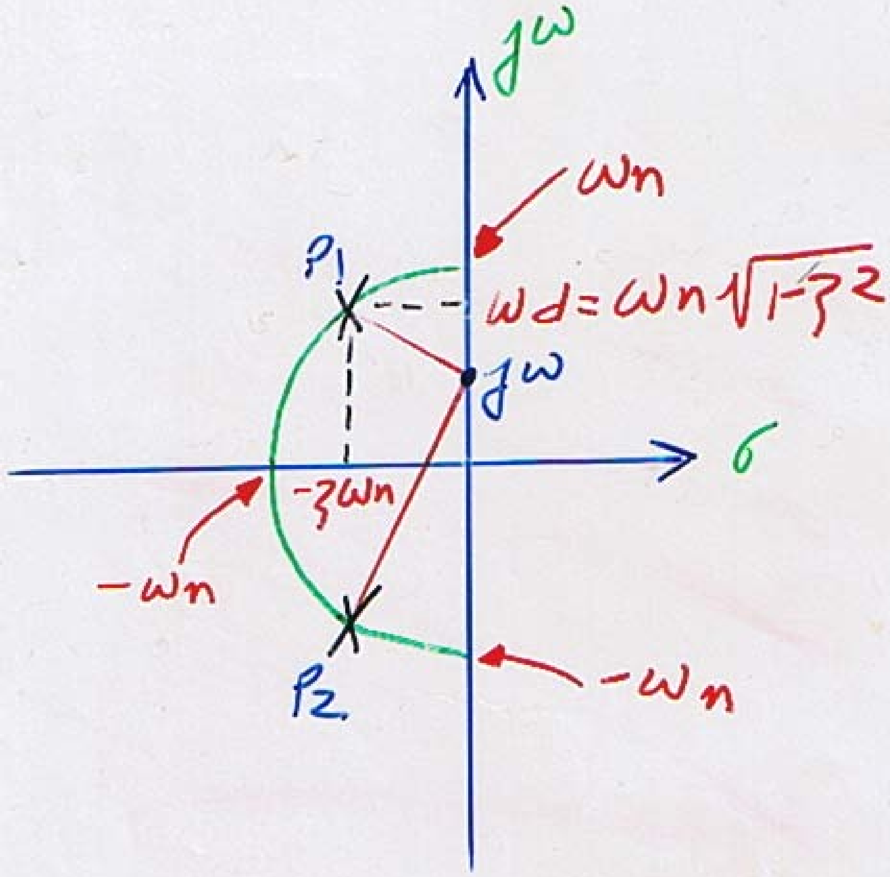
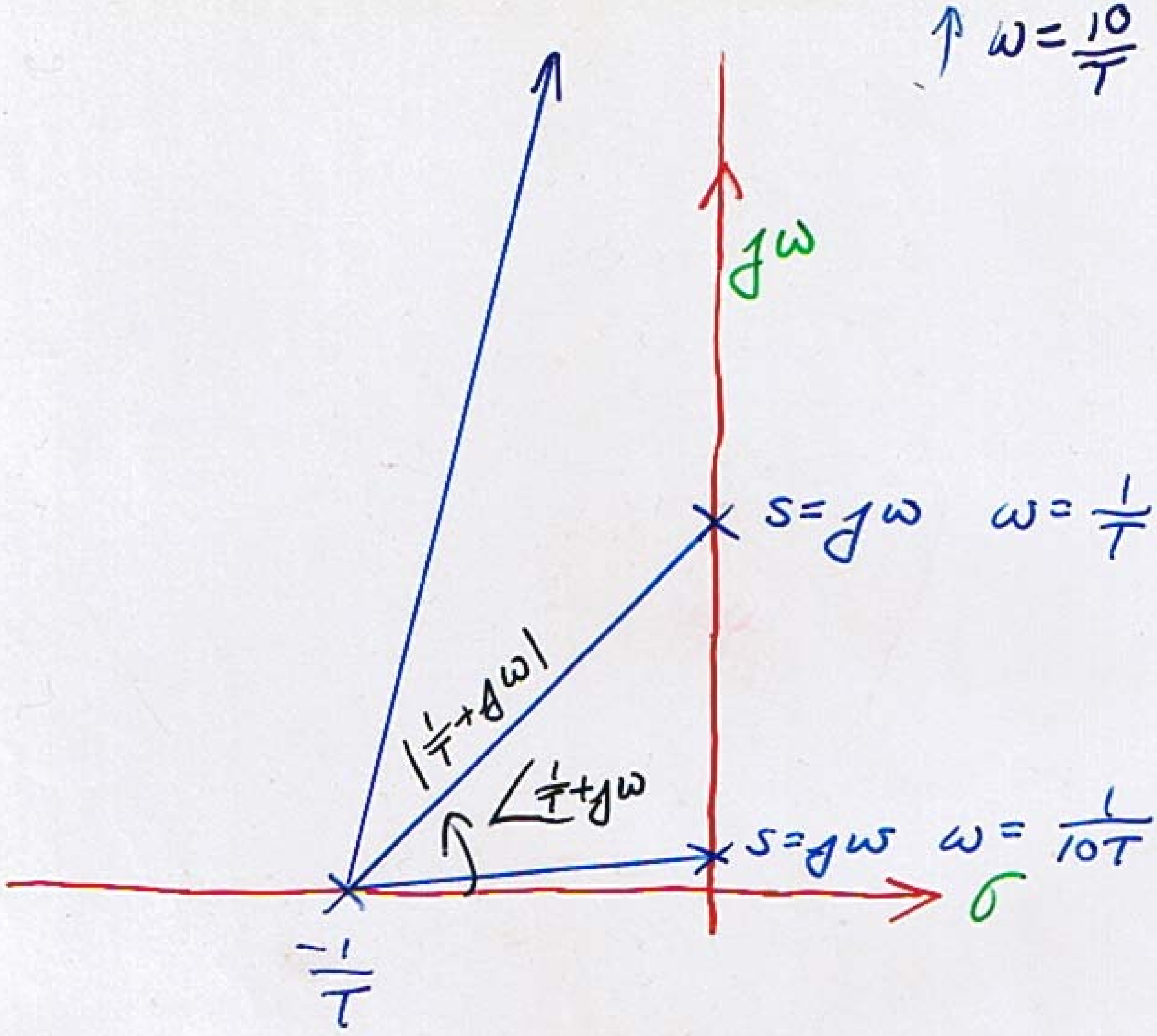


$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$G(j\omega) = \frac{\omega_n^2}{(j\omega - p_1)(j\omega - p_2)}$$

$$|G(j\omega)| = \frac{\omega_n^2}{|j\omega - p_1|\,|j\omega - p_2|}$$



vector representation of

$$\frac{1}{T} + j\omega$$

$$G(s) = \frac{1}{1+sT} \quad \text{or} \quad 1+sT$$

$$G(j\omega) = \frac{1}{1+j\omega T} \quad \text{or} \quad 1+j\omega T$$

$$G(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$G(j\omega) = \frac{\omega_n^2}{(j\omega)^2+j2\zeta\omega_n\omega+\omega_n^2}$$

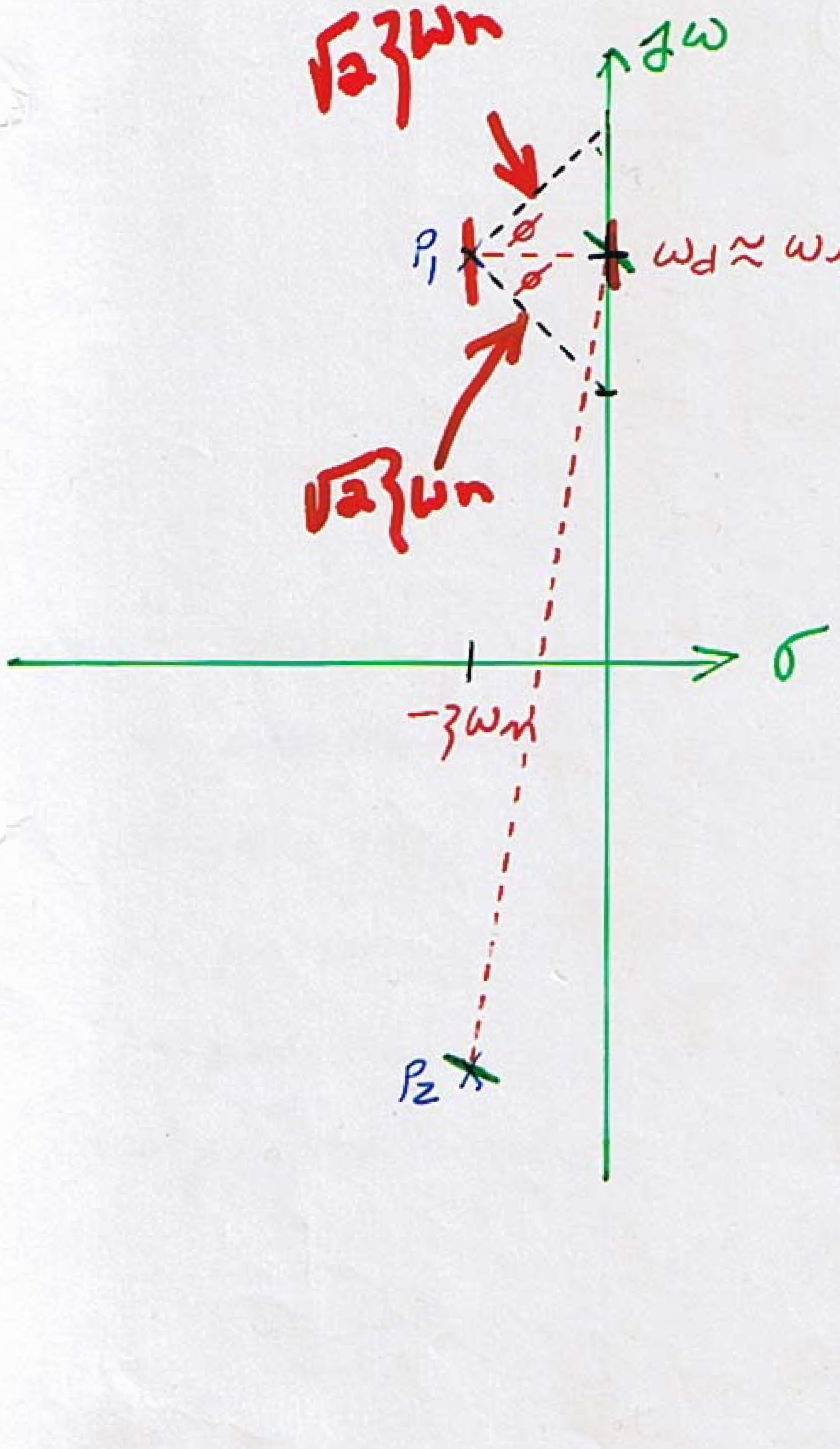
$$= \frac{\omega_n^2}{(\omega_n^2-\omega^2)+j2\zeta\omega_n\omega}$$

let $a = \omega_n^2-\omega^2$ $\quad b = 2\zeta\omega_n\omega$

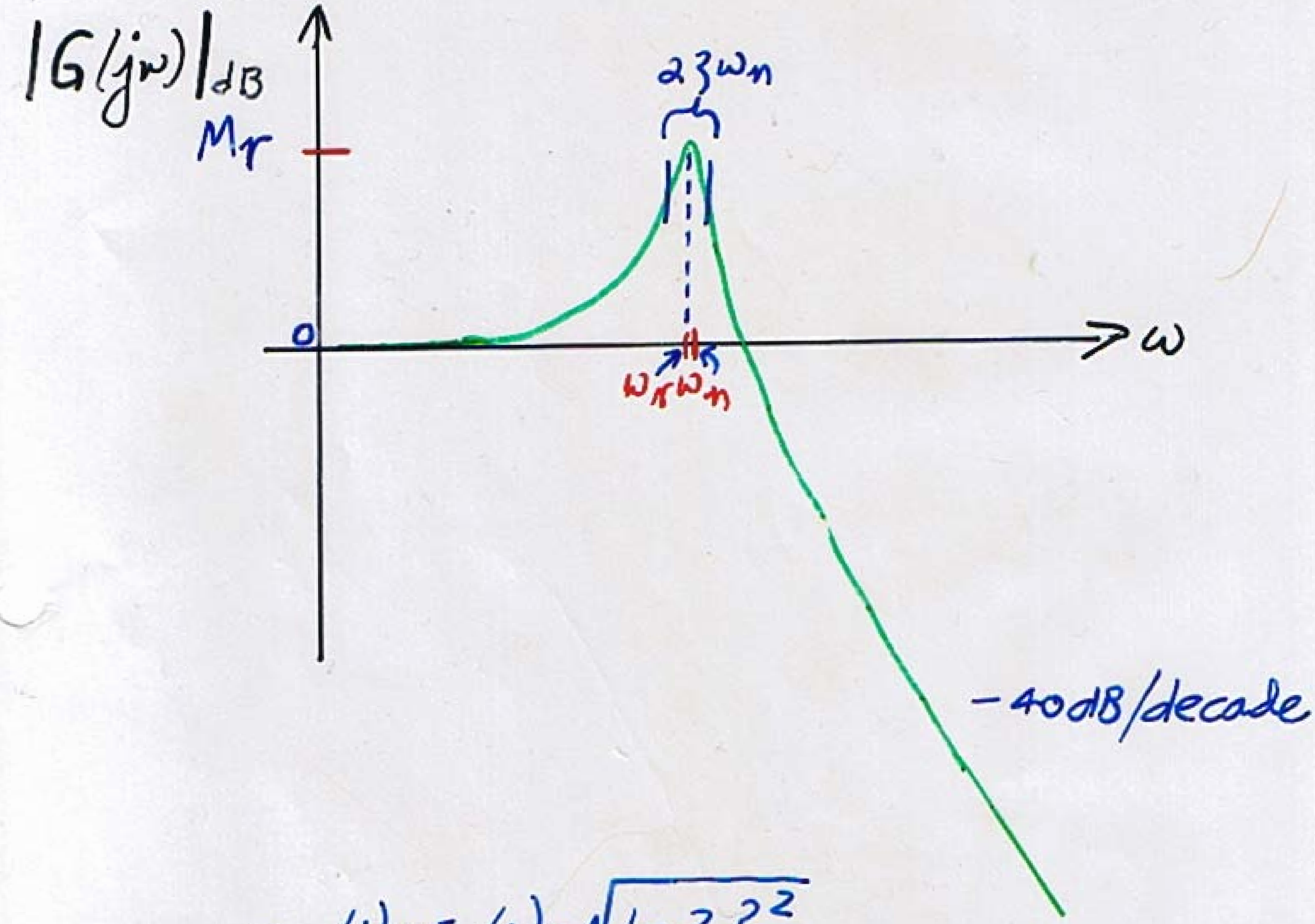
$$|G(j\omega)| = \frac{\omega_n^2}{\sqrt{a^2+b^2}} = \frac{\omega_n^2}{\sqrt{(\omega_n^2-\omega^2)^2+4\zeta^2\omega_n^2\omega^2}}$$

$$\angle G(j\omega) = -\tan^{-1}\left(\frac{b}{a}\right)$$

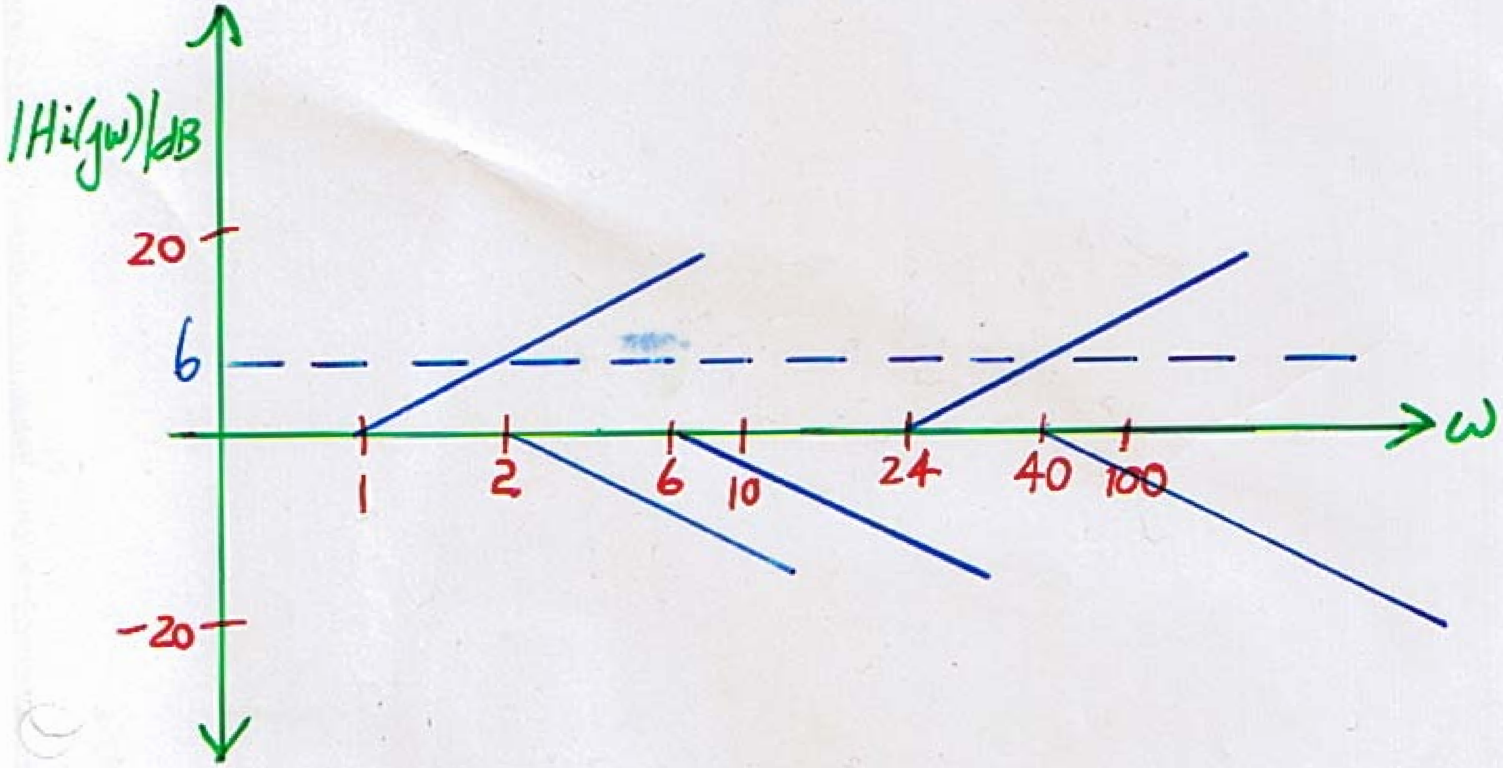
$$= -\tan^{-1}\frac{2\zeta\omega_n\omega}{\omega_n^2-\omega^2}$$


$j\omega$
$\sqrt{2}\zeta\omega_n$
ϕ
ϕ
P_1
$\omega_d \approx \omega_n$
$\sqrt{2}\zeta\omega_n$
σ
$-\zeta\omega_n$
P_2

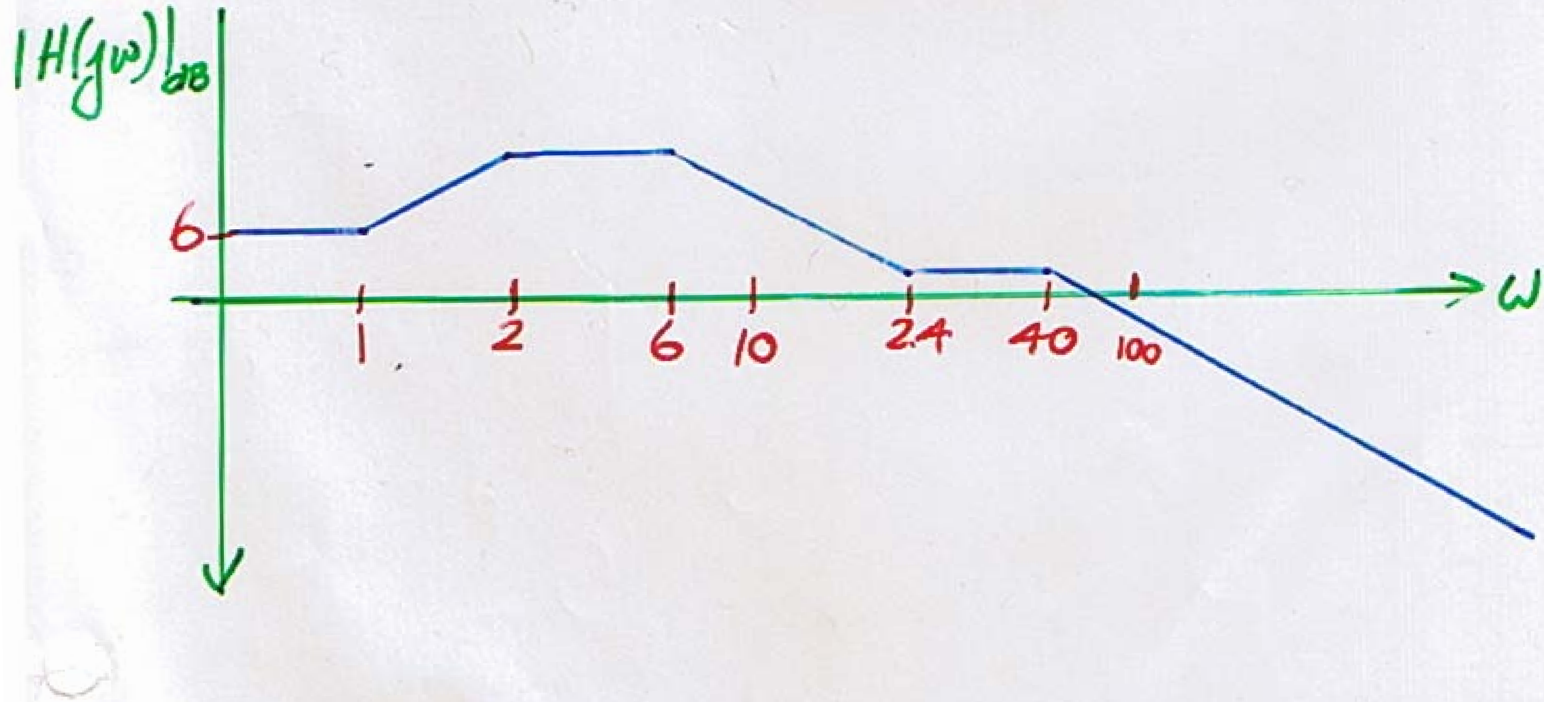
Second Order Systems



- $\omega_r = \omega_n\sqrt{1-2\zeta^2}$
- if $\zeta \ll 1$ $M_r \simeq \frac{1}{2\zeta} = Q$
- otherwise $M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$
- 3dB bandwidth $\cong 2\zeta\omega_n$



|Hi(jω)|dB
20
6
-20
ω
1
2
6
10
24
40
100



|H(jω)|dB
6
ω
1
2
6
10
24
40
100