Design of Frequency-Response-Masking FIR Filters Using SOCP with Coefficient Sensitivity Constraint

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Outline

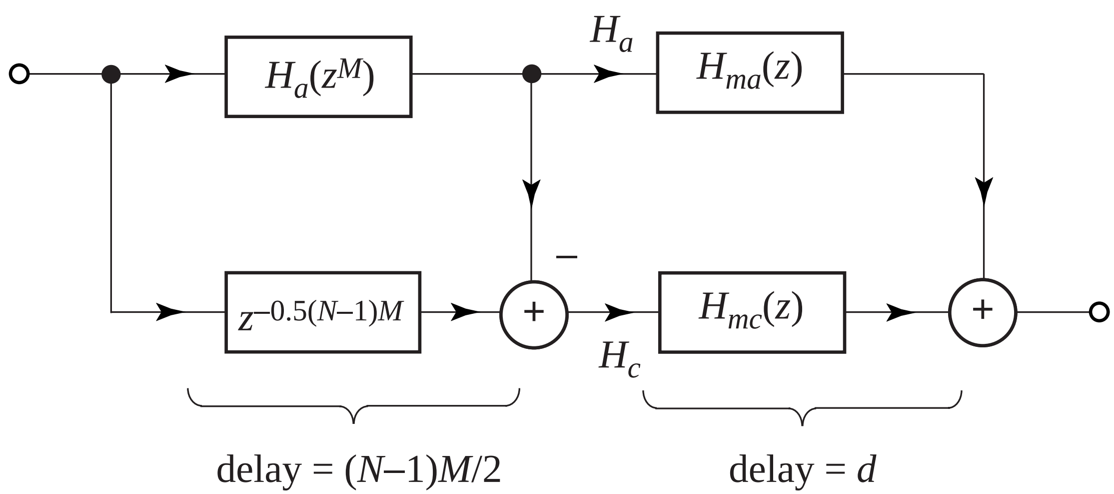
- Early and Recent Work on FRM Filters
- Coefficient Sensitivity (CS) Performance of an Second-Order Cone Programming (SOCP) Algorithm for FRM Filters
- An Enhanced SOCP Algorithm with a CS Constraint
- Experimental Results

Early and Recent Work on FRM Filters

- Lim, 1986.
- Rajan, Neuvo and Mitra, 1988.
- Lim and Lian, 1993.
- Lee and Chen, 1993.
- Lim and Lian, 1994.
- Bellanger, 1996.
- Barcellos, Netto, and Diniz, 2003.
- Saramaki and Lim, 2003.
- Lu and Hinamoto, 2003. (SDP and SOCP techniques)
- Lian and Yang, 2003.
- Lian, 2003.
- Saramaki, Yli-Kaakinen, and Johansson, 2003.
- Lee, Rehbock, and Teo, 2003.
- Gustafsson, Johansson, and Wanhammar, 2003.
- Yu, Teo, Lim, and Zhao, 2005.
- Rodrigues and Pai, 2005.
- Cen and Lian, 2005.
- Lim, Yu, Teo, and Saramaki, 2007. (Coefficient Sensitivity)

CS Performance of an SOCP Algorithm for FRM Filters

• Reference: W.-S. Lu and T. Hinamoto, "Optimal design of frequency-response masking filters using second-order cone programming," *ISCAS'* 2003, vol. 3, pp. 878-881, May 2003.



$$H(\omega, x) = \left[a^T c(\omega)\right]\left[a_a^T c_a(\omega)\right] + \left[1 - a^T c(\omega)\right]\left[a_c^T c_c(\omega)\right], \qquad x = \begin{bmatrix} a \\ a_a \\ a_c \end{bmatrix}$$

$$x_{k+1} = x_k + \delta_k \quad \text{with } \delta_k \text{ "small"}$$

$$H(\omega, x_{k+1}) \approx H(\omega, x_k) + g_k^T(\omega)\delta_k \quad \text{with } g_k(\omega) = \nabla H(\omega, x_k)$$

- SOCP Formulation for an optimal δ_k:

$$\text{minimize} \qquad \eta$$

$$\text{subject to:} \quad W(\omega)\left|H(\omega, x_k) + g_k^T(\omega)\delta - H_d(\omega)\right| \le \eta$$

$$\|\delta\| \le \beta$$

- The coefficient vector of the optimal FRM filter is given

by
$$x^* = x_0 + \sum_{k=0}^{K-1} \delta_k$$

Hence
$$\|x^*\| \le \|x_0\| + \sum_{k=0}^{K-1} \|\delta_k\| \le \|x_0\| + K\beta$$

and $$\|x^* - x_0\| \le \sum_{k=0}^{K-1} \|\delta_k\| \le K\beta$$

Typically K is in the range of 8 to 15 (regardless of filter length).

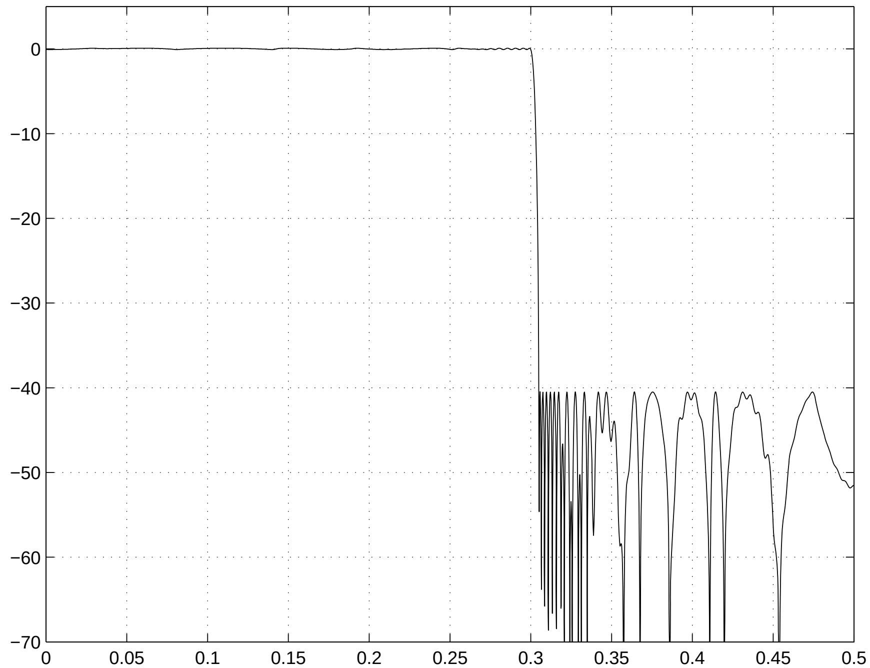
$\Rightarrow$ ♦ $\|x^*\|$ remains small as long as $\|x_0\|$ is small.

♦ Because a small $\|x^*\|$ implies a low CS, **the CS of an SOCP-based solution is low as long as the CS of the initial FRM filter is low**.

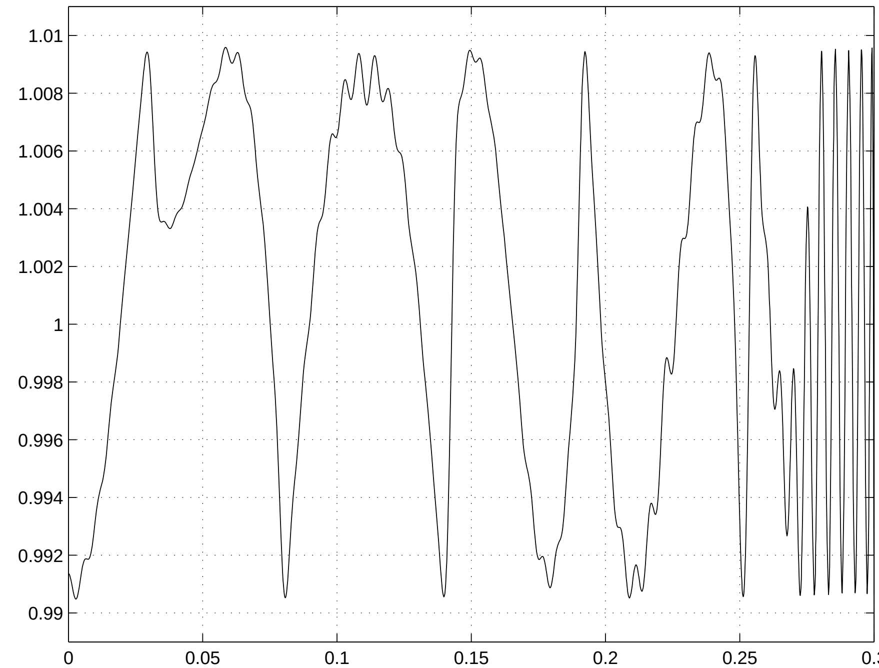
♦ Using SOCP, one can always find an optimal FRM filter ***in a small vicinity*** of a reasonable initial design (obtained e.g. using the method of Lim 1986).

- <u>Example</u> $N = 45$, $N_a = 27$, $N_c = 19$, $M = 9$, $\omega_p = 0.3$, and $\omega_a = 0.305$
  - ♦ CS of the initial FRM filter (Lim 1986): S_1^2 = 24.4532
  - ♦ β was set to 0.1533 (problem size: 44)
  - ♦ the SOCP algorithm converges in 8 iterations
  - ♦ step size for coefficient quantization was set to 2^{-14}
  - ♦ CS of the optimal FRM filter: S_1^2 = 38.9035
  - ♦ peak ripple magnitude in passband: 0.009586
  - ♦ minimum stopband attenuation: 40.4179 dB
  - ♦ coefficient differences in 2-norm:

$$\left\| h^* - h^{(0)} \right\| = 0.5394, \quad \left\| h_a^* - h_a^{(0)} \right\| = 0.0816, \quad \left\| h_c^* - h_c^{(0)} \right\| = 0.1052$$

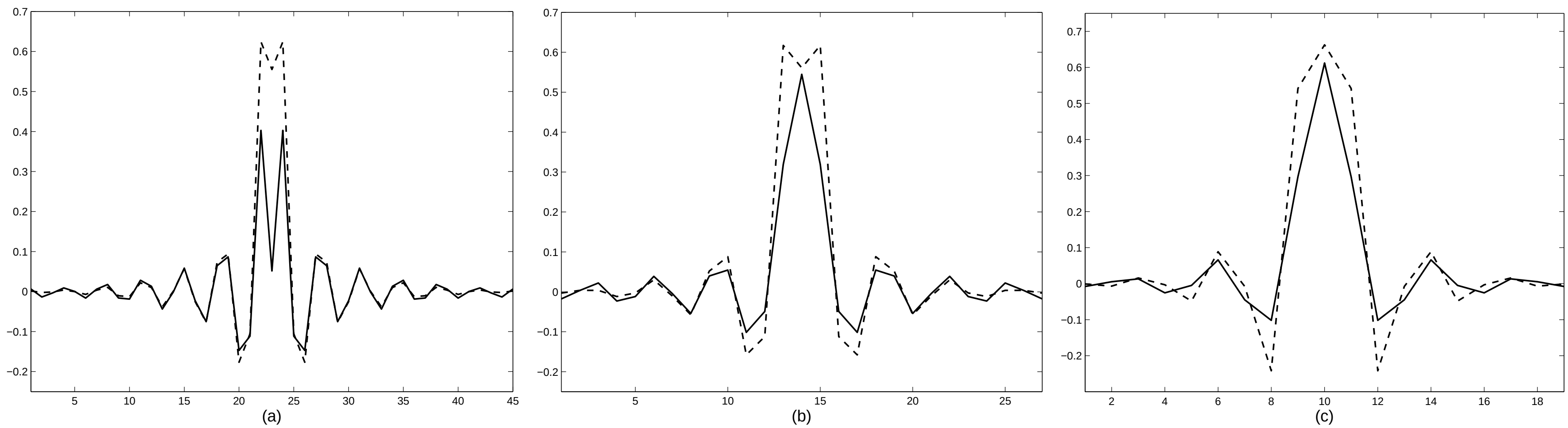


(a)



(b)

- peak ripple magnitude in passband: 0.009586
- minimum stopband attenuation: 40.4179 dB
- CS of the FRM filter: S_1^2 = 38.9035



$$\left\| h^{*} - h^{(0)} \right\| = 0.5394, \quad \left\| h_a^{*} - h_a^{(0)} \right\| = 0.0816, \quad \left\| h_c^{*} - h_c^{(0)} \right\| = 0.1052$$

An Enhanced SOCP Algorithm with a CS Constraint

- The CS measure defined in Lim et al 2007 is given by

$$S_1^2 = \left\| \begin{bmatrix} \sqrt{N_c}\,h \\ \sqrt{N_a}\,h \\ \sqrt{N}(h_a - \hat{h}_c) \end{bmatrix} - \hat{e} \right\|^2$$

with

$$h = \begin{bmatrix} J & 0 & 0 \end{bmatrix} x, \quad h_a = \begin{bmatrix} 0 & J_a & 0 \end{bmatrix} x, \quad h_c = \begin{bmatrix} 0 & 0 & \hat{J}_c \end{bmatrix} x$$

Hence we can write

$$\begin{bmatrix} \sqrt{N_c}\,h \\ \sqrt{N_a}\,h \\ \sqrt{N}(h_a - \hat{h}_c) \end{bmatrix} = A^T x \quad \text{with} \quad A^T = \begin{bmatrix} \sqrt{N_c}\,J & 0 & 0 \\ \sqrt{N_a}\,J & 0 & 0 \\ 0 & \sqrt{N}\,J_a & -\sqrt{N}\,\hat{J}_c \end{bmatrix}$$

If d_{FWL} is the desired upper bound for S_1, then the constraint that S_1 be bound above by d_{FWL} can be cast as

$$\left\| A^T x - \hat{e} \right\| \le d_{FWL}$$

In the kth iteration, $x = x_k + \delta$, the CS constraint becomes

$$\left\| A^T \delta + b_k \right\| \le d_{FWL} \quad \text{with } \; b_k = Ax_k - \hat{e}$$

- Incorporating above constraint on CS into our early SOCP formulation leads an enhanced SOCP problem:

$$\begin{aligned} &\text{minimize} && \eta \\ &\text{subject to:} && W(\omega)\left| H(\omega, x_k) + g_k^T(\omega)\delta - H_d(\omega) \right| \le \eta \\ &&& \left\| \delta \right\| \le \beta \\ &&& \left\| A^T \delta + b_k \right\| \le d_{FWL} \end{aligned}$$

Experimental Results

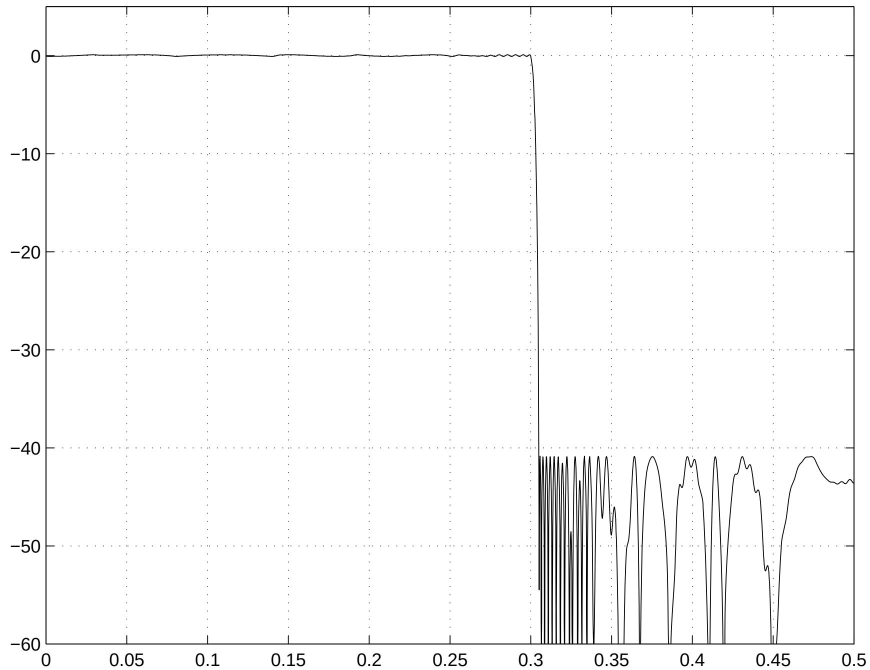
- Design of an FRM filter with $N = 45$, $N_a = 27$, $N_c = 19$, $M = 9$, $\omega_p = 0.3$, and $\omega_a = 0.305$ (same as that in Lim et al, 2007).
- Other design parameters: $\beta = 0.2168$, $d_{FWL} = 5.4 \Leftrightarrow S_1^2 \le 29.16$
- quantization step-size: 2^{-14}
- number of freq. grids in passband and stopband: 2000
- Results: ♦ 8 iterations to converge

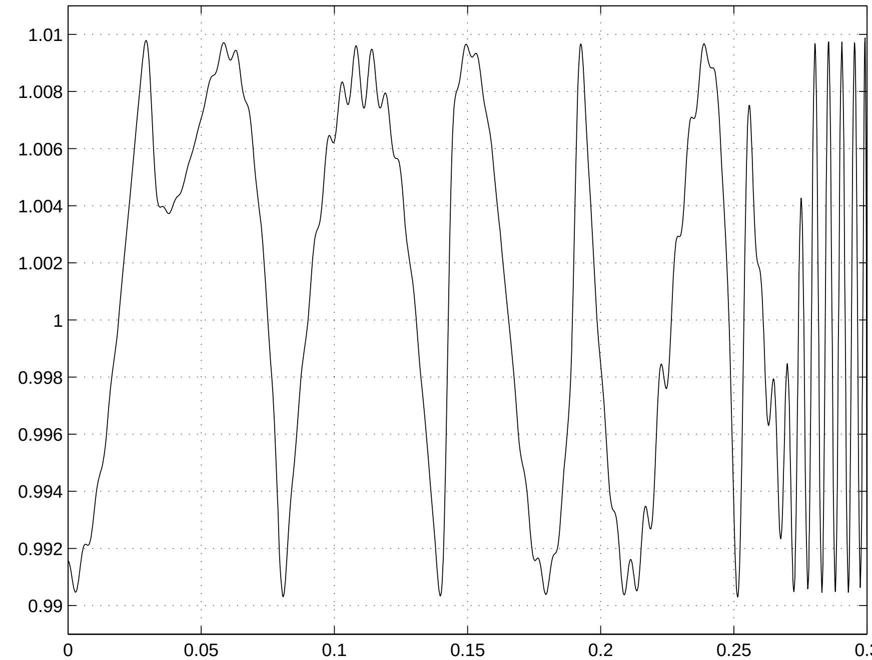
  ♦ peak ripple magnitude in passband: 0.009874

  ♦ minimum stopband attenuation: 40.6479 dB

  ♦ CS of the FRM filter: $S_1^2 = 28.2468$

  ♦ CPU time (3.4 GHz, Pentium 4): 26.91 seconds


0
−10
−20
−30
−40
−50
−60
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
(a)


1.01
1.008
1.006
1.004
1.002
1
0.998
0.996
0.994
0.992
0.99
0
0.05
0.1
0.15
0.2
0.25
0.3
(b)

FRM Filter	Peak passband ripple	Minimum stopband attenuation (dB)	S_1^2
Filter in Table I of Lim et al, 2007	0.009949	40.0674	6.7797 $\times 10^9$
Filter in Table II of Lim et al, 2007	0.010041	39.9628	26.4288
SOCP-based design without CS constraint	0.009586	40.4187	38.9035
SOCP-based design with CS constraint	0.009874	40.6479	28.2468