ELEC 360 Spring 2006

Midterm Test Solutions

1. using $\Sigma F = ma$ we have

$$m\ddot{y}(t) = u(t) - ky(t) - b\dot{y}(t)$$

$$m\ddot{y}(t) + b\dot{y}(t) + ky(t) = u(t)$$

let $x_1 = y \quad x_2 = \dot{y}$

then $\dot{x}_1 = x_2 \quad \dot{x}_2 = \ddot{y} = \frac{u(t)}{m} - \frac{k}{m}x_1 - \frac{b}{m}x_2$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u$$

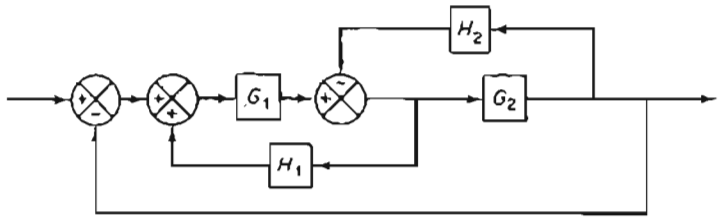
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0]\,u$$

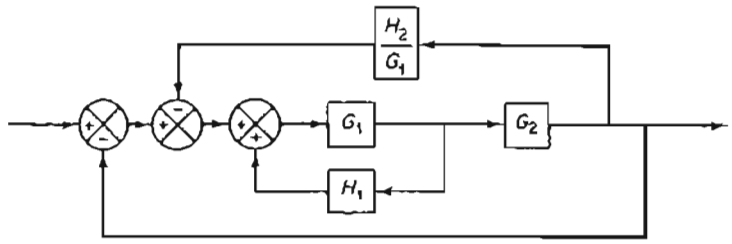
3. a) $Y(s) = U(s)\,G(s) = \frac{1}{s}\left(\frac{1}{1+s}\right) = \frac{1}{s} - \frac{1}{s+1}$

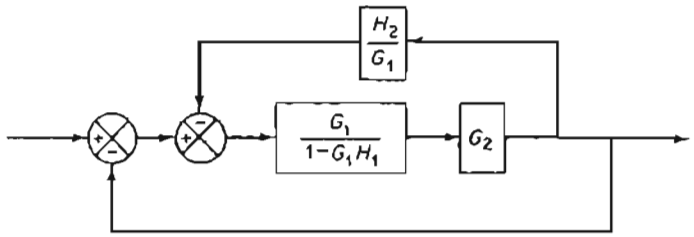
$$y(t) = \mathcal{L}^{-1}[Y(s)] = u(t) - e^{-t} \quad t \geq 0$$

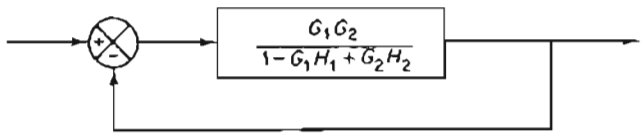
3 b) $$y(\infty) = \lim_{s \to 0} sY(s) = \lim_{s \to 0} \frac{1}{1+s} = 1$$

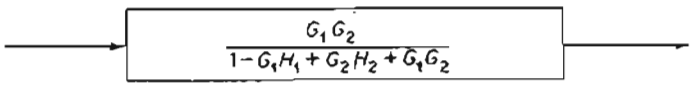
2.











Alternatively Mason's Gain Formula can be used

$P_1 = G_1 G_2 \qquad L_1 = G_1 H_1 \quad L_2 = -G_1 G_2 \quad L_3 = -G_2 H_2$

$\Delta_1 = 1 \qquad \text{o.o} \qquad P = \dfrac{G_1 G_2}{1 - G_1 H_1 + G_2 H_2 + G_1 G_2}$