \Gamma V^T$ where

$$\Gamma = \mathrm{diag}\left\{\left(\gamma_0^2 - \sigma_1^2\right)^{-1/2}, \cdots, \left(\gamma_0^2 - \sigma_{M-N}^2\right)^{-1/2}\right\}.$$

From (15), it follows that $\gamma_0 > \|F_3\|_2 = \sigma_1$, hence, the entries $(\gamma_0^2 - \sigma_i^2)^{-1/2}$ in Γ for $i = 1, \cdots, M - N$ are positive real numbers. Likewise, K_0 in (20c) can be expressed as $K_0 = U\Gamma U^T F_2$. Therefore, $G_0 F_3^T K_0 = F_1 \hat{F}^T F_2$ where $\hat{F} = U\hat{\Sigma}V^T$ with

$$\hat{\Sigma} = \mathrm{diag}\left\{\frac{\sigma_1}{\gamma_0^2 - \sigma_1^2}, \cdots, \frac{\sigma_{M-N}}{\gamma_0^2 - \sigma_{M-N}^2}\right\}. \tag{21}$$

We see that $G_0 F_3^T K_0$ can be evaluated by a) computing the SVD of F_3; b) obtaining $\hat{\Sigma}$ by modifying Σ using (21); c) computing $\hat{F} = U\hat{\Sigma}V^T$; and d) calculating $F_1 \hat{F}^T F_2$.

2) *Linear Phase Property of the Filter Designed:* If the prototype filter has a linear phase response, then the same is true for the filter characterized by the impulse response matrix H in (20a). A proof of this property is given as follows.

Denote $H = \{h_{ij}\}$, $P_{22} = \{p_{ij}\}$, and $G_0 F_3^T K_0 = \{v_{ij}\}$. It follows from (8) and (4) that $p_{i,j} = p_{-i,-j}$. Note that $v_{i,j}$ can be expressed as $v_{ij} = p_i \hat{F} q_j$ where p_i is the ith row of $[P_{21} \quad P_{23}]$, and q_j is the jth column of $[P_{12}^T \quad P_{32}^T]^T$. Using (4) and (8), one concludes that both matrices $[P_{21} \quad P_{23}]$ and $[P_{12}^T \quad P_{32}^T]^T$ are symmetric w.r.t. center, meaning that $p_i = p_{-i}\hat{I}$ where

$$\hat{I} = \begin{bmatrix} 0 & & 1 \\ & \cdot^{\cdot^{\cdot}} & \\ 1 & & 0 \end{bmatrix}_{(M-N)\times(M-N)}$$

and $q_j = \hat{I} q_{-j}$. Moreover, the fact that $\hat{F}$ is symmetric w.r.t. center implies that $\hat{F} = \hat{I}\hat{F}\hat{I}$. It follows that $v_{ij} = v_{-i,-j}$. Since $h_{ij} = p_{ij} + v_{ij}$ we have $h_{ij} = h_{-i,-j}$ which completes the proof.

C. A Case Study (Continued from Section II-C)

We now use the same design example as in Section II-C to illustrate the proposed L_∞ design for $N = 41$. With the order of the prototype filters varying from 43 × 43 to 99 × 99, the suboptimal L_∞ designs are obtained using (20), and evaluated in terms of e_{p2}, e_{s2}, $e_{p\infty}$, and $e_{s\infty}$. Good design results in terms of L_2 error and L_∞ error are included in Table II. Again, for comparison purposes results from direct design using the window and SVD methods are also included in

TABLE II
COMPARISONS OF THE L_∞ DESIGN WITH THE WINDOW AND SVD METHODS

Method	M	e_{p2}	e_{s2}	$e_{p\infty}$	$e_{s\infty}$
The L_∞ design with a prototype filter	59	0.2347	0.2491	0.0215	0.0365
obtained by the window method	63	0.2209	0.3105	0.0230	0.0262
The L_∞ design with a prototype filter	69	0.2302	0.3486	0.0553	0.0566
obtained by the SVD method	83	0.3152	0.3906	0.0374	0.0344
Direct design by the window method	–	0.4963	0.7529	0.0977	0.1087
Direct design by the SVD method	–	0.8667	0.9056	0.1404	0.1570

the Table. It is evident that the proposed method can be used to design linear phase FIR filters with substantial performance improvement over the commonly used direct design methods such as the window and SVD methods.

IV. CONCLUDING REMARKS

We have presented two indirect methods for the design of linear phase, 2-D FIR filters, and have demonstrated through a case study that these methods may lead to substantially improved designs over existing nonoptimal methods such as the SVD method.

Natural extensions of the methods are the weighted L_2 and weighted L_∞ designs. With the weighting function modeled as a certain zero-phase frequency response, the design ideas presented in this paper apply and the design in question can be reduced to a 2-D deconvolution problem. The details of an algorithmic development for these weighted designs will be reported in a separate paper.

REFERENCES

[1] H. Kimura and Y. Honoki, “Balanced approximation of digital FIR filter with linear phase characteristic,” in *Proc. Int. Symp. Circuits Syst.*, 1995, pp. 283–286.

[2] G. Gu and B. A. Shenoi, “A novel approach to the synthesis of recursive digital filters with linear phase,” *IEEE Trans. Circuits Syst.*, vol. 38, pp. 602–612, June 1991.

[3] W.-S. Lu, H.-P. Wang, and A. Antoniou, “Design of two-dimensional digital filters using the singular value decomposition and balanced approximation,” *IEEE Trans. Signal Processing*, vol. 39, pp. 2253–2262, Oct. 1991.

[4] I. Kale, J. Gryka, G. D. Cain, and B. Beliczynski, “On FIR filter order reduction: Balanced model truncation and Hankel-norm optimal approximation,” *IEE Proc. Vis. Image Signal Process.*, 1994, vol. 141, no. 3, pp. 168–174.

[5] J. Gryka, I. Kale, and G. D. Cain, “Complex IIR filter design through balanced model reduction of FIR prototypes,” *Electron. Lett.*, vol. 31, no. 16, pp. 1332–1334, Aug. 1995.

[6] T. Kailath, *Linear Systems*. Englewood Cliffs, NJ: Prentice Hall, 1980.

[7] J. C. Doyle, B. A. Francis, and A. R. Tannenbaum, *Feedback Control Theory*, New York: Macmillan, 1992.

[8] B. C. Moore, “Principal component analysis in linear systems: Controllability, observability and model reduction,” *IEEE Trans. Automat. Contr.*, vol. 26, pp. 17–31, 1981.

[9] K. Glover, “All optimal Hankel norm approximations of linear multivariable systems and their L^∞-error bounds,” *Int. J. Contr.*, vol. 39, no. 6, pp. 1115–1193, 1984.

[10] C. Davis, W. M. Kahan, and H. F. Weinberger, “Norm-preserving dilations and their applications to optimal error bounds,” *SIAM J. Number, Anal.*, vol. 19, pp. 445–469, 1982.

[11] S. Parrott, “On a quotient norm and the Sz-Nagy Foias lifting theorem,” *J. Functional Analysis*, vol, 30, pp. 311–328, 1978.

[12] W.-S. Lu and A. Antoniou, *Two-Dimensional Digital Filters*. New York: Marcel Dekker, 1992.

[13] T. S. Huang, “Two-dimensional windows,” *IEEE Trans. Audio Electroacoust.*, vol. 20, pp. 88–89, Mar. 1972.

Studying the Effects of Mismatching and Clock-Feedthrough in Switched-Current Filters Using Behavioral Simulation

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***Abstract*— Mismatching and clock-feedthrough are sources of error in Switched-Current (SI) circuits producing DC offset, gain errors, and harmonic distortion, which cause deviations in SI filter responses in both the passband and the stopband. This paper develops formulas including these effects in the input–output description of filter building blocks to make possible not only a faster prediction of the filter performance levels but also to analyze tradeoffs at the early design stages. The block input–output descriptions have been derived taking into account the real physical realization of these blocks together with simple error models. Using these behavioral input–output descriptions, we have studied the extent of these effects in the performance of two different SI filter realizations of passive *LC* ladder structures: one using integrators to simulate the signal flow graph (SFG) of the ladder [1], [2], and another based on the use of wave active filter (WAF) structures [3]–[5]. Monte Carlo simulations of a filter example are presented and analyzed.**

***Index Terms*—Switched-current circuits.**

I. INTRODUCTION

Switched-current (SI) circuits are considered an alternative to switched-capacitor (SC) circuits for analog signal processing applications in standard low-voltage CMOS digital process technologies [1]. The basic reason for this, is that time constants in SI circuits relate to transistor dimensions (W/L), while in SC circuits they depend on linear capacitor ratios and require more expensive CMOS technologies. Moreover, the simplicity of SI circuits make them more suitable than SC circuits for low supply voltages and for scaled CMOS technologies. However, SI circuits suffer from fundamental sources of errors, i.e., mismatch of transistors, clock-feedthrough in the switches, finite input–output resistances of current sources and signal-dependent settling time, which limit their level of performance. Previous work has been done in order to model these errors and analyze their effects on the operation of basic SI elements [2], [8], [9], [12]. Understanding of the sources of nonideal behavior of SI circuits has led to the appearance of new circuit ideas which resulted in improved, though not yet error-free, basic element implementations [6], [10], [11]. Although practical realizations of SI filters are based on these circuit techniques, the study of the filter's sensitivity to the remaining errors in its elements is an important task, not only to predict the expected performance levels, but also to enable the designer to analyze trade-offs at early design stages.

The aim of the work presented in this paper is twofold: first, to develop formulas for error modeling in filter building blocks that enable a rapid simulation of the whole filter operation. Error effects are included in the overall input–output relationship of each block derived from its real physical realization. We shall concentrate on matching and clock-feedthrough errors because they are almost impossible to eliminate by circuit techniques in many applications and

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