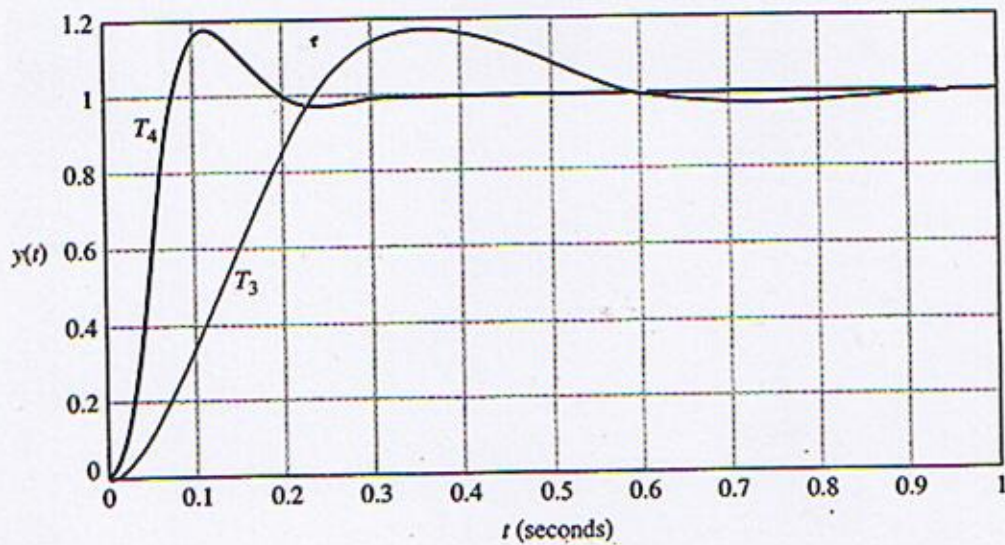


(a)



CLOSED LOOP FREQUENCY RESPONSE

$$G(j\omega) = \vec{OA}$$

$$1 + G(j\omega) = \vec{PA}$$

$$\frac{C(j\omega)}{R(j\omega)} = \frac{G(j\omega)}{1 + G(j\omega)} = \frac{\vec{OA}}{\vec{PA}}$$

$$= \frac{|\vec{OA}|}{|\vec{PA}|} \angle \phi - \theta$$

$$- m - j\alpha$$

CONSTANT MAGNITUDE LOCI (M-CIRCLES)

$$G(j\omega) = X + jY$$

$$M = \frac{|X + jY|}{|1 + X + jY|}$$

$$M^2 = \frac{X^2 + Y^2}{1 + X^2 + Y^2} = \text{CONSTANT}$$

$$X^2(1 - M^2) - 2M^2X - M^2 + (1 - M^2)Y^2 = 0$$

$$M = 1 \quad -2X - 1 = 0 \Rightarrow X = -\frac{1}{2}$$

$$\left(\frac{x + M^2}{M^2 + 1} \right)^2 + Y^2 = \frac{M^2}{(M^2 - 1)^2}$$

circle with center

$$x = \frac{-M^2}{M^2 - 1}, \quad Y = 0$$

radius $\left| \frac{M}{M^2 - 1} \right|$

$$M > 1 \quad M \rightarrow \infty \quad r \rightarrow 0$$

$$\text{center} \rightarrow (-1, 0)$$

$$0 < M < 1 \quad M \rightarrow 0 \quad r \rightarrow 0$$

$$\text{center} \rightarrow (0, 0)$$

CONSTANT PHASE-ANGLE LOCI (N-CIRCLES)

$$\angle e^{j\alpha} = \angle \frac{x+jY}{1+x+jY}$$

$$\alpha = \tan^{-1}\left(\frac{Y}{x}\right) - \tan^{-1}\left(\frac{Y}{1+x}\right)$$

let $N = \tan \alpha = \text{CONSTANT}$

$$x^2 + x + Y^2 - \frac{1}{N}Y = 0$$

$$\left(x + \frac{1}{2}\right)^2 + \left(Y - \frac{1}{2N}\right)^2 = \frac{1}{4} + \left(\frac{1}{2N}\right)^2$$

TWO SOLUTIONS

$$(0,0) \quad (-1,0)$$

circle with center

$$x = -\frac{1}{2} \quad Y = \frac{1}{2N}$$

$$\text{radius} \sqrt{\frac{1}{4} + \left(\frac{1}{2N}\right)^2}$$