Analysis Of Communications Reliability For Smart Grid

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A short review of past work

• Investigating the impact of packet losses on the DR control strategy
  – Heat pump problem
  – Reliability issue

  **Definition:** Reliability here is defined as the ratio of information packets received correctly over total number of packets delivered. In other words, it is the reliability of communications.

• Designing of communication networks
  – Multi-hop networks
  – Packet loss ratio
A short review of past work [1]
A short review of past work

- Topology control: **random networks**
- Performance: **Outage probability**
- Channel: **Fading channel**

\[ P_{out} = P\{snr_i < SNR_{th}\} \]

- Topology control: Clustering-based **random networks**
- Performance: **Average Packet error rate**
- Channel: **AWGN**

- Topology control: Clustering-based **with fixed cluster-header**
- Performance: **Average Packet error rate**
- Channel: **AWGN**
Contributions

• Key issues
  – The outage probability for a single hop
  – The reliability in a one-hop networks
  – The reliability in a multi-hop networks
The outage probability for a single hop

- The outage probability:
  \[ P_o = P\{snr_i < SNR_{th}\} \]

- The signal-to-noise ratio (SNR) of an arbitrary pair
  \[ r = \frac{P_r}{N_0} = \frac{P_t\gamma}{N_0} \]
  - Three random variable:
    - Rayleigh fading
    - shadowing
    - Path-loss

- SNR distribution
  - With/without log-normal shadowing
The outage probability for a single hop

- SNR distribution:
  - Without log-normal shadowing

\[
\bar{r} = \frac{P_r}{N_0} = \frac{P_tKd^{-\epsilon}}{N_0} = \alpha d^{-\epsilon}, \quad r \sim f_{r|d}(r) = \frac{1}{\alpha d^{-\epsilon}} e^{-r/\alpha d^{-\epsilon}} \quad \alpha = \frac{P_tK}{N_0}
\]

\[
P_0(r) = \int_0^\infty f_{r|d}(r) dr \int_0^{d_{\text{max}}} f_d(x) dx
\]

\[
P_0(r) = 1 - I_1(r, \alpha), \quad I_1(r, z) = \int_0^{d_{\text{max}}} e^{-\frac{r}{z}x^{-\epsilon}} f_d(x) dx
\]
The outage probability for a single hop

- SNR distribution:
  - With log-normal shadowing

\[
P_0(r) = \int_0^\infty \int_0^\infty f_{r|\bar{r}}(x)f_{\bar{r}}(y)dydx
\]

\[
P_0(r) = \int_0^\infty \int_0^\infty f_{r|\bar{r}}(x) \int_0^{d_{max}} f_{s|L}(y)f_L(l)dl dydx
\]

\[
P_0(r) = 1 - \int_0^\infty \frac{1}{\sigma \sqrt{2\pi}} e^{-y^2/2\sigma^2} I_1(r, \alpha 10^{y/10})dy
\]

\[
I_1(r, z) = \int_0^{d_{max}} e^{-\frac{r}{z}l^{-\epsilon}} f_d(l)dl
\]
The outage probability for a single hop

• SNR distribution: Numerical Approximation\textsuperscript{[2]}
  – Gaussian Quadrature

\[ \int_{-1}^{1} f(x)dx = \int_{-1}^{1} W(x)g(x)dx = \sum_{i=1}^{N} \omega_i g(x_i) \]

– Gauss Legendre Quadrature
  • When \( W(x)=1 \)

\[ \omega_i = \frac{2}{(1-x_i^2)[P_n'(x_i)]^2} \]
\[ P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}[(x^2 - 1)^n] \]

• For interval \([a,b] \)

\[ \int_{a}^{b} f(x)dx = \frac{b-a}{2} \int_{-1}^{1} f\left(\frac{b-a}{2} x + \frac{a+b}{2}\right)dx \]

\( X_i \) and \( W_i \) with different \( N \) could be found in \[3\]
The outage probability for a single hop

- **SNR distribution: Numerical Approximation**
  - Without log-normal shadowing
    \[
    P_0(r) = 1 - \sum_{i=1}^{N} \omega_i g_1(x_i, r) \begin{cases} 
    g_1(x, r) = \frac{b}{2} \cdot \exp \left\{ -\frac{r}{a} \left( \frac{b}{2} x + \frac{b}{2} \right)^{-\epsilon} \right\} \cdot f_d \left( \frac{b}{2} x + \frac{b}{2} \right) \\
    a = 0, \text{ and } b = d_{\text{max}}
    \end{cases}
    \]
  - With log-normal shadowing
    - Log-normal approximation
      - Given path-loss, approximated as log-normal distribution when \( \sigma > 6\text{dB} \)
    - Numerical approximation
The outage probability for a single hop

• SNR distribution: Numerical Approximation
  – With log-normal shadowing
    • Log-normal approximation\[^{[3]}\]
      – Given path-loss, \( \sigma > 6 \text{dB} \)
      \[
      f_r(x) = \int_0^\infty \frac{1}{r} e^{-r/R} f_s(R) dR
      \]
      \[
      f_r(x) \approx \frac{\eta}{\sigma_a \sqrt{2\pi x}} \exp\left\{ \frac{(10\log_{10} x - \mu_a)^2}{2\sigma_a^2} \right\}
      \]
      \[
      \sigma_a = \sqrt{\sigma^2 + 5.57^2}, \mu_a = \mu - \eta C_e, C_e \approx 0.57721566 \text{ is the Euler’s constant.}
      \]
  • Numerical approximation
The outage probability for a single hop

- **SNR distribution: Numerical Approximation**
  - With log-normal shadowing
    - Log-normal approximation\(^3\)
    - Numerical approximation

\[
P_0(r) = \int_0^\infty \int_0^{d_{\text{max}}} f_{r|L}(x)f_L(y)\,dy\,dx
\]

\[
P_0(r) = \int_0^{d_{\text{max}}} f_d(x)\,dx \int_0^{Y(x)} \frac{e^{-y^2/2\sigma_a^2}}{\sigma_a \sqrt{2\pi}}\,dy
\]

\[
P_0(r) = \int_0^{d_{\text{max}}} f_d(x)[1 - Q[Y(x)]]\,dx
\]

\[
P_0(r) = 1 - \sum_{i=1}^N \omega_ig_2(x_i, r)
\]

\[
Y(x) = 10\log_{10}\frac{rc'}{k\alpha} x^{-\epsilon}, \quad c' = 10^{C_e/\ln 10}
\]

\[
g_2(x, r) = \frac{b}{2} \cdot f_d\left(\frac{b}{2}x + \frac{b}{2}\right) \cdot Q\left[Y\left(\frac{b}{2}x + \frac{b}{2}\right)\right]
\]

\[
a = 0, \text{ and } b = d_{\text{max}}.
\]
The outage probability for a single hop

- Simulation Test
  - Distance distribution in two parallel squares
The outage probability for a single hop

• Simulation Test
  – Comparison with using average pathloss
The outage probability for a single hop

• The reliability in a one-hop networks
  – Probability of reliable delivery
    – Given SNR threshold, $r_o$
      
      $$P_{rel} = P\{\text{snr}_i > r_0\} = 1 - P_o(r_0)$$

  – Given N nodes, the reliable delivery ratio
    – Geometric distribution
      
      $$P_s(i \leq n) = \sum_{i=1}^{n} \binom{N}{i} P_{rel}^i (1 - P_{rel})^{N-i}$$
The outage probability for a multi-hop network

- The reliability in a multi-hop networks
  - Probability of reliable delivery
    - Given SNR threshold $r_o$, j-hops
      \[ P_{rel}(j) = 1 - (1 - P_{rel})^j = 1 - P_o(r_o)^j \]
    - Given N nodes, the reliable delivery ratio
      - Conditional geometric distribution
        \[ P_s(i \leq n) = \sum_{j=1}^{J} P_j \cdot \sum_{i=1}^{n} \binom{N}{i} P_{rel}(j)(1 - P_{rel}(j))^{N-i} \]

where $P_j$ is the probability for j hops determined by the area topology (square, hexagon, circle)
Reference

Thanks!

Questions?