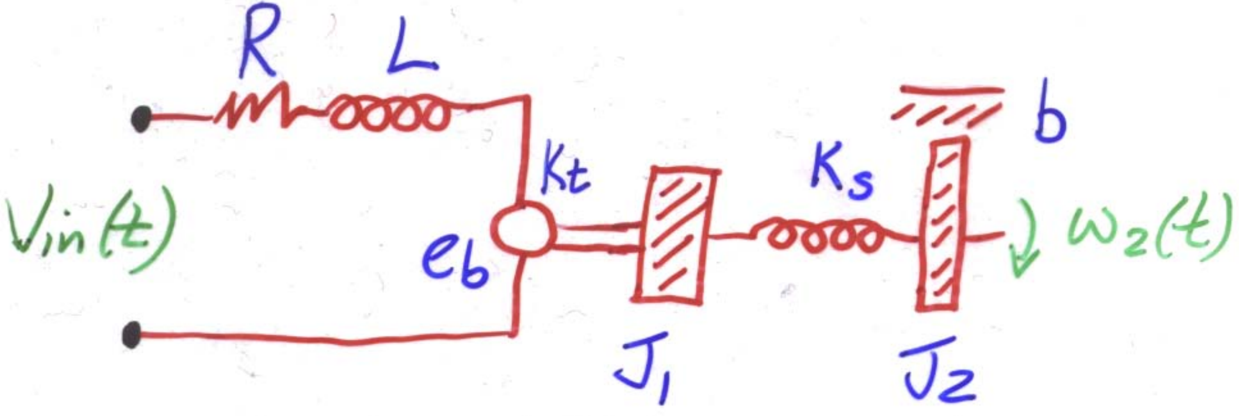
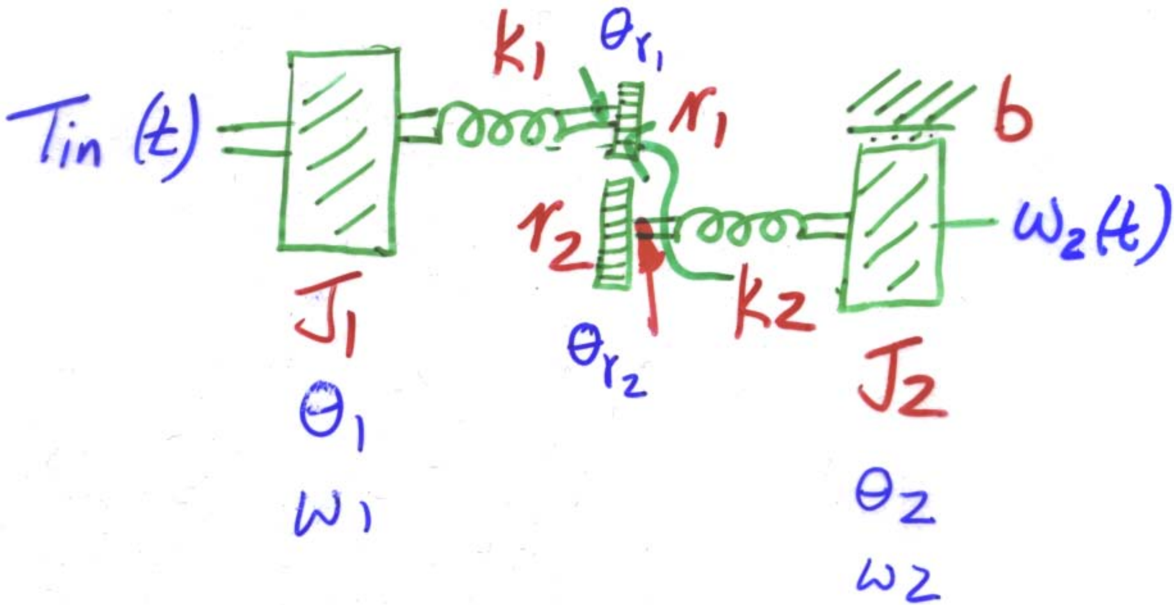
Laplace Transform Pairs

$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-at} x(t)$	$X(s+a)$
$x(t-c)$	$e^{-cs} X(s)$
$t^n x(t)$	$(-1)^n \frac{d^n}{ds^n} X(s)$
$\int_0^t x(\tau)\, d\tau$	$\frac{1}{s} X(s)$
$\dot{x}(t)$	$s X(s) - x(0)$
$x(t) * v(t)$	$X(s) \cdot V(s)$



Example 2

Example 3



$T_{in}(t)$
k_1
θ_{r_1}
r_1
b
r_2
$\omega_2(t)$
k_2
J_1
θ_1
ω_1
θ_{r_2}
J_2
θ_2
ω_2

$$J_1 \dot{\omega}_1 = T_{in} - k_1 \Delta\theta_1$$

$$\dot{\omega}_1 = \frac{T_{in}}{J_1} - \frac{k_1}{J_1} \Delta\theta_1 \qquad (1)$$

$$k_1 \Delta\theta_1 \frac{r_2}{r_1} = k_2 \Delta\theta_2$$

$$\Delta\theta_2 = \frac{k_1}{k_2} \frac{r_2}{r_1} \Delta\theta_1$$

$$\Delta\dot{\theta}_2 = \frac{k_1}{k_2} \frac{r_2}{r_1} \Delta\dot{\theta}_1$$

$$\Delta\dot{\theta}_1 = \omega_1 - \omega r_1$$

$$\omega r_1 = \frac{r_2}{r_1} \omega r_2 = \frac{r_2}{r_1} \left(\omega_2 + \Delta\dot{\theta}_2 \right)$$

$$= \frac{r_2}{r_1} \left(\omega_2 + \frac{k_1}{k_2} \frac{r_2}{r_1} \Delta\dot{\theta}_1 \right)$$

$$\Delta\dot{\theta}_1 = \omega_1 - \frac{r_2}{r_1} \omega_2 - \frac{k_1}{k_2} \left(\frac{r_2}{r_1} \right)^2 \Delta\dot{\theta}_1$$

$$\Delta\dot{\theta}_1 = \frac{\omega_1 - \frac{r_2}{r_1}\omega_2}{1 + \frac{k_1}{k_2}\left(\frac{r_2}{r_1}\right)^2} \quad (2)$$

$$J_2 \dot{\omega}_2 = k_2 \Delta\theta_2 - b\,\omega_2$$

$$\dot{\omega}_2 = \frac{k_2}{J_2}\Delta\theta_2 - \frac{b}{J_2}\omega_2$$

$$\dot{\omega}_2 = \frac{k_1}{J_2}\frac{r_2}{r_1}\Delta\theta_1 - \frac{b}{J_2}\omega_2 \quad (3)$$

$$A = \begin{bmatrix} 0 & 0 & -\dfrac{k_1}{J_1} \\ 0 & -\dfrac{b}{J_2} & \dfrac{k_1}{J_2}\dfrac{r_2}{r_1} \\ \dfrac{1}{1+\frac{k_1}{k_2}\left(\frac{r_2}{r_1}\right)^2} & \dfrac{-r_2/r_1}{1+\frac{k_1}{k_2}\left(\frac{r_2}{r_1}\right)^2} & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} \dfrac{1}{J_1} \\ 0 \\ 0 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 0 \end{bmatrix}$$