ELEC 619A
Selected Topics in Digital Communications: Information Theory

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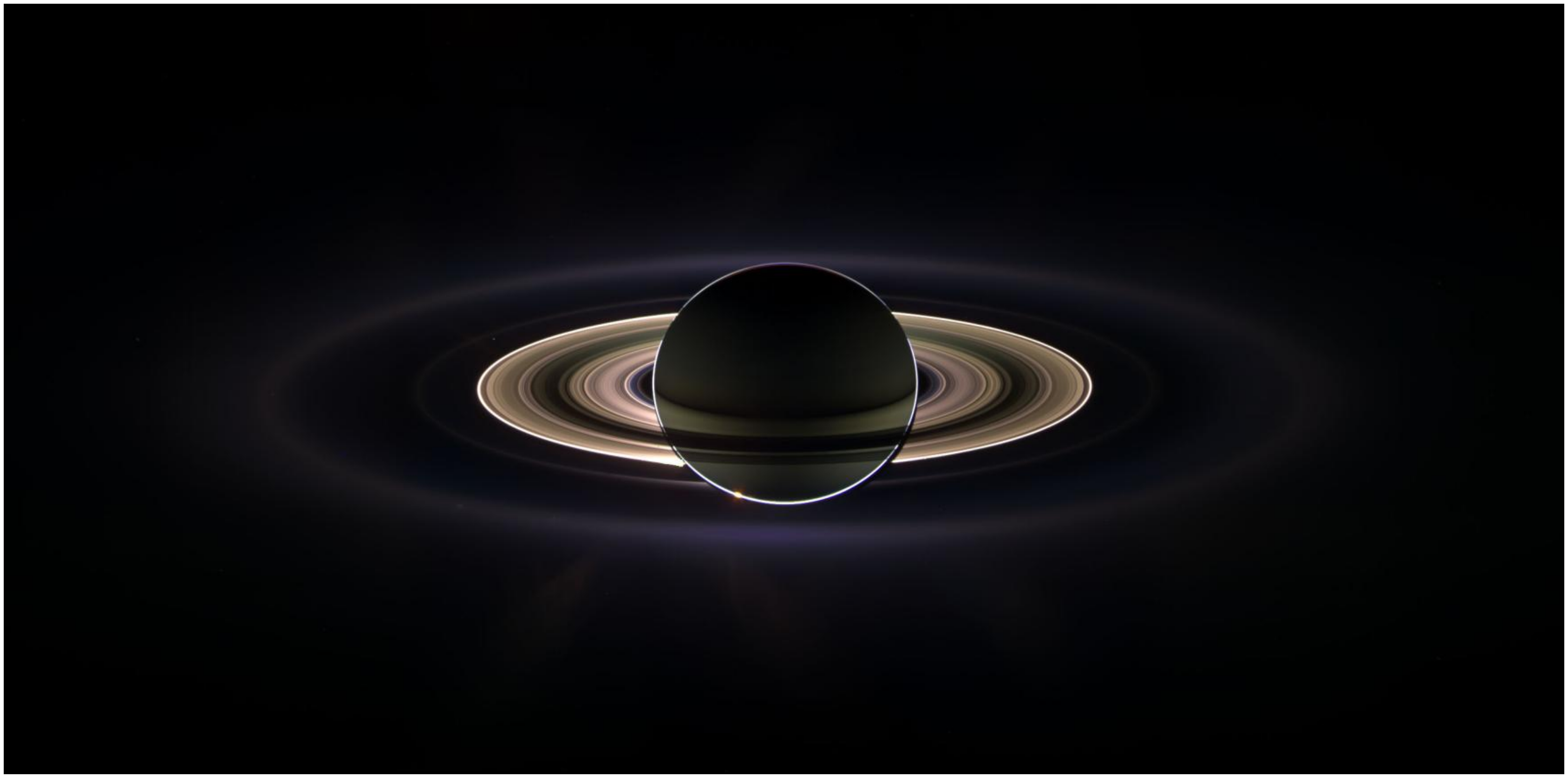
Syllabus

- Introduction and Background
- Measure of Information
  - Entropy
  - Mutual Information
- Distortionless Source Coding
- Channel Coding for Noisy Channels
- Rate Distortion Theory

Deep Space Communications



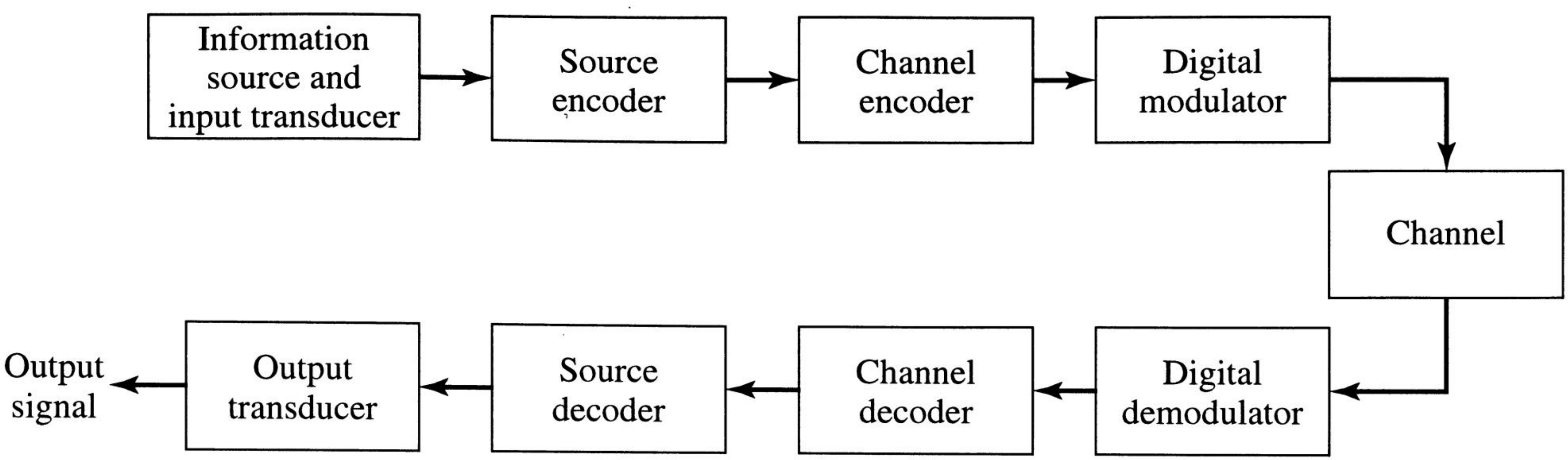


Figure 1.2 Basic elements of a digital communication system.

Transmitter

- Matches the message to the channel
- Operations: Amplification, Modulation
- Modulation encodes the message into the amplitude, phase or frequency of the carrier signal (PSK, FSK, QAM, OFDM,PAM,PPM)
- Advantages:
  - Reduce noise and interference
  - Multiplexing
  - Channel assignment
- Examples: TV station, 802.11, cellphone BS

Receiver

- Extracts message from the received signal
- Operations: Amplification, Demodulation, Filtering
- Goal: The receiver output is a scaled, delayed version of the message signal (ideal transmission)
- Examples: cellphone

Channel

- Physical medium that that the signal is transmitted through
- Examples: Air, wires, coaxial cables, fiber optic cables
- Every channel introduces some amount of distortion, noise and interference

Noise

- Internal and External Noise
- Internal Noise: Generated by components within a communication system (thermal noise)
- External Noise:
  - Atmospheric noise (electrical discharges)
  - Man-made noise (ignition noise)
  - Interference (multiple transmission paths - multipath)

Channel Capacity

- An important question for a communication channel is the maximum rate at which it can transfer information.
- There is a theoretical maximum rate below which information can be transmitted error free over the channel, called the channel capacity **C**.
- The channel capacity **C** is given by

$$C=W*\log(1+(P/N)) \text{ b/s}$$

where W is the bandwidth, P/N is the signal-to-noise ratio.

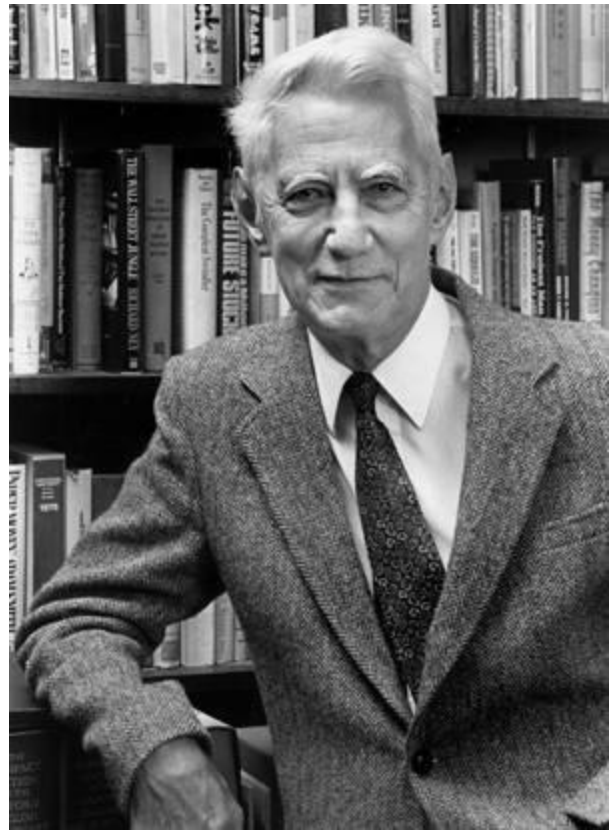
Fundamental Limitations

- There are two factors that determine the capacity of a channel:
  - Bandwidth (W)
  - Signal-to-Noise Ratio (SNR)
    - Ratio of the average signal power to the average noise power

Applications of Information Theory

- Communication systems
- Statistical physics
- Computer science - complexity
- Probability and statistics
- Economics and Investing; Gambling
- Mathematics - inequalities

Claude Shannon (1916-2001)



A Mathematical Theory of Communications, BSTJ July, 1948

``The fundamental problem of communication is that of reproducing at one point exactly or approximately a message selected at another point. … If the channel is noisy it is not in general possible to reconstruct the original message or the transmitted signal with certainty by any operation on the received signal.''

Types of Information

- Syntactic Information
  - Related to the symbols that make up the message
- Semantic Information
  - Related to the meaning of the message
- Pragmatic Information
  - Related to the usage and effect of the message

Example

1. John was brought to the airport by a taxi.
2. A taxi brought John to the airport.
3. There was an accident on Highway 1 between Duncan and Nanaimo.
4. There was an accident on Highway 1.

How is Information Measured?

- John hit a tree with a rock.
- John hit a tree with a rock from a distance of 300m.

Information Theory Topics

- Measurement of information
- Limits on the amount of information that can be transmitted
- Limits on the compression of information
- How to construct systems which approach these limits

History of Information Theory

1838	S.F.B. Morse
1924	H. Nyquist
1928	R.V.L. Hartley
1940	A. Turing
1946	R.E. Hamming
1948	C.E. Shannon
1951	S. Kullback and R.Leibler

TABLE 1.1 MORSE CODE

A	·—	N	—·
B	—···	O	———
C	—·—·	P	·——·
D	—··	Q	——·—
E	·	R	·—·
F	··—·	S	···
G	——·	T	—
H	····	U	··—
I	··	V	···—
J	·———	W	·——
K	—·—	X	—··—
L	·—··	Y	—·——
M	——	Z	——··

(a) Letters

1	·————
2	··———
3	···——
4	····—
5	·····
6	—····
7	——···
8	———··
9	————·
0	—————

(b) Numbers

Period (·)	·—·—·—	Wait sign (AS)	·—···
Comma (,)	——··——	Double dash (break)	—···—
Interrogation (?)	··——··	Error sign	········
Quotation Mark (")	·—··—·	Fraction bar (/)	—··—·
Colon (:)	———···	End of message (AR)	·—·—·
Semicolon (;)	—·—·—·	End of transmission (SK)	···—·—
Parenthesis ( )	—·——·—		

(c) Punctuation and Special Characters

Information Measures

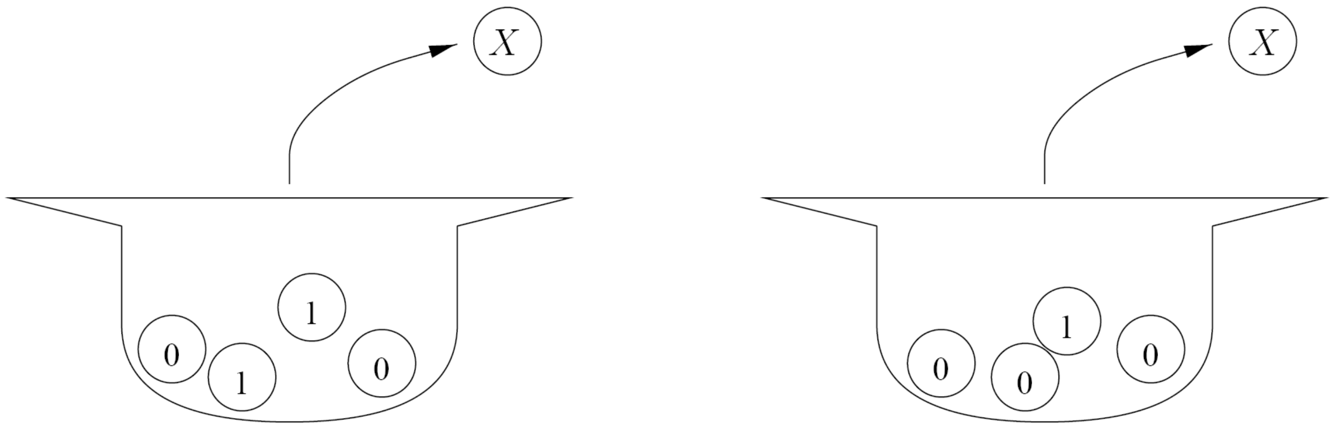
$b = 2$ $\quad$ *Sh* (*shannons*) or *bit* (*binary digit*)

$b = e$ $\quad$ *logons* or *nats* (*natural units*)

$b = 10$ $\quad$ *hartleys* (in honor of R.V.L. Hartley, pioneer in communication and information theory)

$$
\begin{aligned}
1\ logon &= \frac{1}{\ln 2} &&= \log_2 e &&= 1.443\ Sh \\
1\ hartley &= \frac{1}{\log_{10} 2} &&= \log_2 10 &&= 3.322\ Sh \\
1\ hartley &= \frac{1}{\log_{10} e} &&= \ln 10 &&= 2.303\ logons
\end{aligned}
$$

USA Standard Code for Information Exchange (USASCII)								
	b_7 b_6 b_5	b_7 b_6 b_5	b_7 b_6 b_5	b_7 b_6 b_5	b_7 b_6 b_5	b_7 b_6 b_5	b_7 b_6 b_5	b_7 b_6 b_5
b_4 b_3 b_2 b_1	0 0 0	0 0 1	0 1 0	0 1 1	1 0 0	1 0 1	1 1 0	1 1 1
0 0 0 0	NUL	DLE	SP	0	@	P	`	p
0 0 0 1	SOH	DC1	!	1	A	Q	a	q
0 0 1 0	STX	DC2	″	2	B	R	b	r
0 0 1 1	ETX	DC3	#	3	C	S	c	s
0 1 0 0	EOT	DC4	$	4	D	T	d	t
0 1 0 1	ENQ	NAK	%	5	E	U	e	u
0 1 1 0	ACK	SYN	&	6	F	V	f	v
0 1 1 1	BEL	ETB	´	7	G	W	g	w
1 0 0 0	BS	CAN	(	8	H	X	h	x
1 0 0 1	HT	EM	)	9	I	Y	i	y
1 0 1 0	LF	SUB	*	:	J	Z	j	z
1 0 1 1	VT	ESC	+	;	K	[	k	{
1 1 0 0	FF	FS	,	$<$	L	\	l	$\vert$
1 1 0 1	CR	GS	-	=	M	]	m	}
1 1 1 0	SO	RS	.	$>$	N	$\wedge$	n	$\sim$
1 1 1 1	SI	US	/	?	O	_	o	DEL



(a) (b)

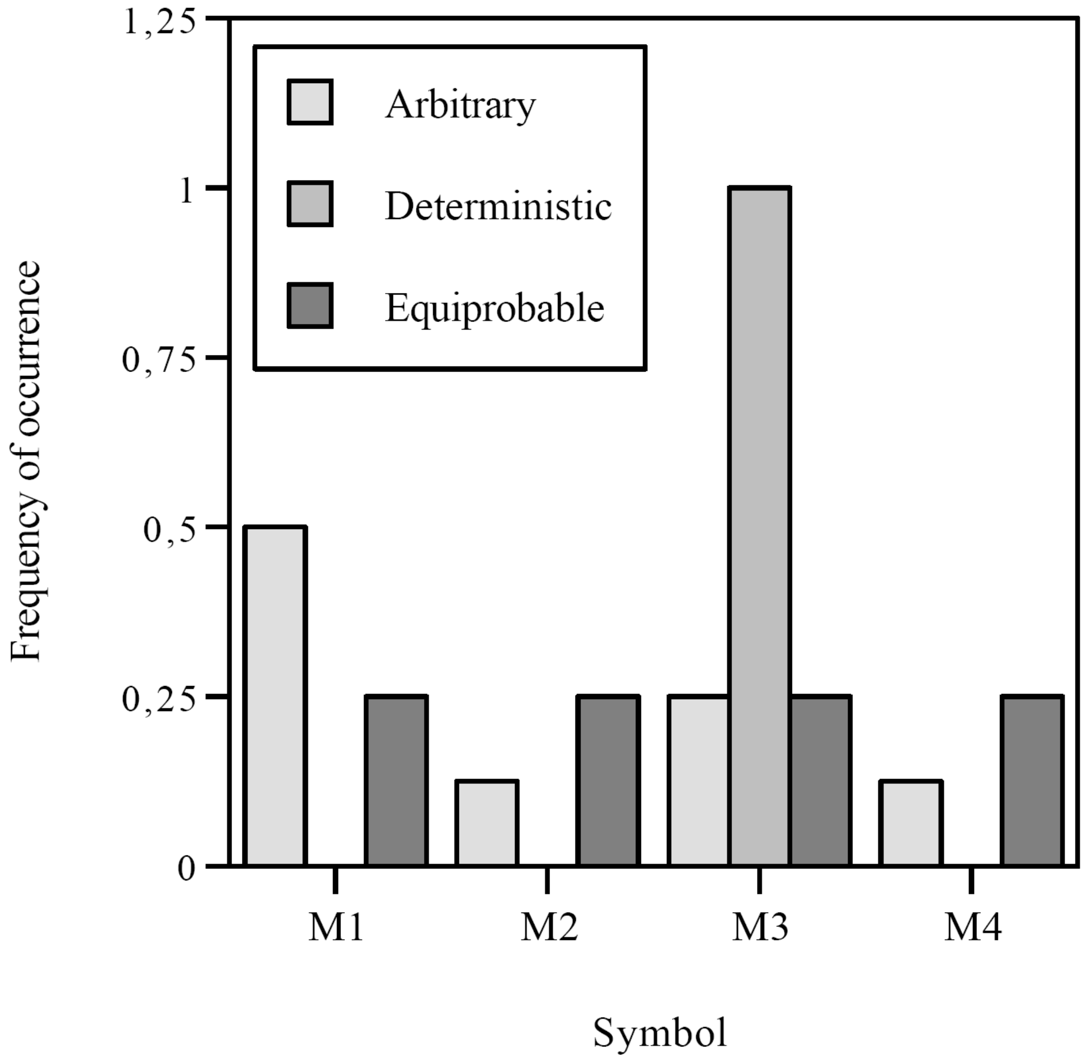
Self Information

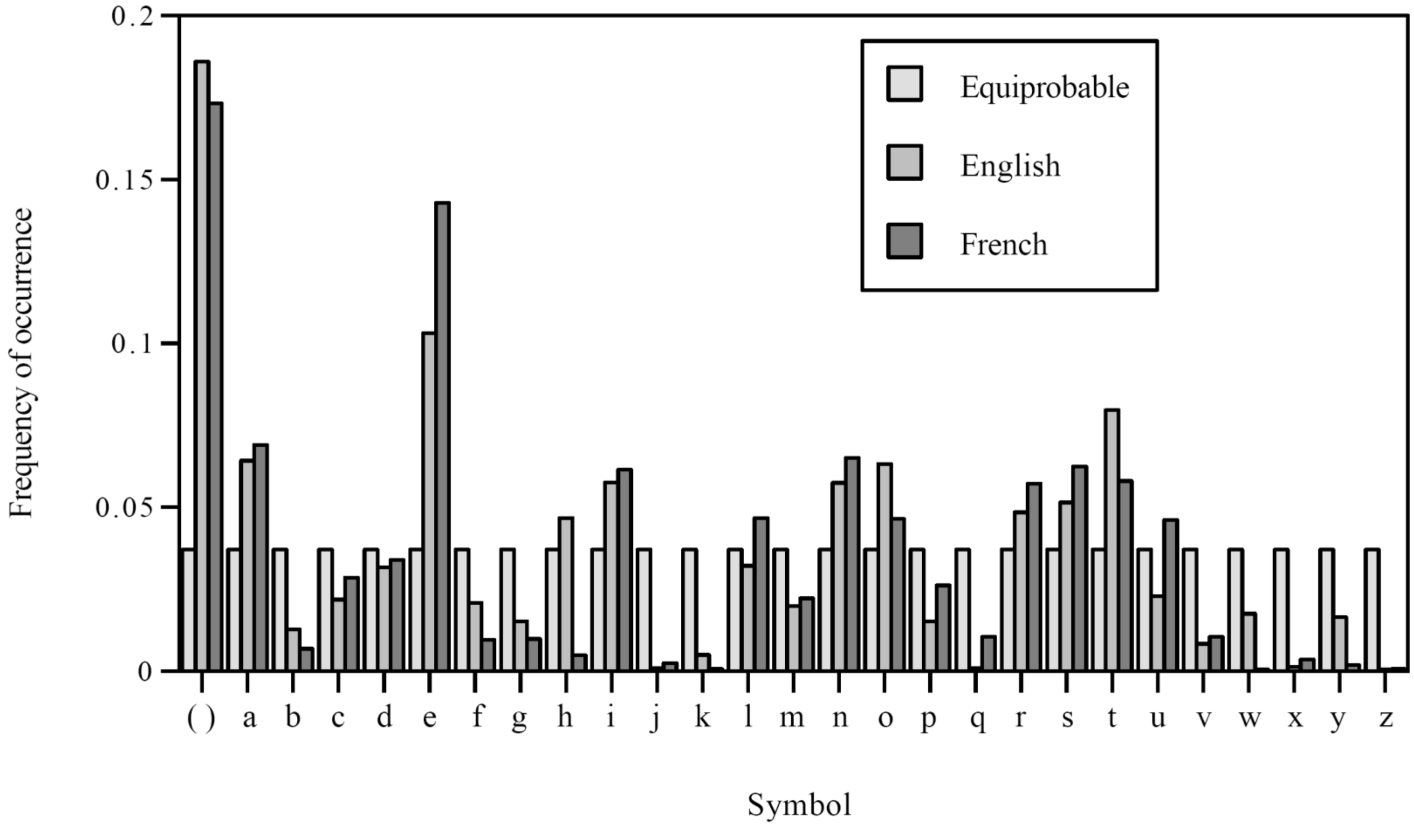
$$\boxed{I_X(x_i) = -\ \log_b p(x_i)}$$

Entropy

$$\boxed{H(X) = -\sum_{i=1}^{N} p(x_i)\ \log_b p(x_i)}$$

Quaternary source distribution




0.2
0.15
0.1
0.05
0
Frequency of occurrence
Equiprobable
English
French
( )
a
b
c
d
e
f
g
h
i
j
k
l
m
n
o
p
q
r
s
t
u
v
w
x
y
z
Symbol

Letter	Equiprobable		English language		French language	
x_i	$p(x_i) = \frac{1}{27}$	$-\log_2 \frac{1}{27}$	$p(x_i)$	$-\log_2 p(x_i)$	$p(x_i)$	$-\log_2 p(x_i)$
□	0.0370	4.7549	0.1859	2.4274	0.1732	2.5295
a	0.0370	4.7549	0.0642	3.9613	0.0690	3.8573
b	0.0370	4.7549	0.0127	6.2990	0.0068	7.2002
c	0.0370	4.7549	0.0218	5.5195	0.0285	5.1329
d	0.0370	4.7549	0.0317	4.9794	0.0339	4.8826
e	0.0370	4.7549	0.1031	3.2779	0.1428	2.8079
f	0.0370	4.7549	0.0208	5.5873	0.0095	6.7179
g	0.0370	4.7549	0.0152	6.0398	0.0098	6.6730
h	0.0370	4.7549	0.0467	4.4204	0.0048	7.7027
i	0.0370	4.7549	0.0575	4.1203	0.0614	4.0256
j	0.0370	4.7549	0.0008	10.2877	0.0024	8.7027
k	0.0370	4.7549	0.0049	7.6730	0.0006	10.7027
l	0.0370	4.7549	0.0321	4.9613	0.0467	4.4204
m	0.0370	4.7549	0.0198	5.6584	0.0222	5.4933
n	0.0370	4.7549	0.0574	4.1228	0.0650	3.9434
o	0.0370	4.7549	0.0632	3.9839	0.0464	4.4297
p	0.0370	4.7549	0.0152	6.0398	0.0261	5.2598
q	0.0370	4.7549	0.0008	10.2877	0.0104	6.5873
r	0.0370	4.7549	0.0484	4.3688	0.0572	4.1278
s	0.0370	4.7549	0.0514	4.2821	0.0624	4.0023
t	0.0370	4.7549	0.0796	3.6511	0.0580	4.1078
u	0.0370	4.7549	0.0228	5.4548	0.0461	4.4391
v	0.0370	4.7549	0.0083	6.9127	0.0104	6.5873
w	0.0370	4.7549	0.0175	5.8365	0.0005	10.9658
x	0.0370	4.7549	0.0013	9.5873	0.0035	8.1584
y	0.0370	4.7549	0.0164	5.9302	0.0018	9.1178
z	0.0370	4.7549	0.0005	10.9658	0.0006	10.7027

$$H(X) = -\left[p(x_1)\log_b p(x_1) + (1 - p(x_1))\log_b (1 - p(x_1))\right]$$