

USEFUL FORMULAE AND OTHER INFORMATION

	x	$\cos x$	$\sin x$
$e^{j\theta} = \cos(\theta) + j \sin(\theta)$	0	1	0
$\cos(\theta) = \frac{1}{2} (e^{j\theta} + e^{-j\theta})$	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
$\sin(\theta) = \frac{1}{2j} (e^{j\theta} - e^{-j\theta})$	$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\text{sinc}(t) = \sin(t)/t$	$\frac{\pi}{2}$	0	1
	$\frac{3\pi}{4}$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
	π	-1	0

$$\begin{aligned}
 x(t) &= \sum_{k=-\infty}^{\infty} c_k e^{j(2\pi/T)kt} & \mathcal{F}x(\omega) = X(\omega) &= \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt & X(\omega) &= \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0) \\
 c_k &= \frac{1}{T} \int_T x(t) e^{-j(2\pi/T)kt} dt & \mathcal{F}^{-1}X(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega & X(\omega) &= \sum_{k=-\infty}^{\infty} \omega_0 X_T(k\omega_0) \delta(\omega - k\omega_0) \\
 S(\omega) &= \frac{\omega_s}{2\pi} \sum_{k=-\infty}^{\infty} X(\omega - k\omega_s) & H(\omega) &= \frac{2\pi}{\omega_s} \text{rect}\left(\frac{\omega}{\omega_s}\right) & a_k &= \frac{1}{T} X_T(k\omega_0)
 \end{aligned}$$

Fourier Transform Properties

Property	Time Domain	Frequency Domain
Linearity	$a_1 x_1(t) + a_2 x_2(t)$	$a_1 X_1(\omega) + a_2 X_2(\omega)$
Time-Domain Shifting	$x(t - t_0)$	$e^{-j\omega t_0} X(\omega)$
Frequency-Domain Shifting	$e^{j\omega_0 t} x(t)$	$X(\omega - \omega_0)$
Time/Frequency-Domain Scaling	$x(at)$	$\frac{1}{ a } X\left(\frac{\omega}{a}\right)$
Conjugation	$x^*(t)$	$X^*(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Time-Domain Convolution	$x_1 * x_2(t)$	$X_1(\omega) X_2(\omega)$
Frequency-Domain Convolution	$x_1(t) x_2(t)$	$\frac{1}{2\pi} X_1 * X_2(\omega)$
Time-Domain Differentiation	$\frac{d}{dt} x(t)$	$j\omega X(\omega)$
Frequency-Domain Differentiation	$tx(t)$	$j \frac{d}{d\omega} X(\omega)$
Time-Domain Integration	$\int_{-\infty}^t x(\tau) d\tau$	$\frac{1}{j\omega} X(\omega) + \pi X(0) \delta(\omega)$
Parseval's Relation	$\int_{-\infty}^{\infty} x(t) ^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) ^2 d\omega$	

Fourier Transform Pairs

Pair	$x(t)$	$X(\omega)$
1	$\delta(t)$	1
2	$u(t)$	$\pi \delta(\omega) + \frac{1}{j\omega}$
3	1	$2\pi \delta(\omega)$
4	$\text{sgn}(t)$	$\frac{2}{j\omega}$
5	$e^{j\omega_0 t}$	$2\pi \delta(\omega - \omega_0)$
6	$\cos(\omega_0 t)$	$\pi [\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
7	$\sin(\omega_0 t)$	$\frac{\pi}{j} [\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
8	$\text{rect}\left(\frac{t}{T}\right)$	$ T \text{sinc}\left(\frac{T\omega}{2}\right)$
9	$\text{sinc}(Bt)$	$\frac{\pi}{ B } \text{rect}\left(\frac{\omega}{2B}\right)$
10	$e^{-at} u(t), \text{Re}\{a\} > 0$	$\frac{1}{a + j\omega}$
11	$t^{n-1} e^{-at} u(t), \text{Re}\{a\} > 0$	$\frac{(n-1)!}{(a + j\omega)^n}$
12	$e^{-at} \cos(\omega_0 t) u(t), \text{Re}\{a\} > 0$	$\frac{a + j\omega}{(a + j\omega)^2 + \omega_0^2}$
13	$e^{-at} \sin(\omega_0 t) u(t), \text{Re}\{a\} > 0$	$\frac{\omega_0}{(a + j\omega)^2 + \omega_0^2}$
14	$e^{at} u(-t), \text{Re}\{a\} > 0$	$\frac{1}{a - j\omega}$