LINEAR TIME INVARIANT SYSTEMS

- output is a LINEAR function of the input

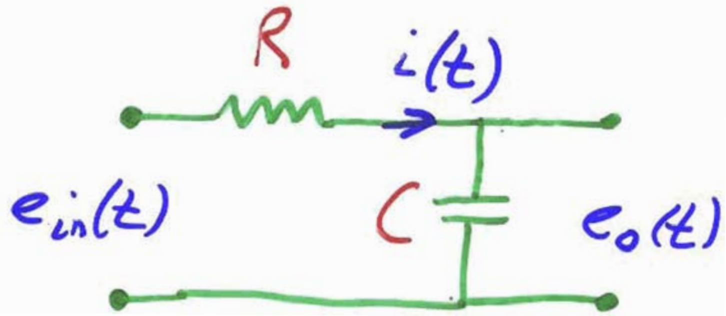
- can be represented by LINEAR differential equations
  - no product terms
  - no power terms
  - no time-varying parameters

$$A\ddot{y}(t) + B\dot{y}(t) + Cy(t) = x(t)$$

- obey the principle of superposition

Analagous Systems

Electrical



$$e_{in}(t) = R\,i(t) + \frac{1}{C}\int i(t)\,dt$$

$$i(t) = \dot{q}(t)$$

$$e_o(t) = \frac{1}{C}\,q(t)$$

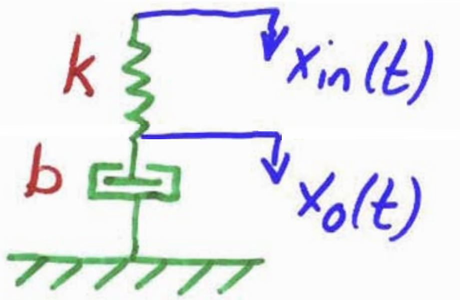
$$e_{in}(t) = R\,\dot{q}(t) + \frac{1}{C}\,q(t)$$

$$i(t) = C\,\dot{e}_o(t)$$

$$e_{in}(t) = RC\,\dot{e}_o(t) + e_o(t)$$

$$\dot{e}_o(t) + \frac{1}{RC}\,e_o(t) = \frac{1}{RC}\,e_{in}(t)$$

Mechanical



$$k(x_{in}(t) - x_o(t)) = b\,\dot{x}_o(t)$$

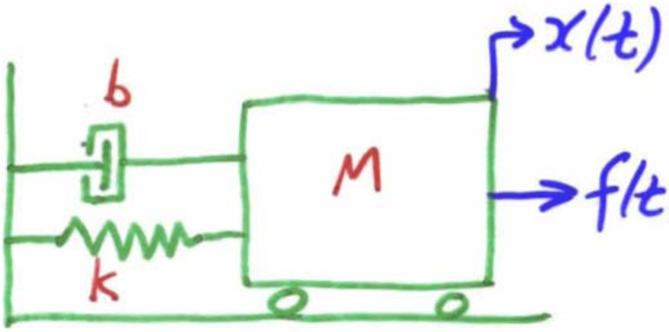
$$b\,\dot{x}_o(t) + k\,x_o(t) = k\,x_{in}(t)$$

$$\dot{x}_o(t) + \frac{k}{b}\,x_o(t) = \frac{k}{b}\,x_{in}(t)$$

system dynamics are identical if

$$R = b \qquad \frac{1}{C} = k$$

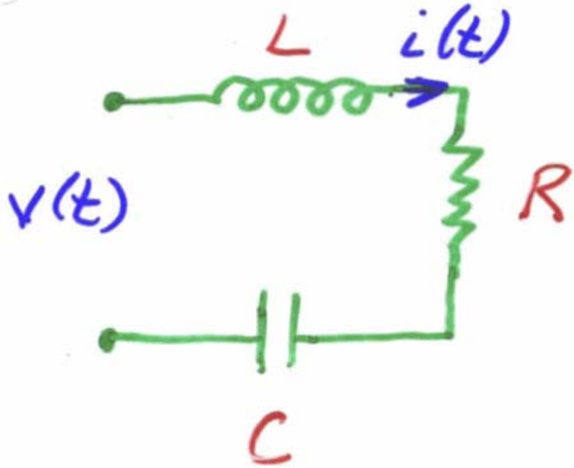
Second Order Systems



$$F = Ma$$

$$f(t) - b\dot{x}(t) - kx(t) = M\ddot{x}(t)$$

$$M\ddot{x}(t) + b\dot{x}(t) + kx(t) = f(t)$$



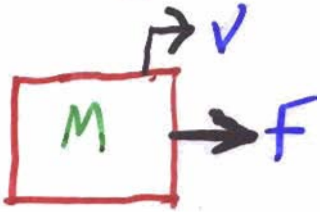
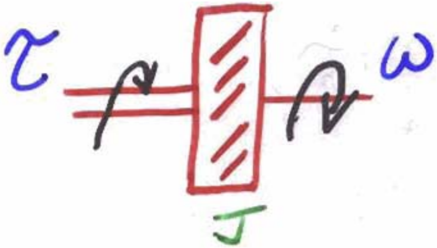
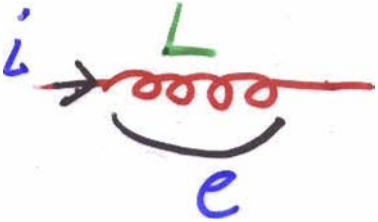
$$L\dot{i}(t) + Ri(t) + \frac{1}{C}\int i(t)\,dt = v(t)$$

$$L\ddot{\underline{q}}(t) + R\dot{\underline{q}}(t) + \frac{1}{C}\underline{q}(t) = v(t)$$

TABLE 4–1 FORCE–VOLTAGE ANALOGY

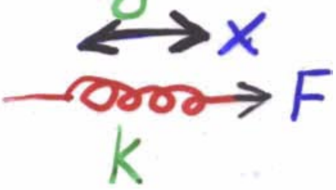
Mechanical systems	Electrical systems
Force p (torque T)	Voltage e
Mass m (moment of inertia J)	Inductance L
Viscous-friction coefficient b	Resistance R
Spring constant k	Reciprocal of capacitance, $1/C$
Displacement x (angular displacement θ)	Charge q
Velocity $\dot{x}$ (angular velocity $\dot{\theta}$)	Current i

Inertial Elements

Diagram	state variable	equation
	v, velocity	$M\dot{v} = F$, $\dot{v} = \frac{1}{M} F$
	ω, angular velocity	$J\dot{\omega} = \tau$
	i, current	$L\dot{i} = e$

All store Energy

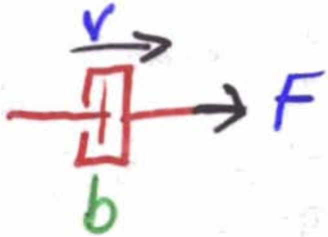
Capacitive Elements

Diagram	state variable	equation
	x, compression or extension (Δx)	$F = kx$
	θ twist ($\Delta\theta$)	$T = k\theta$
	q, charge on C	$e = \frac{1}{C} q$ $e = \frac{1}{C}\int i(\lambda)\, d\lambda$

All store Energy

Dissipative Elements

Diagram | equation

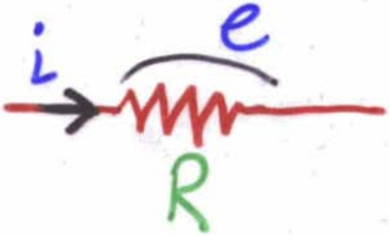


$$F = bv = b\dot{x}$$

$$T = b\dot{\theta} = b\omega$$

$$T = b\omega$$

viscous (bearing) friction



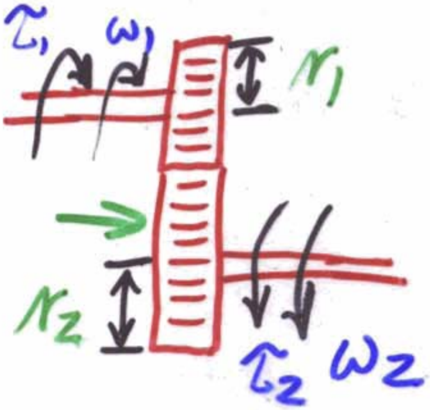
$$e = Ri$$

$$V = Ri$$

All dissipate Energy (heat)

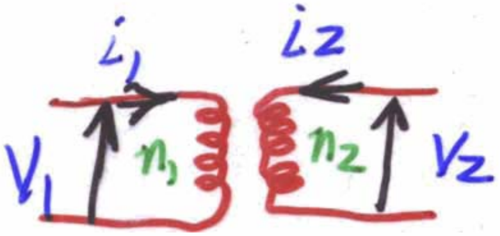
Transformer Elements

Diagram | equation



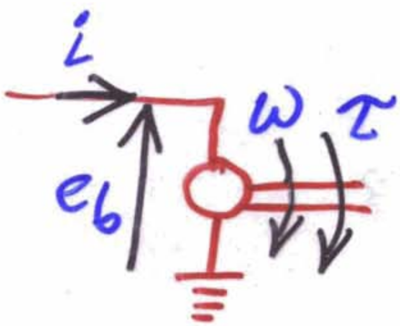
$$\tau_2 = \frac{r_2}{r_1}\tau_1 \qquad \omega_2 = \frac{r_1}{r_2}\omega_1$$

$$\tau_1\omega_1 = \tau_2\omega_2$$



$$V_2 = \frac{n_2}{n_1}V_1 \qquad i_2 = \frac{n_1}{n_2}i_1$$

$$V_1 i_1 = V_2 i_2$$

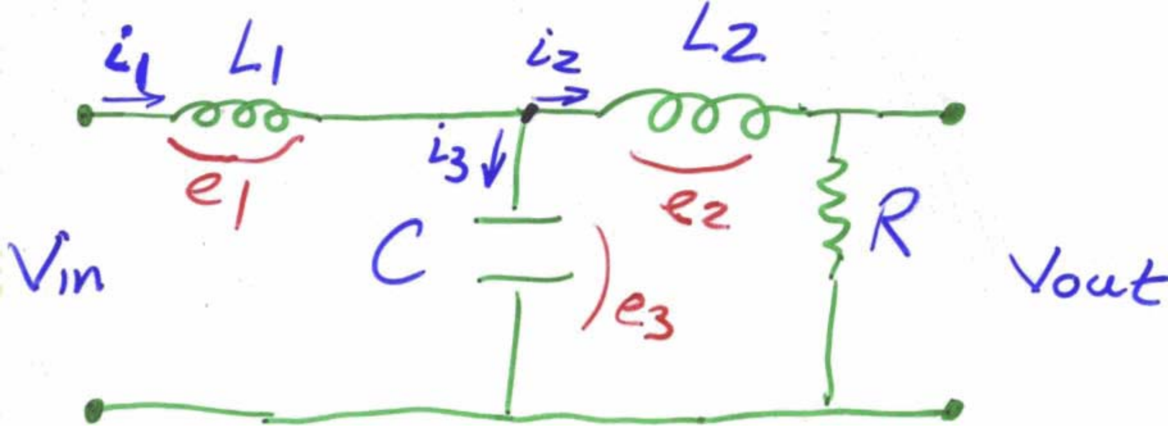


$$\tau = k_t i \qquad e_b = k_t\omega$$

$$e_b i = \tau\omega$$

These devices neither store nor dissipate energy

Example 1



KVL loop 1

$$V_{in} = \dot{i}_1 L_1 + q/C \Rightarrow \dot{i}_1 = \frac{-q}{L_1 C} + \frac{V_{in}}{L_1}$$

node currents

$$i_1 = i_2 + i_3 \Rightarrow i_3 = i_1 - i_2 \Rightarrow \dot{q} = i_1 - i_2$$

KVL loop 2

$$L_2 \dot{i}_2 + i_2 R - q/C = 0 \Rightarrow \dot{i}_2 = \frac{q}{L_2 C} - \frac{i_2 R}{L_2}$$

In matrix form

$$\underbrace{\begin{bmatrix} \dot{i}_1 \\ \dot{q} \\ \dot{i}_2 \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} 0 & -1/L_1C & 0 \\ 1 & 0 & -1 \\ 0 & 1/L_2C & -R/L_2 \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} i_1 \\ q \\ i_2 \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 1/L_1 \\ 0 \\ 0 \end{bmatrix}}_{B} \underbrace{V_{in}}_{u}$$

$$\underbrace{V_{out}}_{y} = \underbrace{\begin{bmatrix} 0 & 0 & R \end{bmatrix}}_{C} \cdot \underbrace{\begin{bmatrix} i_1 \\ q \\ i_2 \end{bmatrix}}_{x}$$

Alternate form

$$\begin{bmatrix} \dot{V}_o \\ \ddot{V}_o \\ \dddot{V}_o \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{-R}{L_1 L_2 C} & \frac{-(L_1+L_2)}{L_1 L_2 C} & -R/L_2 \end{bmatrix} \cdot \begin{bmatrix} V_o \\ \dot{V}_o \\ \ddot{V}_o \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{R}{L_1 L_2 C} \end{bmatrix}$$

$$V_o = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} V_o \\ \dot{V}_o \\ \ddot{V}_o \end{bmatrix}$$