Fast Algorithms for Restoration of Color Wireless Capsule Endoscopy Images

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Outline

- Introduction
- Background and Related Work
- A Fast Algorithm for Denoising Color Images
- Performance Evaluation
- Concluding Remarks

1. Introduction

- Wireless capsule endoscopy (WCE) is a state-of-the-art technology for diagnosing gastrointestinal tract diseases without invasiveness.

- It acquires images during a slow squirm process and transmits them from inside of the body by a wireless transmitter.

- Raw WCE images are often blurred and noise corrupted due to less ideal environment and conditions for image transmission, imposing difficulties for accurate and effective diagnosis.

- This paper reports some new development for the denoising of color WCE images in a total variation (TV) minimization framework.

2. Background and Related Work

Image Model

$$\boldsymbol{u}_0 = \mathcal{A}\boldsymbol{u} + \boldsymbol{w} \qquad (1)$$

with $\mathcal{A}$ a blurring operator and $\boldsymbol{w}$ Gaussian white noise.

- The restoration problem is to recover image $\boldsymbol{u}$ given the observation $\boldsymbol{u}_0$.
- The problem can be treated as an unconstrained convex problem

$$\underset{\boldsymbol{u}}{\text{minimize}} \quad \mu\|\boldsymbol{u}\|_{\mathrm{TV}} + \frac{1}{2}\|\mathcal{A}\boldsymbol{u} - \boldsymbol{u}_0\|_F^2 \qquad (2)$$

where the TV norm is defined by

$$\|\boldsymbol{u}\|_{\mathrm{TV}} = \sum_{i=1}^{n_1-1}\sum_{j=1}^{n_2-1}\sqrt{\left(u_{i,j}-u_{i+1,j}\right)^2+\left(u_{i,j}-u_{i,j+1}\right)^2}$$
$$+\sum_{i=1}^{n_1-1}\left|u_{i,n_2}-u_{i+1,n_2}\right|+\sum_{j=1}^{n_2-1}\left|u_{n_1,j}-u_{n_1,j+1}\right| \tag{3}$$

A Channel-by-Channel (CBC) TV Minimization Approach

The approach leads to three independent EL equations

$$\nabla\circ\left(\frac{\nabla\boldsymbol{u}^{(i)}}{\left\|\boldsymbol{u}^{(i)}\right\|}\right)-\lambda\left(\boldsymbol{u}^{(i)}-\boldsymbol{u}_0^{(i)}\right)=0 \tag{4}$$

For discrete images, the TV formulation related to these ELs is given by

$$\underset{\boldsymbol{u}}{\text{minimize}}\quad \mu\left\|\boldsymbol{u}^{(i)}\right\|_{\mathrm{TV}}+\frac{1}{2}\left\|\boldsymbol{u}^{(i)}-\boldsymbol{u}_0^{(i)}\right\|_F^2 \tag{5}$$

for i = 1, 2, 3, where μ is inversely related to the Lagrange multiplier λ.

A TV Norm for Color Images

- Let $\{\boldsymbol{u}^{(1)}, \boldsymbol{u}^{(2)}, \boldsymbol{u}^{(3)}\}$ be a color image with $\boldsymbol{u}^{(1)}, \boldsymbol{u}^{(2)}, \boldsymbol{u}^{(3)}$ the red, green and blue components of $\boldsymbol{u}$. The color-TV (CTV) norm of $\boldsymbol{u}$ is defined as

$$\|\boldsymbol{u}\|_{\text{CTV}} = \left(\sum_{i=1}^{3} \left\|\boldsymbol{u}^{(i)}\right\|_{\text{TV}}^{2} \right)^{1/2} \tag{6}$$

The Euler-Lagrange equations associated with CTV norm are

$$\frac{\left\|\boldsymbol{u}^{(i)}\right\|_{\text{TV}}}{\|\boldsymbol{u}\|_{\text{CTV}}} \nabla \circ \left(\frac{\nabla \boldsymbol{u}^{(i)}}{\left\|\boldsymbol{u}^{(i)}\right\|} \right) - \lambda \left(\boldsymbol{u}^{(i)} - \boldsymbol{u}_0^{(i)} \right) = 0 \tag{7}$$

The EL equations in (7) are coupled via the ratio of the TV norms:

$$r_i(\boldsymbol{u}) = \frac{\left\|\boldsymbol{u}^{(i)}\right\|_{\text{TV}}}{\|\boldsymbol{u}\|_{\text{CTV}}} \tag{8}$$

3. A Fast Algorithm for Denoising Color Images

Analysis

· High correlation between channel components $\boldsymbol{u}^{(i)}$ exists, hence we are interested in a discrete formulation that is related to the CTV-based EL equations (7), (8) which can be written as

$$\nabla \circ \left(\frac{\nabla \boldsymbol{u}^{(i)}}{\left\| \boldsymbol{u}^{(i)} \right\|} \right) - \underbrace{\overbrace{\frac{\lambda}{r_i(\boldsymbol{u})}}}_{\lambda_i(\boldsymbol{u})} \left(\boldsymbol{u}^{(i)} - \boldsymbol{u}_0^{(i)} \right) = 0$$

i.e.

$$\nabla \circ \left(\frac{\nabla \boldsymbol{u}^{(i)}}{\left\| \boldsymbol{u}^{(i)} \right\|} \right) - \lambda_i(\boldsymbol{u}) \left(\boldsymbol{u}^{(i)} - \boldsymbol{u}_0^{(i)} \right) = 0 \qquad (9)$$

where $\lambda_i(\boldsymbol{u}) = \dfrac{\lambda}{r_i(\boldsymbol{u})}$ through which the EL eqs. are coupled.

· The EL in (9) leads to a TV minimization formulation as

$$\underset{\boldsymbol{u}}{\text{minimize}} \quad \mu_i(\boldsymbol{u})\left\|\boldsymbol{u}^{(i)}\right\|_{\mathrm{TV}} + \frac{1}{2}\left\|\boldsymbol{u}^{(i)} - \boldsymbol{u}_0^{(i)}\right\|_F^2 \tag{10}$$

where $\mu_i(\boldsymbol{u}) \propto 1/\lambda_i(\boldsymbol{u})$ and assumes the form

$$\mu_i(\boldsymbol{u}) = \mu \cdot r_i(\boldsymbol{u}) = \mu \cdot \frac{\left\|\boldsymbol{u}^{(i)}\right\|_{\mathrm{TV}}}{\left\|\boldsymbol{u}\right\|_{\mathrm{CTV}}} \tag{11}$$

<u>Solving Minimization Problem (10)</u>

We take an iterative approach where in the *k*th iteration (10) is approximated by

$$\underset{\boldsymbol{u}}{\text{minimize}} \quad \mu_i(\boldsymbol{u}_{k-1})\left\|\boldsymbol{u}^{(i)}\right\|_{\mathrm{TV}} + \frac{1}{2}\left\|\boldsymbol{u}^{(i)} - \boldsymbol{u}_0^{(i)}\right\|_F^2 \tag{12}$$

for i = 1, 2, 3. Note that (12) can be solved efficiently using a fast algorithm like MFISTA.

A Bisection Technique for Determining Optimal μ

· From image model $\boldsymbol{u}_0 = \boldsymbol{u} + \boldsymbol{w}$, we have

$$\|\boldsymbol{u} - \boldsymbol{u}_0\|_F^2 = \|\boldsymbol{w}\|_F^2 \approx n_1 n_2 \sigma^2 \qquad (13)$$

· Following (5), parameter μ controls the trade-off between the TV norm of the image and the closeness of $\boldsymbol{u}$ to $\boldsymbol{u}_0$ in Frobenius norm: a μ too large puts a heavier weight on the TV norm leading to a too-large $\frac{1}{2}\|\boldsymbol{u} - \boldsymbol{u}_0\|_F^2$ exceeding $n_1 n_2 \sigma^2$ thus violating (13); a μ too small puts a heavier weight on $\frac{1}{2}\|\boldsymbol{u} - \boldsymbol{u}_0\|_F^2$, leading to a $\|\boldsymbol{u} - \boldsymbol{u}_0\|_F^2$ smaller than $n_1 n_2 \sigma^2$.

· Consequently $\|\boldsymbol{u} - \boldsymbol{u}_0\|_F^2$ as a function of μ is monotonic that increases with μ, and a near optimal value of μ can be identified as one that satisfies (13).

· Based on the analysis, a bisection technique is developed to identify such a μ.

Step 1: Set an initial iterate, say, to the noisy observation $\boldsymbol{u}_0$ and identify an interval $[\mu_L, \mu_U]$ containing the optimal μ. Set a tolerance ε, and $k = 1$.

Step 2: Set $\mu_k = (\mu_L + \mu_U)/2$, solve (12) for *i* = 1, 2, 3, and form $\boldsymbol{u}_k = \left\{\boldsymbol{u}^{(1)}, \boldsymbol{u}^{(2)}, \boldsymbol{u}^{(3)}\right\}$.

Step 3: If $\left\|\boldsymbol{u}_k - \boldsymbol{u}_0\right\|_F^2 > n_1 n_2 \sigma^2$, set $\mu_U = \mu_k$; otherwise set $\mu_L = \mu_k$.

Step 4: If $\mu_U - \mu_L < \varepsilon$, output optimal solution $\boldsymbol{u}_k$ and stop; otherwise set k = $k + 1$ and repeat from Step 2.

4. Performance Evaluation

· Data: A color WCE image of size 140 $\times$ 122, see Fig. 1a.

The image was corrupted by additive Gaussian white noise with $\sigma = 0.05$, see Fig. 1b.

· The initial interval $[\mu_L, \mu_U]$ was identified as $\mu_L = 0$ and $\mu_U = 0.2$.

· A modified MFISTA/FGP algorithm (A. Beck and M. Teboulle, 2009) based on the proposed CTV formulation, equipped with the proposed bisection technique was applied to the above noise-corrupted WCE image.

· A Windows XP laptop PC with an Intel CPU P8700@2.53 GHz with 2 GB of RAM, equipped with MATLAB 7.8.0 was used to run the code. It took the algorithm 10 iterations to converge with 1.24 seconds of elapsed time. The SNR was improved from 18.09 dB to 25.1278 dB. The restored image is shown in Fig. 1c.

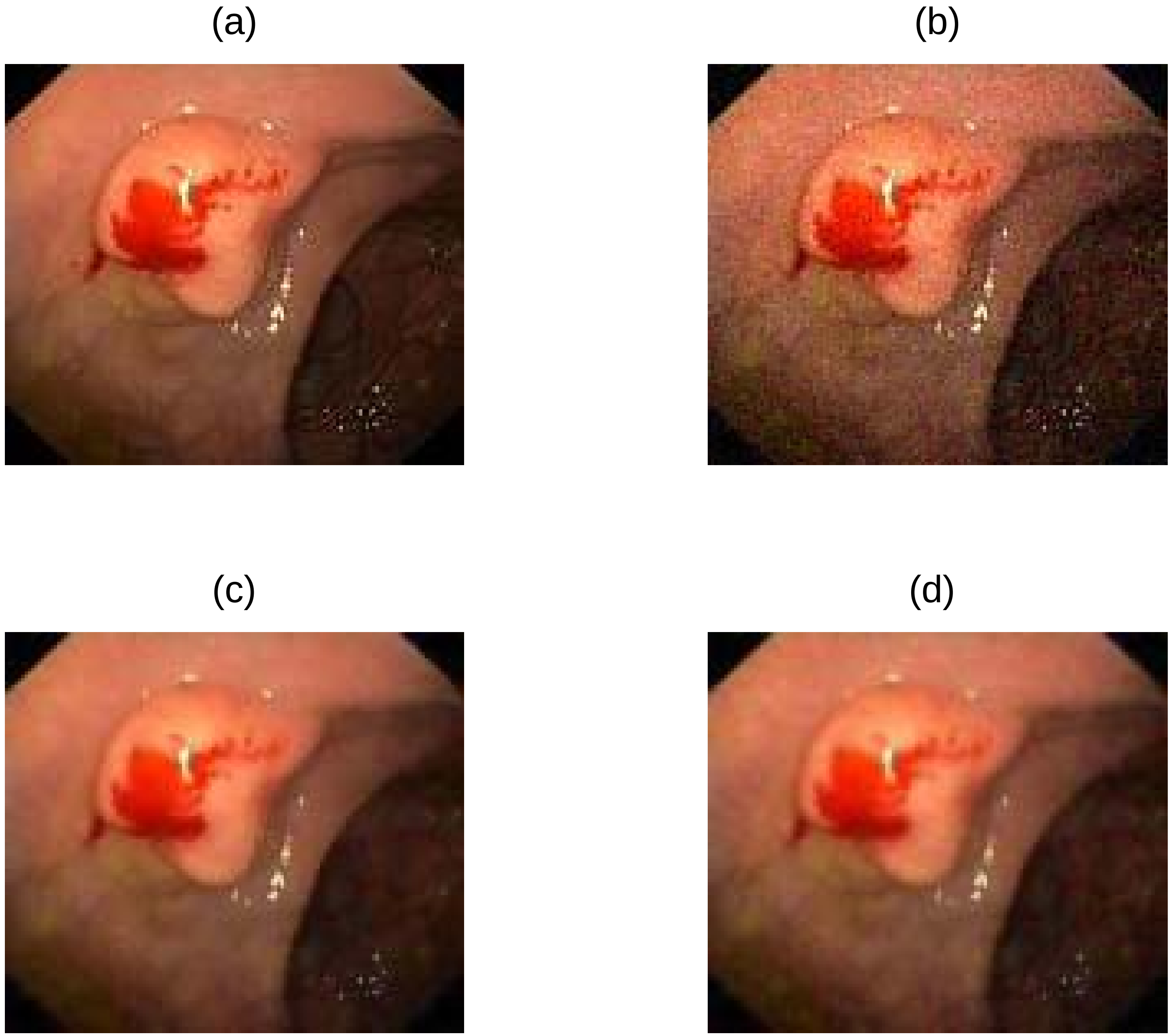


Fig. 1

· Fig. 2 shows profiles of $\left\{\mu_i(\boldsymbol{u}_{k-1}) = \mu_k \cdot r_i(\boldsymbol{u}_{k-1}), \text{ for } k = 1,\ldots,10\right\}$ for the three color channels.

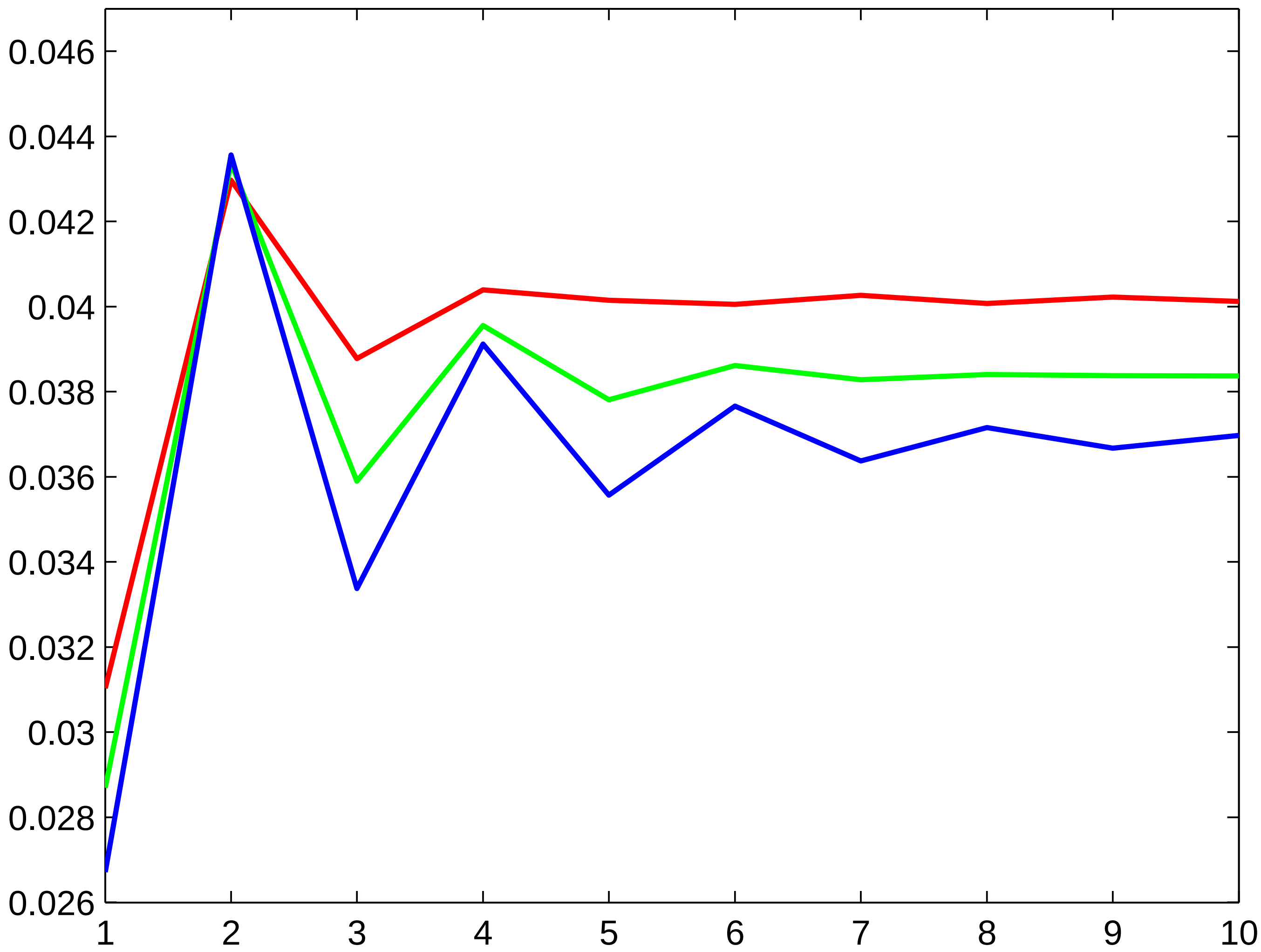


Fig. 2

· To justify the bisection technique, Fig. 3 depicts a profile of the values of $\{\mu_k, \text{for } k = 1, 2, ..., 10\}$ generated in the iterations. We observe that μ_k converges to a value of 0.0667 in 10 iterations.

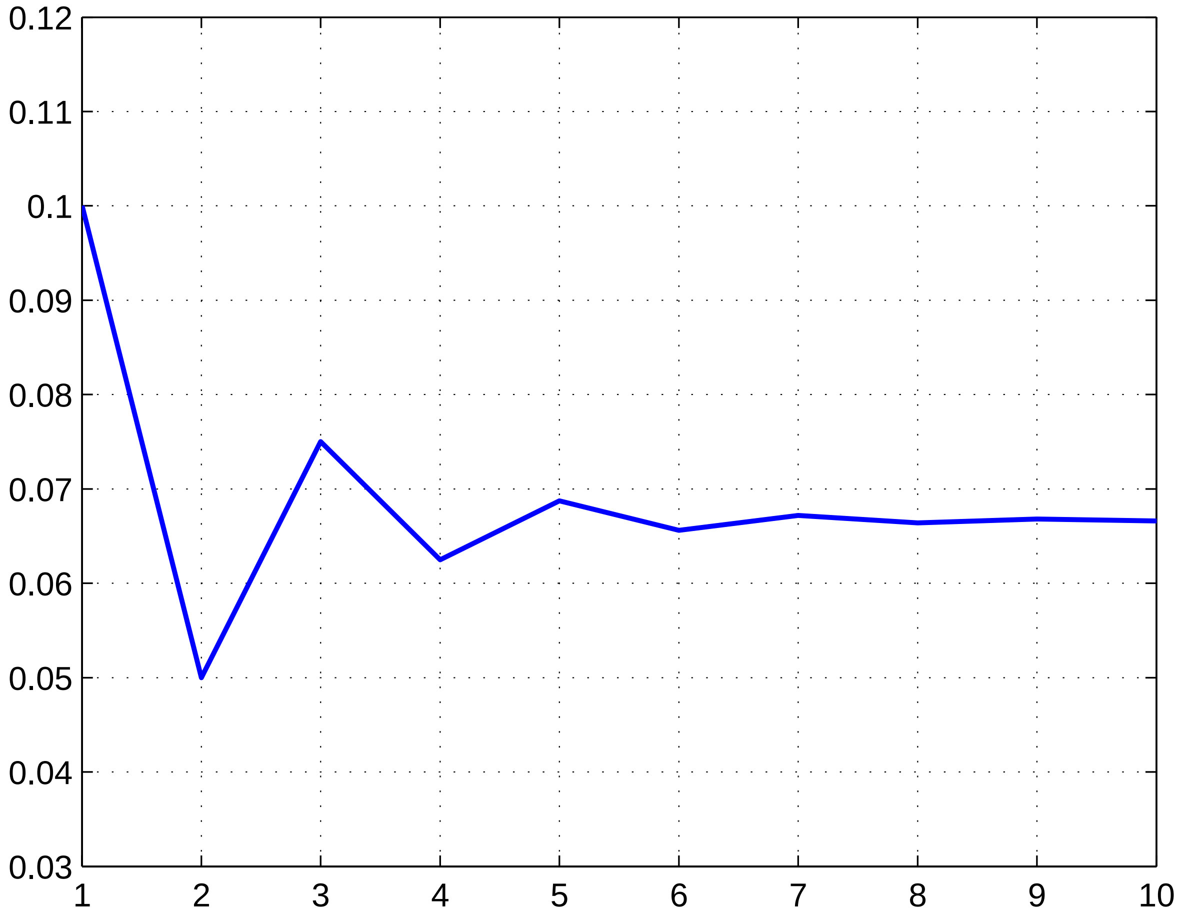
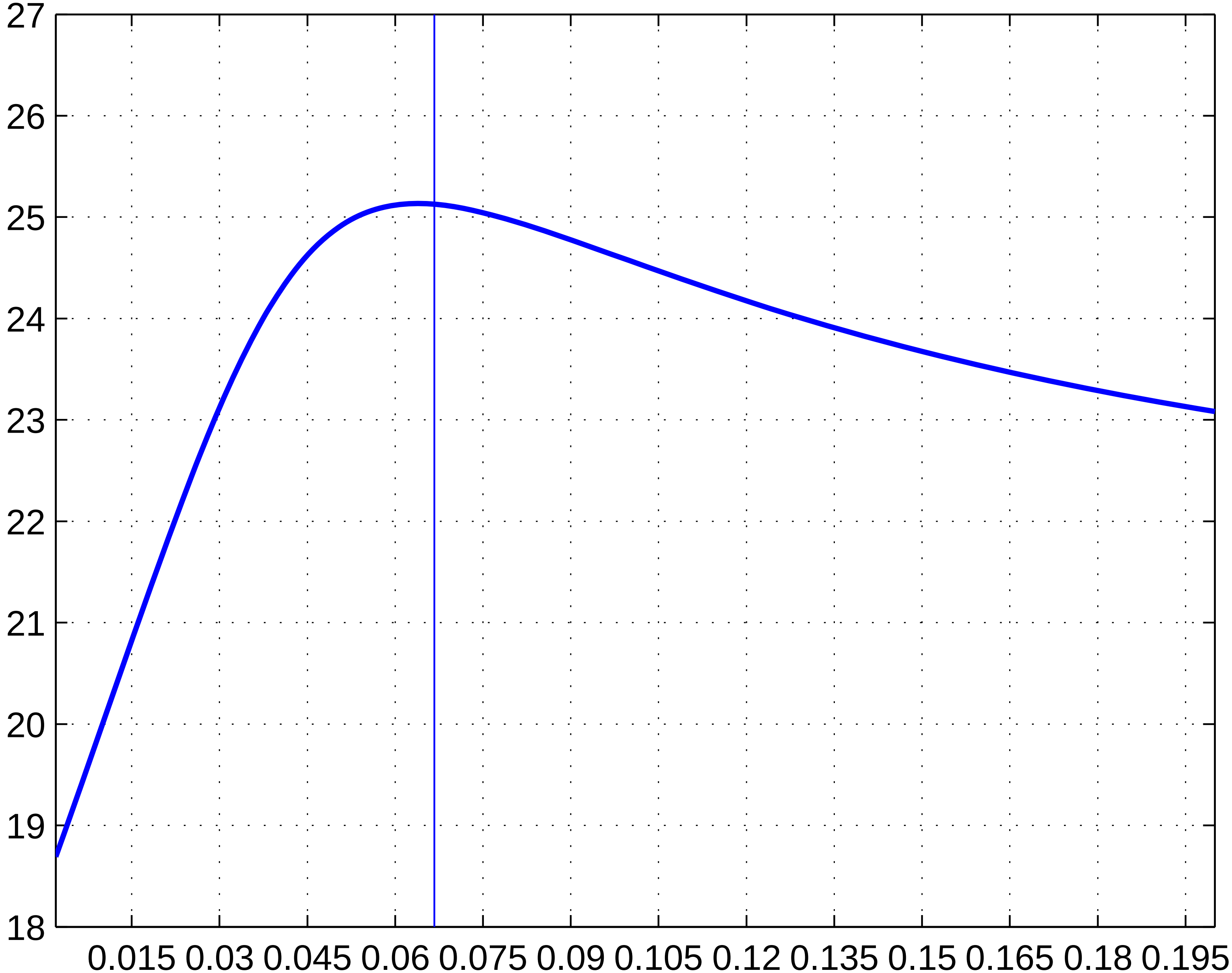


Fig. 3

· Fig. 4 shows the SNR of solution $\boldsymbol{u}$ with respect to parameter μ over an interval $[\mu_L, \mu_U]$. We see that the SNR of the solution produced by the proposed algorithm with μ = 0.0667 is very close to the maximum SNR achievable by a solution of (10) which is 25.1352 dB when μ is taken to be 0.0640.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0.015 0.03 0.045 0.06 0.075 0.09 0.105 0.12 0.135 0.15 0.165 0.18 0.195

Fig. 4

· For comparison, a modified MFISTA/FGP algorithm based on CBC-TV formulation (5), equipped with the bisection technique, was also applied to the above noise-corrupted image. It took the algorithm 10 iterations to converge to a solution.

· The SNR achieved was found to be 24.3148, a gain that was 0.8130 dB less than that obtained by the CTV-based algorithm. The restored image is shown in Fig. 1d.

5. Concluding Remarks

- An algorithm for removing within-channel white noise from multi-channel images has been proposed.
- The algorithm is built on a concept of color total variation (CTV) in an MFISTA/FGP framework.
- The algorithm is enhanced by incorporating a bisection technique that helps identify a near optimal value of the regularization parameter μ.
- Simulation results have demonstrated the effectiveness of the proposed algorithm for restoring color WCE images.