University of Victoria

Department of Electrical and Computer Engineering

ECE 405/511 **Error Control Coding** **Spring 2026**

MIDTERM TEST

INSTRUCTIONS:

One sheet of $8.5'' \times 11''$ paper and a calculator are allowed. There are 20 marks total. State any assumptions that may be necessary. Please write clearly.

Question 1 [2 marks]

Find a basis for the dual space to the binary vectors spanned by $\{(0111), (1010)\}$.

Question 2 [12 marks total]

Consider the binary linear code C_1 with the following generator matrix

$$G_1 = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

a) [2 marks] What is the minimum distance of this code?

b) [1 mark] What is the maximum weight for which the detection of all errors is guaranteed?

c) [1 mark] What is the maximum weight for which the correction of all errors is guaranteed?

d) [4 marks] Convert C_1 to an equivalent code with a systematic generator matrix.

e) [4 marks] Find a parity check matrix for the code defined by the generator matrix in d).

Question 3 [6 marks total]

Consider the binary linear code C_2 with generator matrix

$$G_2 = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

a) [4 marks] Determine a syndrome decoding table for this code.

b) [2 marks] Using the syndromes in a), decode the following received vector $r = 110$.

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1. 0111 and 1010 are linearly independent
therefore they form a two dimensional subspace of V_4

the vectors orthogonal to this subspace are

0000
0101
1011
1110

a basis for this subspace is

$$\begin{bmatrix} 0101 \\ 1011 \end{bmatrix}$$

2. code C_1 with generator matrix

$$G_1 = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

the nonzero codewords are

	w
1 0 1 0 1	3
1 1 0 1 0	3
0 1 1 1 1	4

a) $d_{min} = 3$

b) $v = d_{min} - 1 = 2$

c) $t = \left\lfloor \frac{d_{min}-1}{2} \right\rfloor = \left\lfloor \frac{3-1}{2} \right\rfloor = 1$

d) interchange columns 1 and 3

$$G_1' = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} = [I \ P]$$

e) $H_1' = [P^T \; I]$

$$= \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

3. $G_2 = [111]$ (3,1,3) repetition code $t=1$

a) $H_2 = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$s_0 = (0\,0\,0)H_2^T = 00$

$s_1 = (1\,0\,0)H_2^T = 11$

$s_2 = (0\,1\,0)H_2^T = 10$

$s_3 = (0\,0\,1)H_2^T = 01$

b) $r = 110$ $\quad rH_2^T = 01 = s_3$

$\therefore$ $e = 001$ and $c = r + e = 111$

a) $H_2' = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

$s_0 = (000)\, H_2'^T = 00$

$s_1 = (100)\, H_2'^T = 10$

$s_2 = (010)\, H_2'^T = 01$

$s_3 = (001)\, H_2'^T = 11$

b) $r = 110 \qquad rH_2'^T = 11$

$\therefore \quad e = 001$ and $c = r + e = 111$