Poles and Zeros for Examples and Additional Results and Examples: R. C. Nongpiur, D. J. Shpak, and A. Antoniou, "Design of IIR Digital Differentiators Using Constrained Optimization," *IEEE Trans. on Signal Pro*cessing (in press).

This document provides the poles and zeros for Examples 1 to 10 in the above paper and also additional examples and results.

Example 1

Proposed Method 1 (Example 1)	
Poles	**Zeros**
-0.802939710229065 -0.241380950498653 0.083893496449488 + 0.146793964733408i 0.083893496449488 - 0.146793964733408i	1.685398737456287 + 6.071796610826690i 1.685398737456287 - 6.071796610826690i 1.000000000000001 -0.782457582401973
Ho	0.029122624790710

Proposed Method 2 (Example 1)	
Poles	**Zeros**
-0.927649925869125 -0.650119824864531 0.100763481534592 + 0.133421456705378i 0.100763481534592 - 0.133421456705378i -0.132834931374619 -0.132595352473694	4.969200549370092 -1.214579621718989 + 4.448766658563339i -1.214579621718989 - 4.448766658563339i 1.000000000000002 -0.924835080249195 -0.624598789038767
Ho	-0.010937362221720

Example 2

Proposed Method 1 (Example 2)	
Poles	**Zeros**
-0.761663688370332 -0.094596243487183	1.000000000000000 -0.725564150323285
Ho	-1.173370446979618

Proposed Method 2 (Example 2)	
Poles	**Zeros**
-0.869635736897934 -0.127994742174087 0.020819765180775	11.125020236118004 1.000000000000000 -0.862698322974249
Ho	-0.104018813668788

Example 3

Proposed Method 1 (Example 3)	
Poles	**Zeros**
-0.736764971286088 -0.107044029734546	1.000000000000000 -0.693865746550412
Ho	1.168706485627214

Proposed Method 2 (Example 3)	
Poles	**Zeros**

-0.867395089454792 -0.173180694762977 0.069176941669491	12.888748038196136 1.000000000000000 -0.856503834132135
Ho	-0.089528242053933

Example 4

Proposed Method (Example 4)	
Poles	**Zeros**
-0.562359925296005 + 0.551713315560392i -0.562359925296005 - 0.551713315560392i 0.122522727708173 + 0.156211319017677i 0.122522727708173 - 0.156211319017677i	5.071836543495142 1.000000000000001 -0.809042883798810 + 0.411496961528866i -0.809042883798810 - 0.411496961528866i
Ho	-0.154047458887425

Example 5

Proposed Method 1 (Example 5)	
Poles	**Zeros**
0.324266352462174 + 0.684489022581523i 0.324266352462174 - 0.684489022581523i 0.478053004887125 + 0.225065123823776i 0.478053004887125 - 0.225065123823776i	2.585554855000773 1.000000000000000 -0.260795397088261 + 0.804615265544004i -0.260795397088261 - 0.804615265544004i
Ho	0.083005158062897

Proposed Method 2 (Example 5)	
Poles	**Zeros**
0.387268767496249 + 0.612679644602625i 0.387268767496249 - 0.612679644602625i 0.587097851836742 0.493610476289837 + 0.310539128286949i 0.493610476289837 - 0.310539128286949i	5.480915299019374 1.705766374319428 + 0.935683687788366i 1.705766374319428 - 0.935683687788366i -0.929595031773406 1.000000000000000
Ho	-0.009348821397799

Example 6

Proposed Method 1 (Example 6)	
Poles	**Zeros**
0.351967642455433 + 0.499074142965067i 0.351967642455433 - 0.499074142965067i 0.462890092806419	2.534817269382062 -0.960640904275117 1.000000000000000
Ho	-0.115230030359596

Proposed Method 2 (Example 6)	
Poles	**Zeros**
0.160500130044039 + 0.508476009641183i 0.160500130044039 - 0.508476009641183i 0.328174454900780 + 0.244073231585899i 0.328174454900780 - 0.244073231585899i	2.618838341239818 1.000000000000001 -0.561086821019342 + 0.588068082140887i -0.561086821019342 - 0.588068082140887i
Ho	-0.105636564605152

Example 7

Proposed Method 1 (Example 7)	
Poles	**Zeros**
0.007537208087305 + 0.444257353375315i 0.007537208087305 - 0.444257353375315i 0.238881865812303	3.584485697669815 -0.938885431418600 1.000000000000000
Ho	-0.168807340853109

Proposed Method 2 (Example 7)	
Poles	**Zeros**
-0.106334797631461 + 0.456775124822759i -0.106334797631461 - 0.456775124822759i 0.175724163833827 + 0.277347893289393i 0.175724163833827 - 0.277347893289393i 0.253695536784501	5.247928490609988 + 5.796417784918430i 5.247928490609988 - 5.796417784918430i 5.373578132560946 -1.023627467289299 1.000000000000000
Ho	-0.001748944832111

Example 8

Proposed Method 1 (Example 8)	
Poles	**Zeros**
0.137128164507020 -0.051259186497668 + 0.115630480341598i -0.051259186497668 - 0.115630480341598i -0.101892319175950	-12.306077060448622 4.084369004067208 + 6.189515360016281i 4.084369004067208 - 6.189515360016281i 1.000000000000000
Ho	0.001818092120882

Proposed Method 2 (Example 8)	
Poles	**Zeros**
0.098688835405006 + 0.910061119941870i 0.098688835405006 - 0.910061119941870i -0.979999999999966 -0.129076995580145	-0.980010437479793 1.000000000000000 0.098764485256241 + 0.910128372219393i 0.098764485256241 - 0.910128372219393i
Ho	1.231706761798416

Example 9

Proposed Method (Example 9)	
Poles	**Zeros**
0.408655892534579 + 0.795779919914023i 0.408655892534579 - 0.795779919914023i 0.327867344860575 + 0.367231776543552i 0.327867344860575 - 0.367231776543552i 0.157023475896400	1.000000000000000 0.298120016672825 + 0.789886604521385i 0.298120016672825 - 0.789886604521385i -0.635203537971680 -0.205443879443335
Ho	0.221136590142179

Example 10

Proposed Method (Example 10)	
Poles	**Zeros**
0.312471615100270 + 0.809891020539611i 0.312471615100270 - 0.809891020539611i 0.172837483827109 0.172837420508986	0.999999999999999 -0.867231144955044 0.121055731858053 + 0.831530607921186i 0.121055731858053 - 0.831530607921186i
Ho	0.292359933087689

Example 13

Proposed Method (Example 13)	
Poles	**Zeros**
-0.714916715171425 -0.115001808853094	1.000000000000000 -0.665947510077671
Ho	1.163793160164963

Table XXIII
Design results for Example 13 (Fullband differentiator)

Starting lower frequency = 0 radians			
Parameters	Proposed method	Method in [18] c.f. eqn(4)	Method in [18] c.f. eqn(5)
Filter order	2	2	2
Max. rel. error, δ_r	0.019	∞	∞
Avg. group delay, $\bar{\tau}$	0.5	1	0
Max. phase error, ξ_ϕ	9.12	90	90
Starting lower frequency = 0.1% of Nyquist			
Parameters	Proposed method	Method in [18] c.f. eqn(4)	Method in [18] c.f. eqn(5)
Filter order	2	2	2
Max. rel. error, δ_r	0.019	3.3	3.44
Avg. group delay, $\bar{\tau}$	0.5	1.06	0.061
Max. phase error, ξ_ϕ	9.12	78.89	79.04
Starting lower frequency = 1% of Nyquist			
Parameters	Proposed method	Method in [18] c.f. eqn(4)	Method in [18] c.f. eqn(5)
Filter order	2	2	2
Max. rel. error, δ_r	0.019	0.18	0.15
Avg. group delay, $\bar{\tau}$	0.5	1.35	0.35
Max. phase error, ξ_ϕ	9.12	69.5	27.11

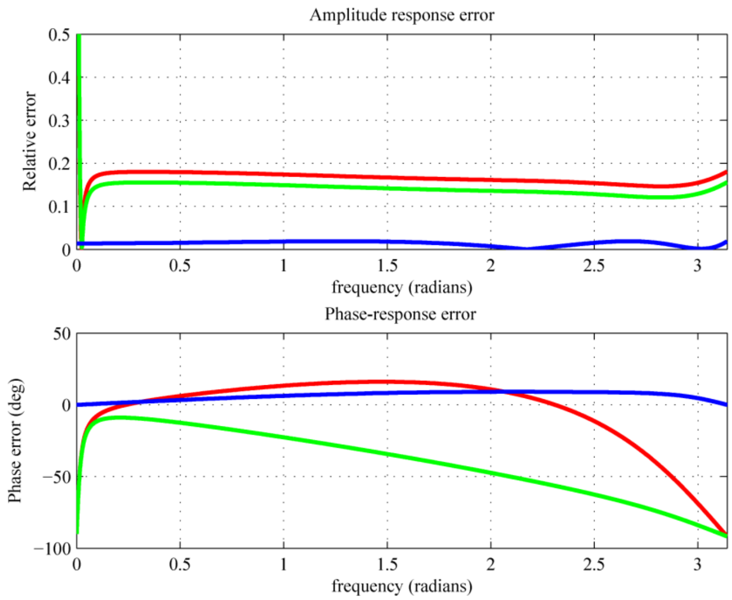


Figure 3: Plots of the amplitude and phase-response errors for Example13 for the proposed method (blue curve), eqn. (4) of [18] (red curve), and eqn (5) of [18] (green curve). The ideal phase response of a differentiator for the evaluation of the phase-response error is obtained by taking the average of the average group delay as in (50). Using some other group-delay value could reduce the phase-response error at some frequencies but the maximum phase-response error , ξ_{ϕ}, would not be reduced.

Example 14

Table XXIV
Design results for Example 14 (Fullband differentiator)

Starting lower frequency = 0 radians			
Parameters	Proposed method (same as Example 13)	Method in [15] c.f. eqn(5)	Method in[15] c.f. eqn(6)
Filter order	2	2	2
Max. rel. error, δ_r	0.019	∞	∞
Avg. group delay, $\bar{\tau}$	0.5	0	0
Max. phase error, ξ_{ϕ}	9.12	90	90
Starting lower frequency = 0.1% of Nyquist			
Parameters	Proposed method (same as Example 13)	Method in [15] c.f. eqn(5)	Method in[15] c.f. eqn(6)
Filter order	2	2	2
Max. rel. error, δ_r	0.019	0.029	0.39
Avg. group delay, $\bar{\tau}$	0.5	0.42	0.26
Max. phase error, ξ_{ϕ}	9.12	14.16	44.03
Starting lower frequency = 1% of Nyquist			
Parameters	Proposed method (same as Example 13)	Method in [15] c.f. eqn(5)	Method in[15] c.f. eqn(6)
Filter order	2	2	2
Max. rel. error, δ_r	0.019	0.018	0.035
Avg. group delay, $\bar{\tau}$	0.5	0.49	0.47
Max. phase error, ξ_{ϕ}	9.12	10.75	10.25

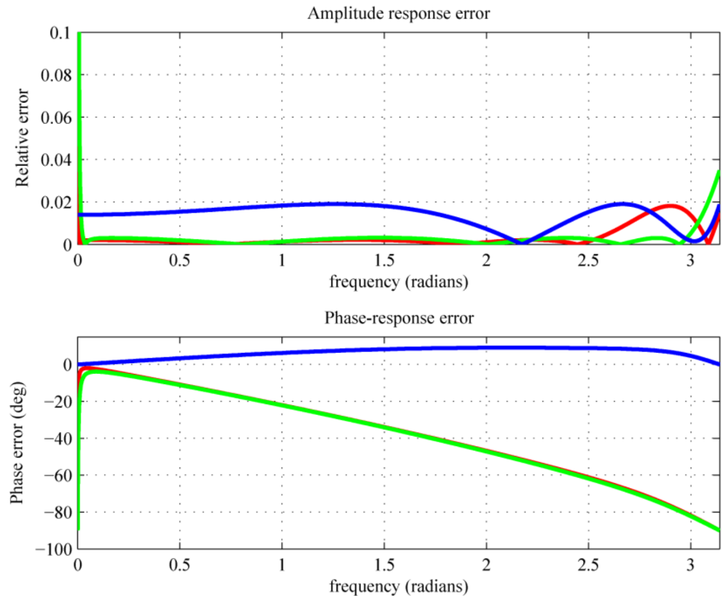


Figure 4: Plots of the amplitude and phase-response errors for Example14 for the proposed method (blue), eqn. (5) of [15] (red curve), and eqn. (6) of [15] (green curve).

Example 15

Table XXV
Design results for Example 15 (Fullband differentiator)

Starting lower frequency = 0 radians		
Parameters	Proposed method (same as Example 13)	Method in [16] (first example)
Filter order	2	8
Max. rel. error, δ_r	0.019	∞
Avg. group delay, $\bar{\tau}$	0.5	4
Max. phase error, ξ_ϕ	9.12	90
Starting lower frequency = 0.1% of Nyquist		
Parameters	Proposed method (same as Example 13)	Method in [16] (first example)
Filter order	2	8
Max. rel. error, δ_r	0.019	0.87
Avg. group delay, $\bar{\tau}$	0.5	3.83
Max. phase error, ξ_ϕ	9.12	58.66
Starting lower frequency = 1% of Nyquist		
Parameters	Proposed method (same as Example 13)	Method in [16] (first example)
Filter order	2	8
Max. rel. error, δ_r	0.019	0.023
Avg. group delay, $\bar{\tau}$	0.5	3.55
Max. phase error, ξ_ϕ	9.12	9.2

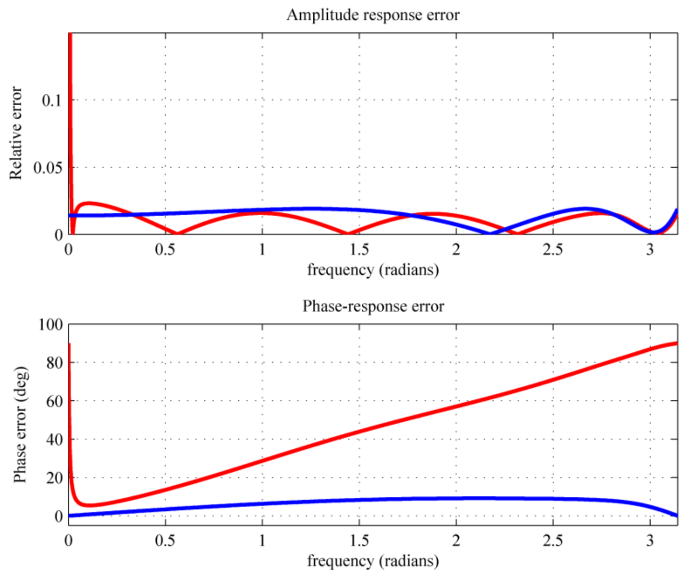


Figure 5: Plots of the amplitude and phase-response errors for Example15 for the proposed method (blue), example 1 of [16] (red).

Example 16

Table XXVI
Design results for Example 16 (Fullband differentiator)

Parameters	Proposed method (same as Example 13)	Method in [14] c.f. eqn (4)
Filter order	2	2
Max. rel. error, δ_r	0.019	0.04
Avg. group delay, $\bar{\tau}$	0.5	1.5
Max. phase error, ξ_ϕ	9.12	70.6

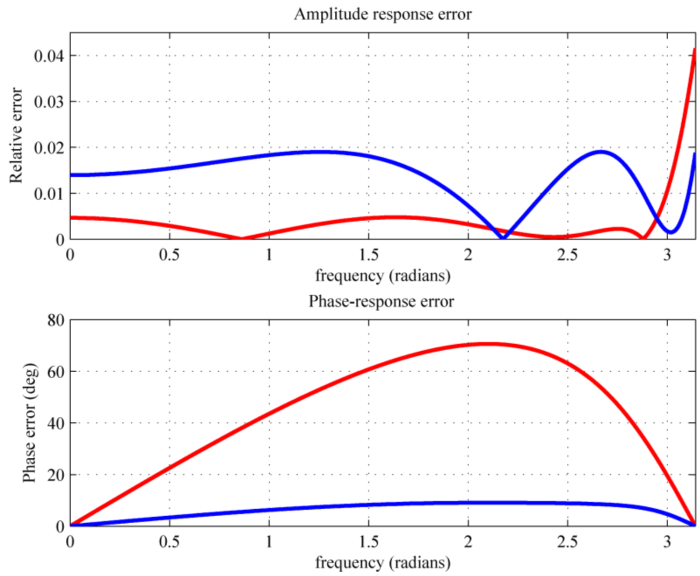
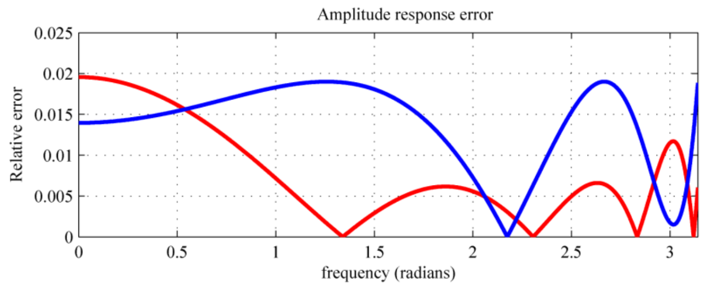


Figure 6: Plots of the amplitude and phase-response errors for Example16 for the proposed method (blue curve) and eqn. (4) of [14] (red curve).

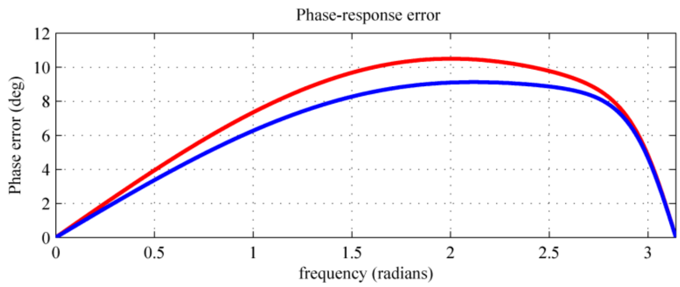
Example 17

Table XXVII
Design results for Example 17 (Fullband differentiator)

Parameters	Proposed method (same as Example 13)	Method in [10] (first example)
Filter order	2	2
Max. rel. error, δ_{r}	0.019	0.0196
Avg. group delay, $\bar{\tau}$	0.5	0.5
Max. phase error, ξ_{ϕ}	9.12	10.49



(a)



(b)

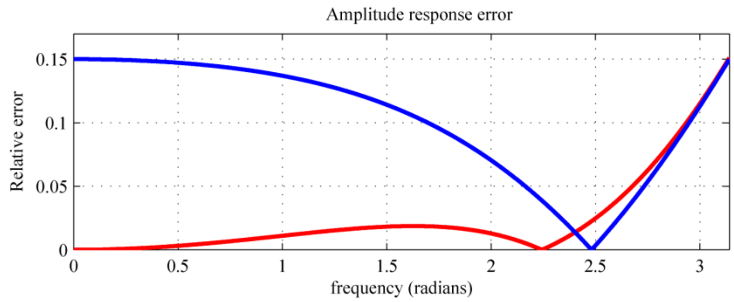
Figure 1: Plots of the amplitude and phase-response errors for Example16 for the proposed method (blue curve) and first example of [10] (red curve).

Example 18

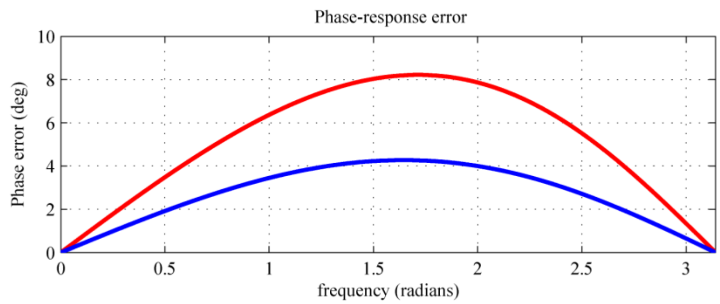
Proposed Method (Example 18)	
Poles	**Zeros**
-0.074512554591850	1
Ho	1.235689437780627

Table XXVIII
Design results for Example18 (Fullband differentiator)

Parameters	Proposed method	Method in [11] (last example)
Filter order	1	1
Max. rel. error, δ_r	0.15	0.15
Avg. group delay, $\bar{\tau}$	0.5	0.5
Max. phase error, ξ_ϕ	4.27	8.21



(a)



(b)

Figure 2: Plots of the amplitude and phase-response errors for Example16 for the proposed method (blue curve) and last example of [11] (red curve).