Structures for LTI systems (4)

Signal flow graphs cont'd

Reading: 6.3, 6.5 (without 6.5.3)

Basic Structures for IIR and FIR systems

- For any given rational system function, a wide variety of equivalent sets of difference equations or network structures exist. How do we choose among those?

1. Computational complexity
   - Multiplication is time consuming
     - Reduction in no of multipliers = increase in speed
   - Delay is implemented by a memory register
     - Reduction in no of delay units=reduction in memory requirements
2. VLSI implementations: modularity and regularity of structure
3. Effects of a finite register length and finite precision arithmetic

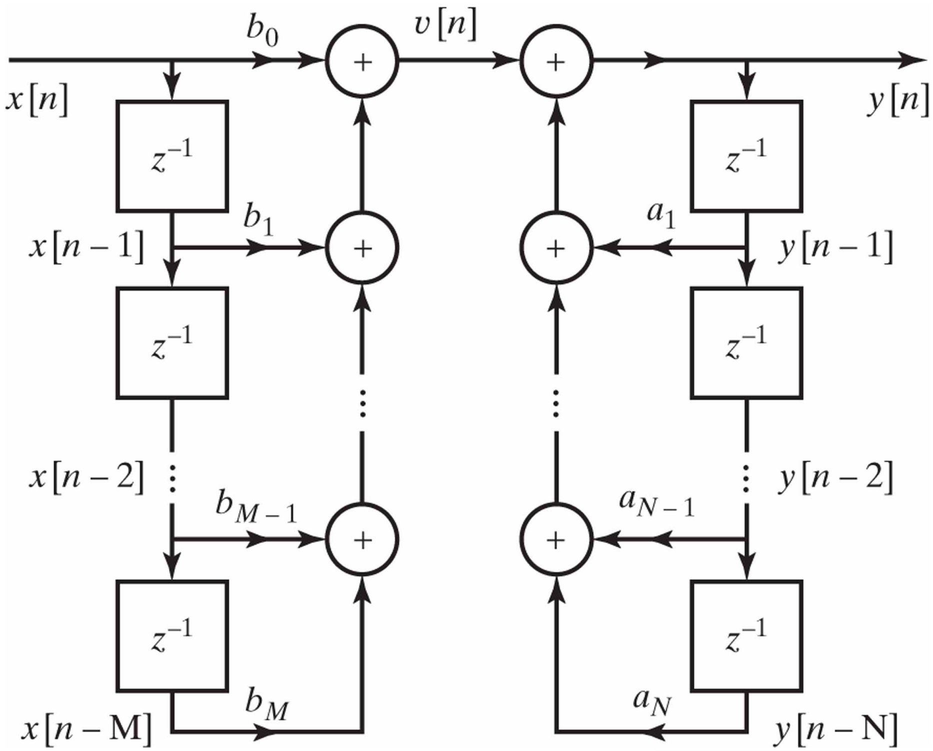
IIR versus FIR filters

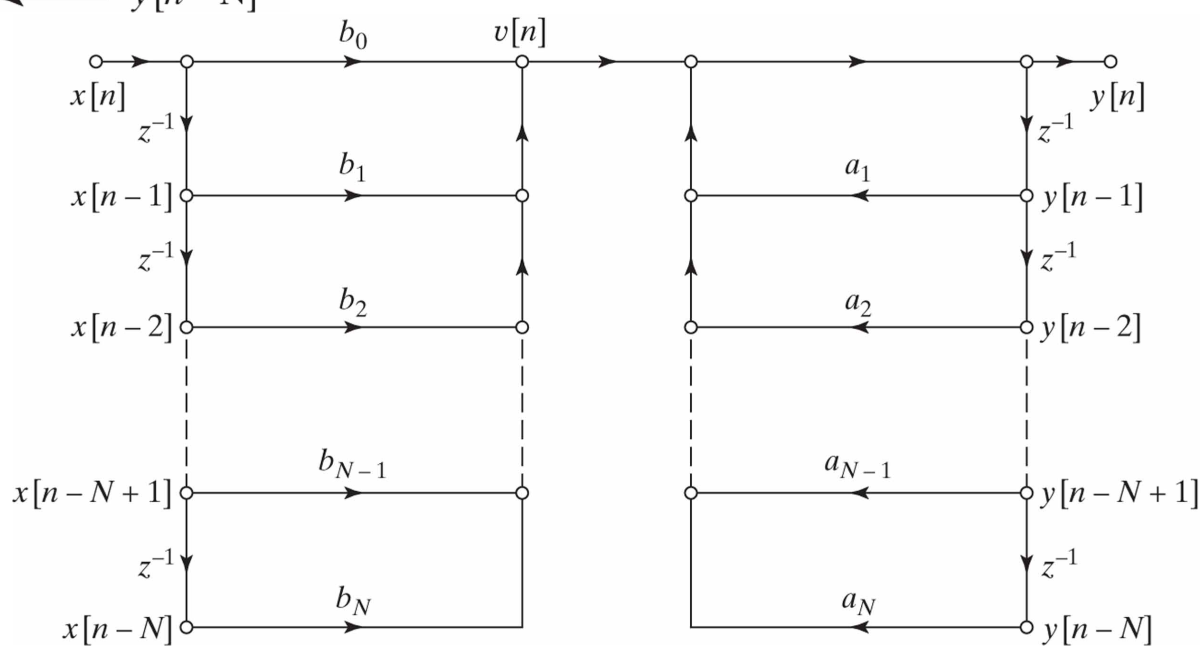
- IIR = Infinite Impulse Response
  - h[n] is a causal signal of infinite duration
- FIR=Finite Impulse Response
  - h[n] is a causal signal of finite duration (finite no of samples)

Basic Structures for IIR filters

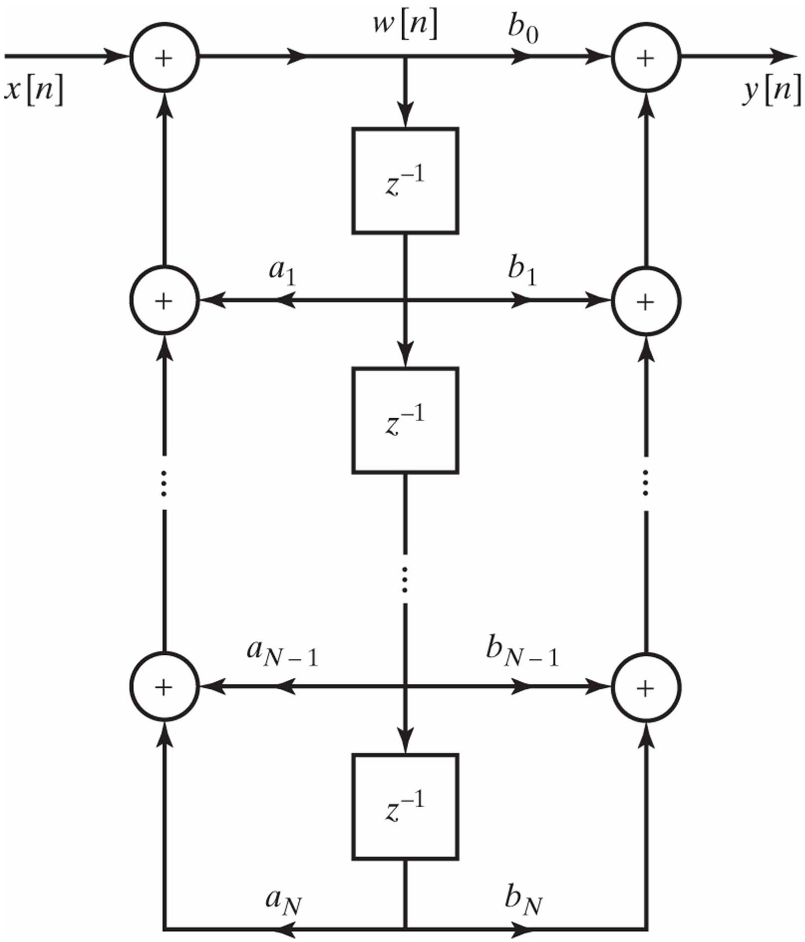
- Direct Forms (I and II)
- Cascade forms
- Parallel forms
- Transposed Forms

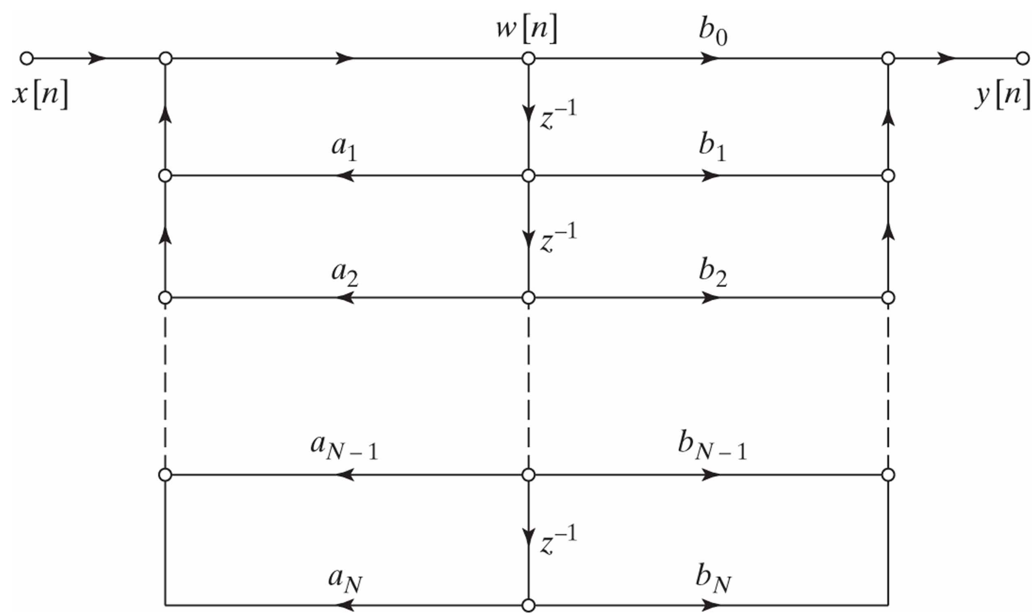
IIR Direct Form I





IIR Direct Forms II





Cascade Forms

- Instead of deriving the signal flow graph directly from the system function (direct forms), we could also factor the denominator and numerator of the system function into first-order and second-order subsystems:

$$H(z) = A \frac{\prod_{k=1}^{M_1}(1 - f_k z^{-1}) \prod_{k=1}^{M_2}(1 - g_k z^{-1})(1 - g_k^* z^{-1})}{\prod_{k=1}^{N_1}(1 - c_k z^{-1}) \prod_{k=1}^{N_2}(1 - d_k z^{-1})(1 - d_k^* z^{-1})}, \qquad (6.29)$$

- This is based on the fact that causal systems with a rational H(z) with real coefficients have their complex poles coming in complex conjugate pairs

Cascade forms (cont'd)

- To implement a cascade with a minimum of storage and computation: combine pairs of real factors and complex conjugate pairs into second order factors

$$H(z) = \prod_{k=1}^{N_s} \frac{b_{0k} + b_{1k}z^{-1} + b_{2k}z^{-2}}{1 - a_{1k}z^{-1} - a_{2k}z^{-2}},$$

where $N_s = \lfloor (N+1)/2 \rfloor$ is the largest integer contained in $(N+1)/2$.

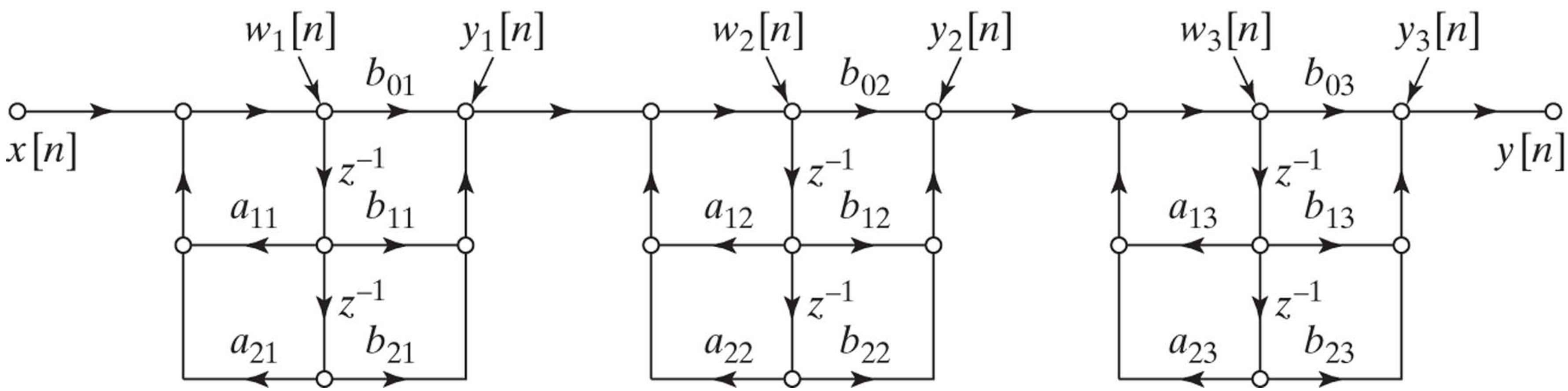


Figure 6.18 Cascade structure for a 6th-order system with a direct form II realization of each 2nd-order subsystem.

Parallel forms

- Expressing the transfer function as a sum using partial fraction expansion gives a parallel structure:

$$H(z) = \sum_{k=0}^{N_p} C_k z^{-k} + \sum_{k=1}^{N_1} \frac{A_k}{1 - c_k z^{-1}} + \sum_{k=1}^{N_2} \frac{B_k(1 - e_k z^{-1})}{(1 - d_k z^{-1})(1 - d_k^* z^{-1})},$$

$$H(z) = \sum_{k=0}^{N_p} C_k z^{-k} + \sum_{k=1}^{N_s} \frac{e_{0k} + e_{1k} z^{-1}}{1 - a_{1k} z^{-1} - a_{2k} z^{-2}} .$$

Parallel forms (cont'd)

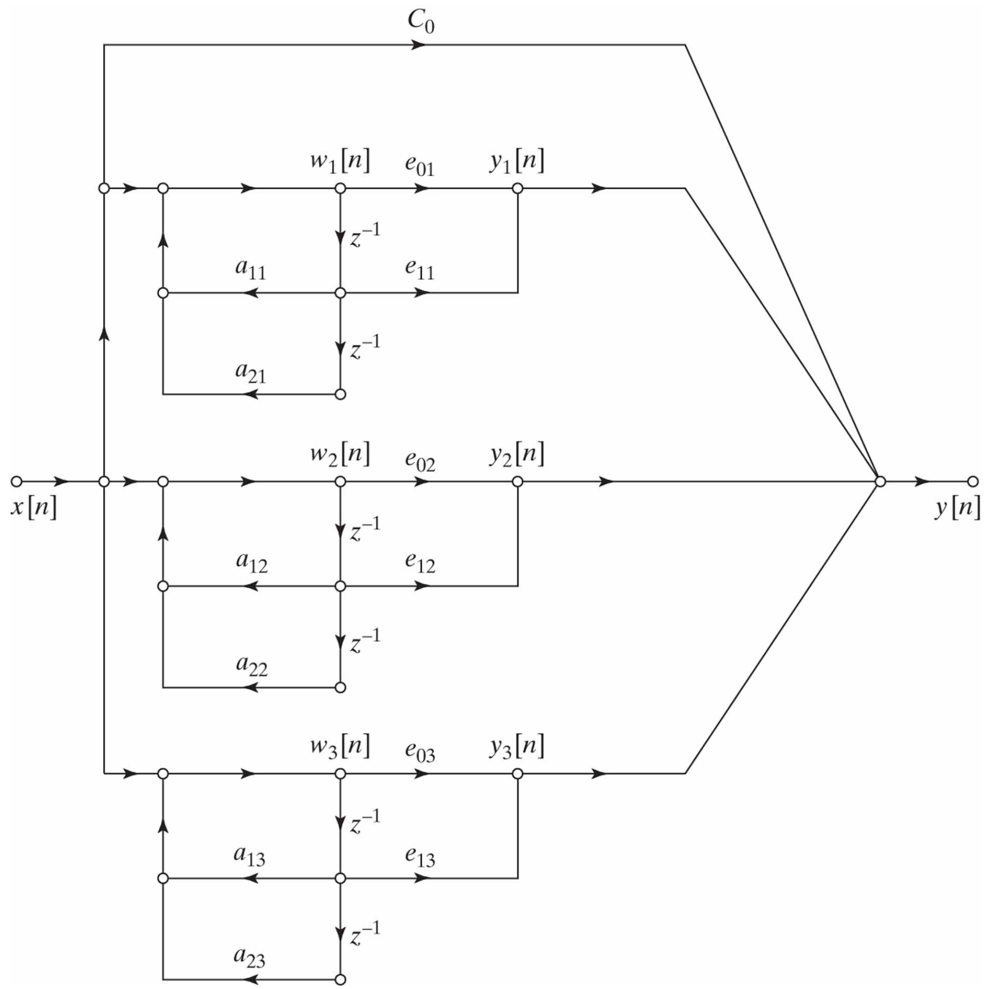


Figure 6.20 Parallel form structure for 6th-order system ($M = N = 6$) with the real and complex poles grouped in pairs.

Transposed forms

- Using signal flow graphs, we can transform a given system into a different network structure while maintaining the same system function. One such transformation is transposition.

Transposition Theorem

- Reverse direction of all branches
- Interchange input and output

- For single-input single-output systems, interchanging the input and output nodes after reversing the flow graph gives the same transfer function as the original system.

Transposed forms (cont'd)

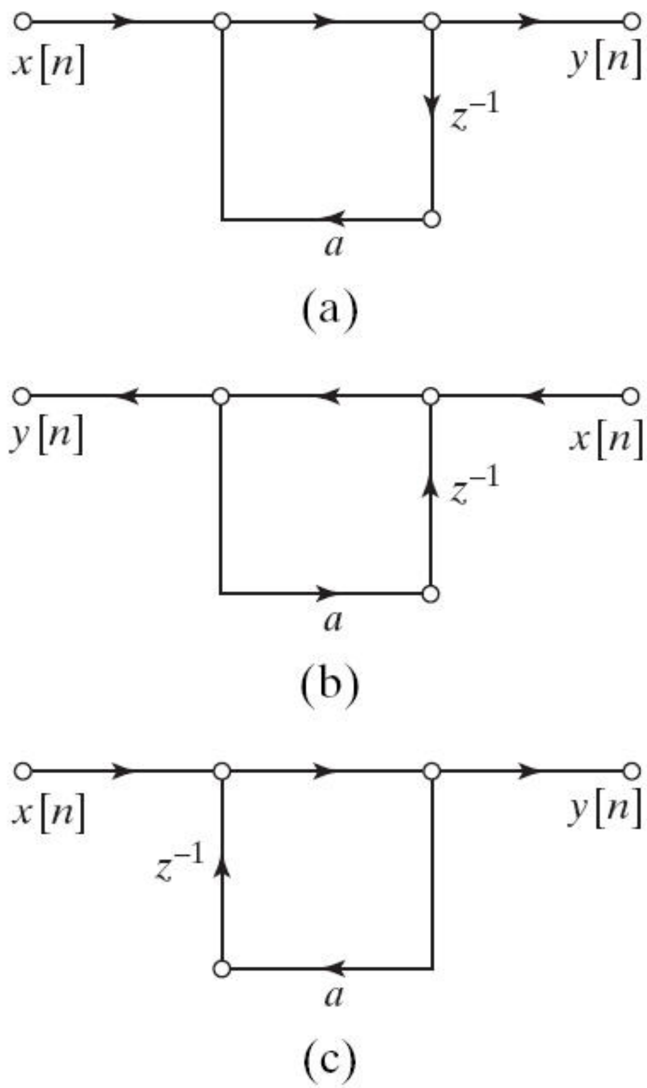


Figure 6.24 (a) Flow graph of simple 1st-order system. (b) Transposed form of (a). (c) Structure of (b) redrawn with input on left.

FIR filter structures

- FIR systems are special cases of IIR systems, which can be structured into direct, cascade, or parallel forms.
- additional forms specific to FIR systems exist
- Direct Form: *tap-delay line filter structure or a transversal filter structure.*
- The implementation of FIR systems is not necessarily always nonrecursive, since pole-zero cancellation may exist.

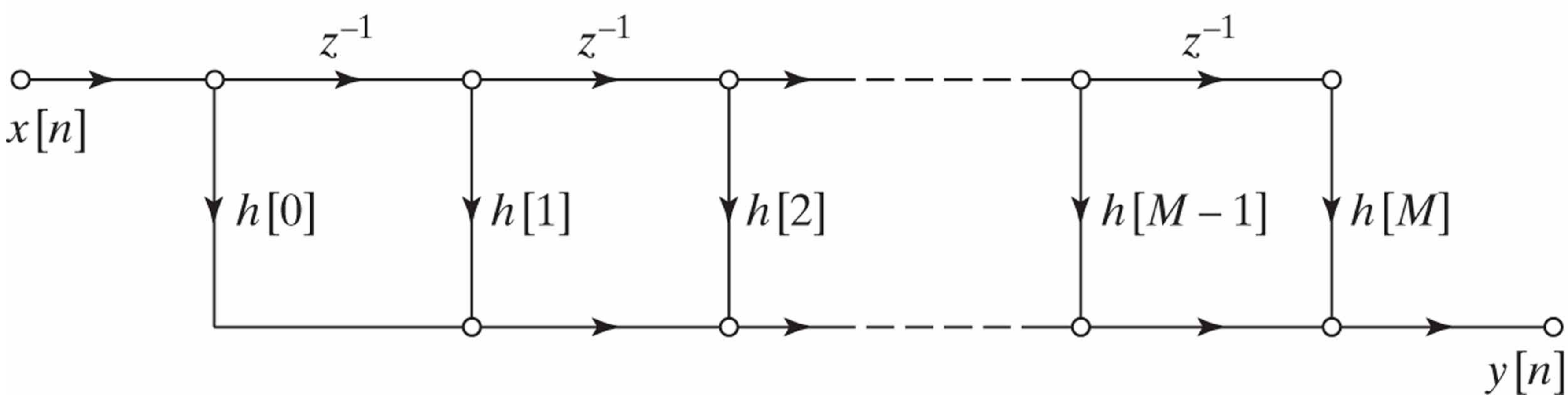


Figure 6.29 Direct form realization of an FIR system.

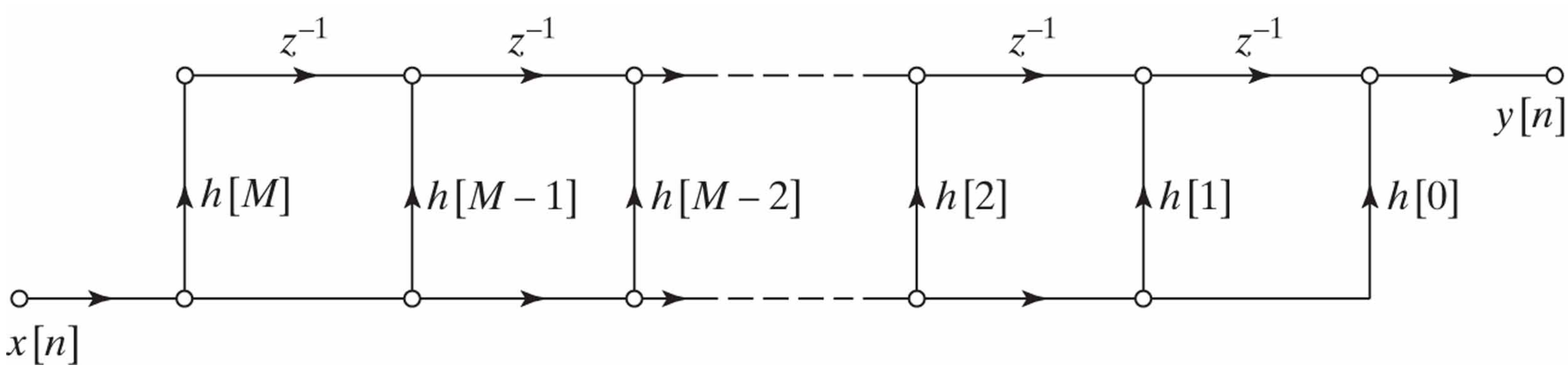
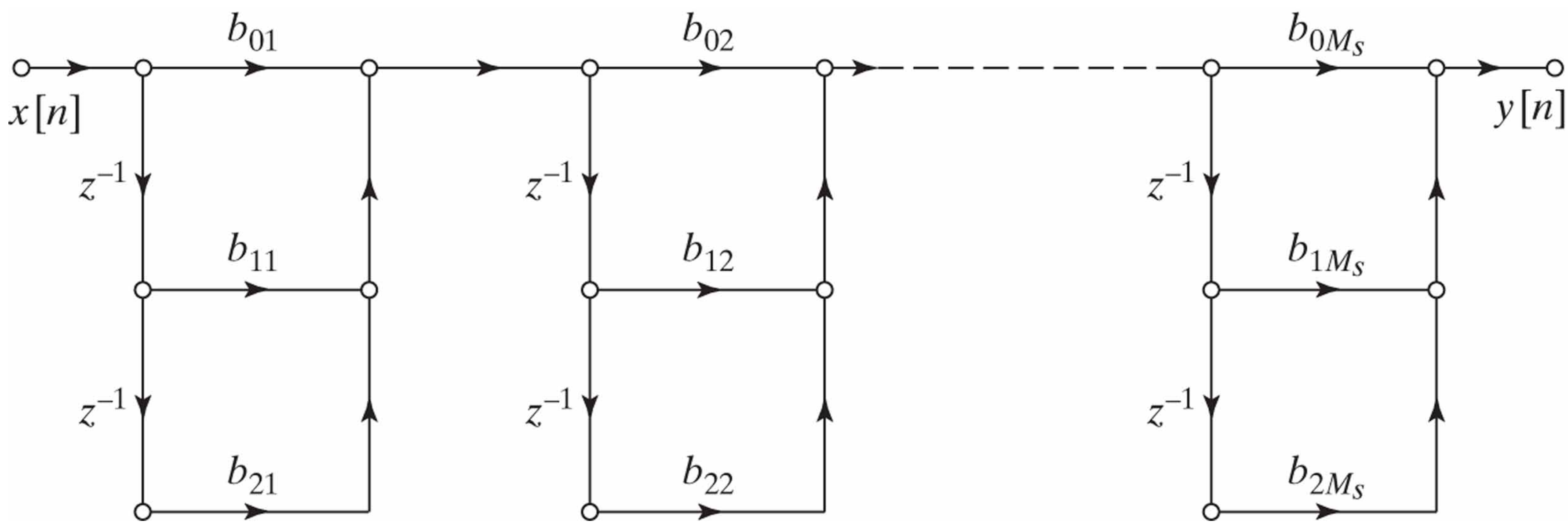


Figure 6.30 Transposition of the network of Figure 6.29.

Cascade Form for FIR

$$H(z) = \sum_{n=0}^{M} h[n]z^{-n} = \prod_{k=1}^{M_s}(b_{0k} + b_{1k}z^{-1} + b_{2k}z^{-2}),$$

where $M_s = \lfloor (M+1)/2 \rfloor$ is the largest integer contained in $(M+1)/2$.



Example

6.26. A causal LTI system has system function given by the following expression:

$$H(z) = \frac{1}{1 - z^{-1}} + \frac{1 - z^{-1}}{1 - z^{-1} + 0.8z^{-2}}.$$

(a) Is this system stable? Explain briefly.
(b) Draw the signal flow graph of a parallel form implementation of this system.
(c) Draw the signal flow graph of a cascade form implementation of this system as a cascade of a 1^{st}-order system and a 2^{nd}-order system. Use a transposed direct form II implementation for the 2^{nd}-order system.